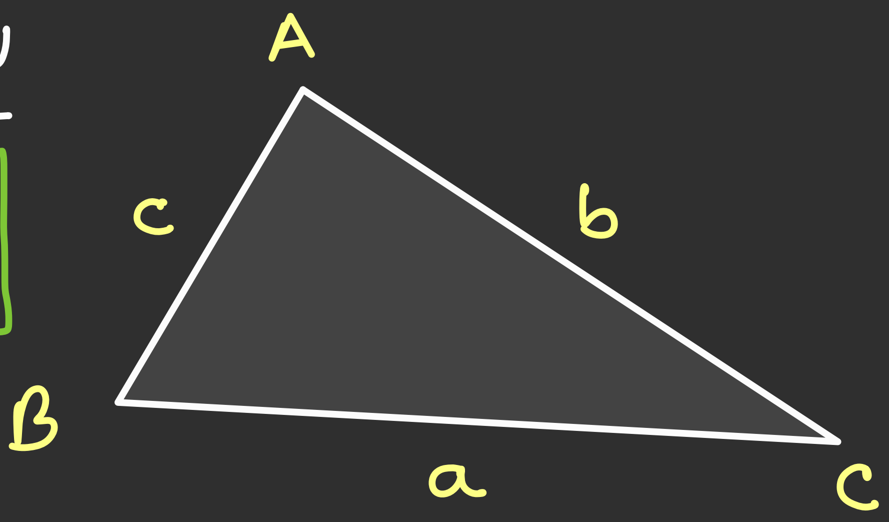
Length Chase

- Jubair

Sin law
Cos Law
Angle Bisector
Centroid Median [2:1]
Parallel Line + Ratio
Ceva
Manelaus

Sin Law

$$\boxed{\Delta : \frac{1}{2} ab \sin C}$$

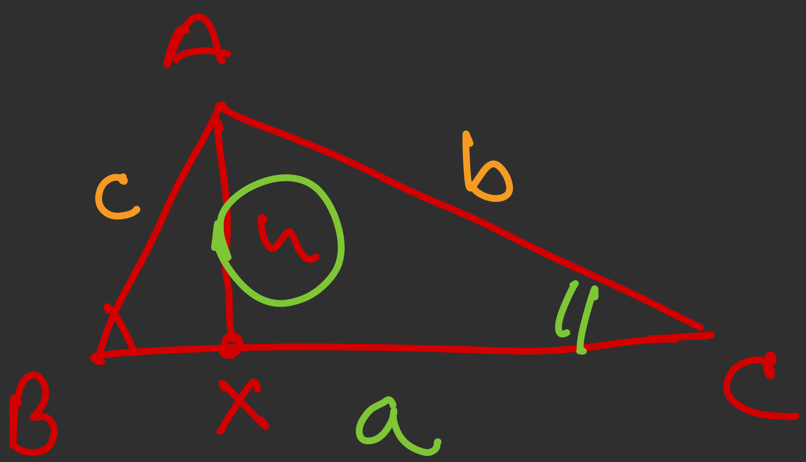


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$BC = a$

$CA = b$

$AB = c$



$$\sin B = \frac{AX}{AB} = \frac{h}{c}$$

$$h = c \sin B$$

$$\sin C = \frac{AX}{AC} = \frac{h}{b}$$

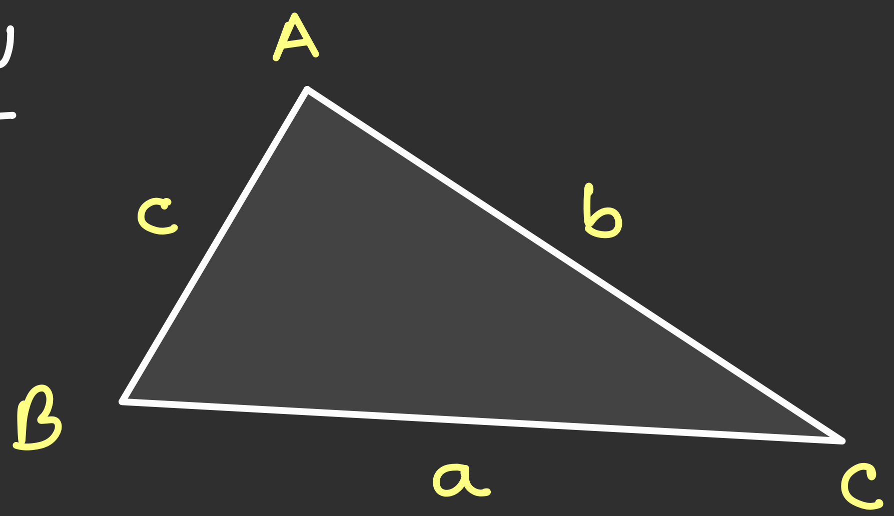
$$AX = AB \sin B = \underline{c \sin B}$$

$$AX = AC \sin C = \underline{b \sin C}$$

$$\boxed{h = b \sin C}$$

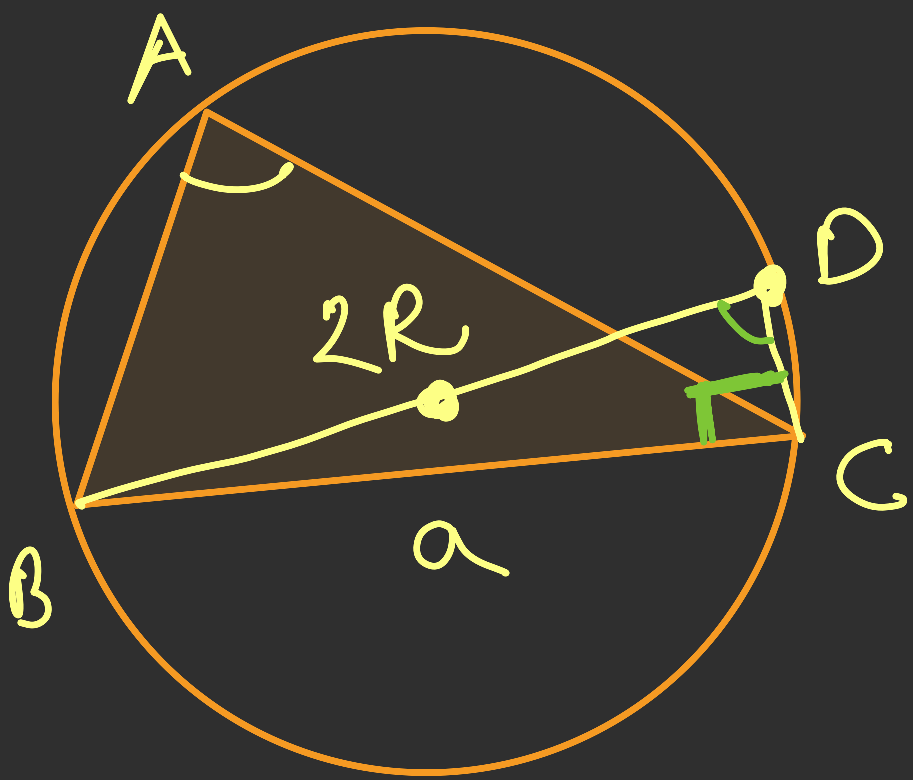
$$(\Delta ABC) = \frac{1}{2} ah = \frac{1}{2} ab \sin C = \frac{1}{2} ac \sin B$$

Sin Law



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

$BC = a$
$CA = b$
$AB = c$

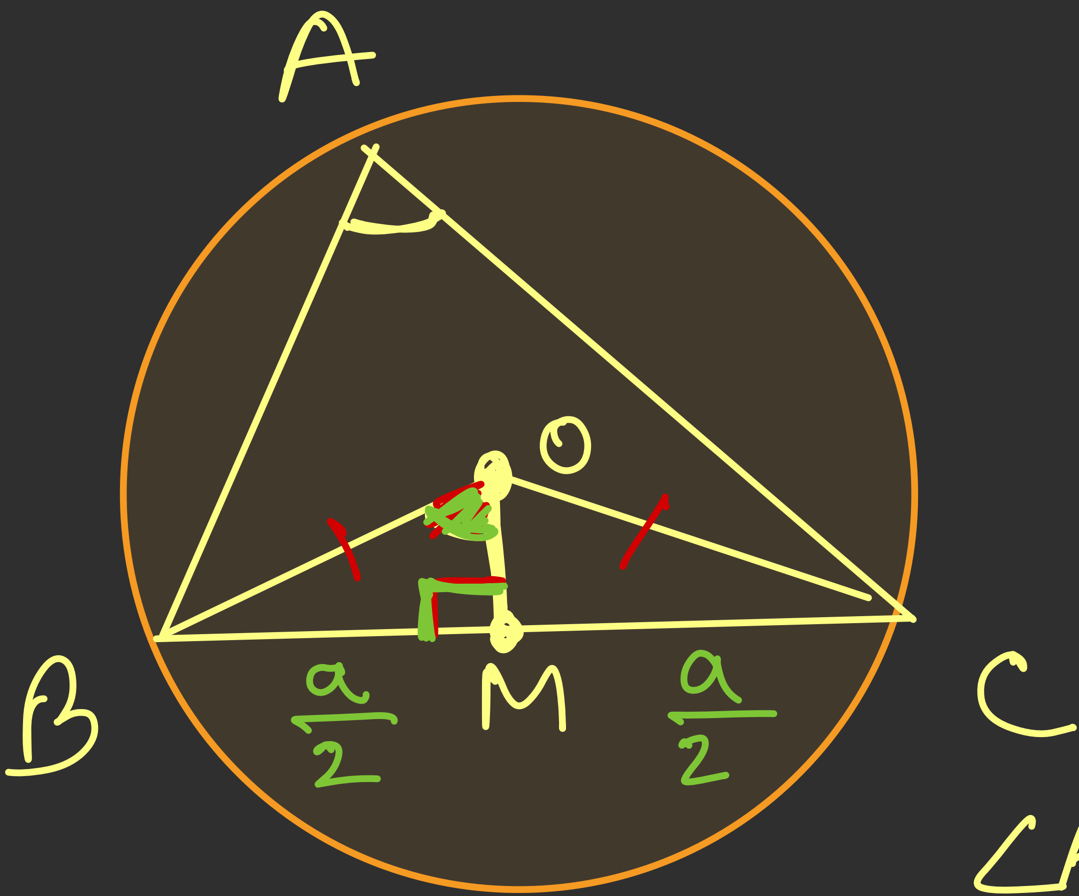


$BD = 2R$

$\angle BAC = \angle A$
$\angle BDC$

$$\sin A = \sin \angle BDC = \frac{a}{2R}$$

$$\frac{a}{\sin A} = 2R$$



$\angle A =$

$\angle BOC = 2\angle A$

$\sin\angle BOM = \dfrac{BM}{BO}$

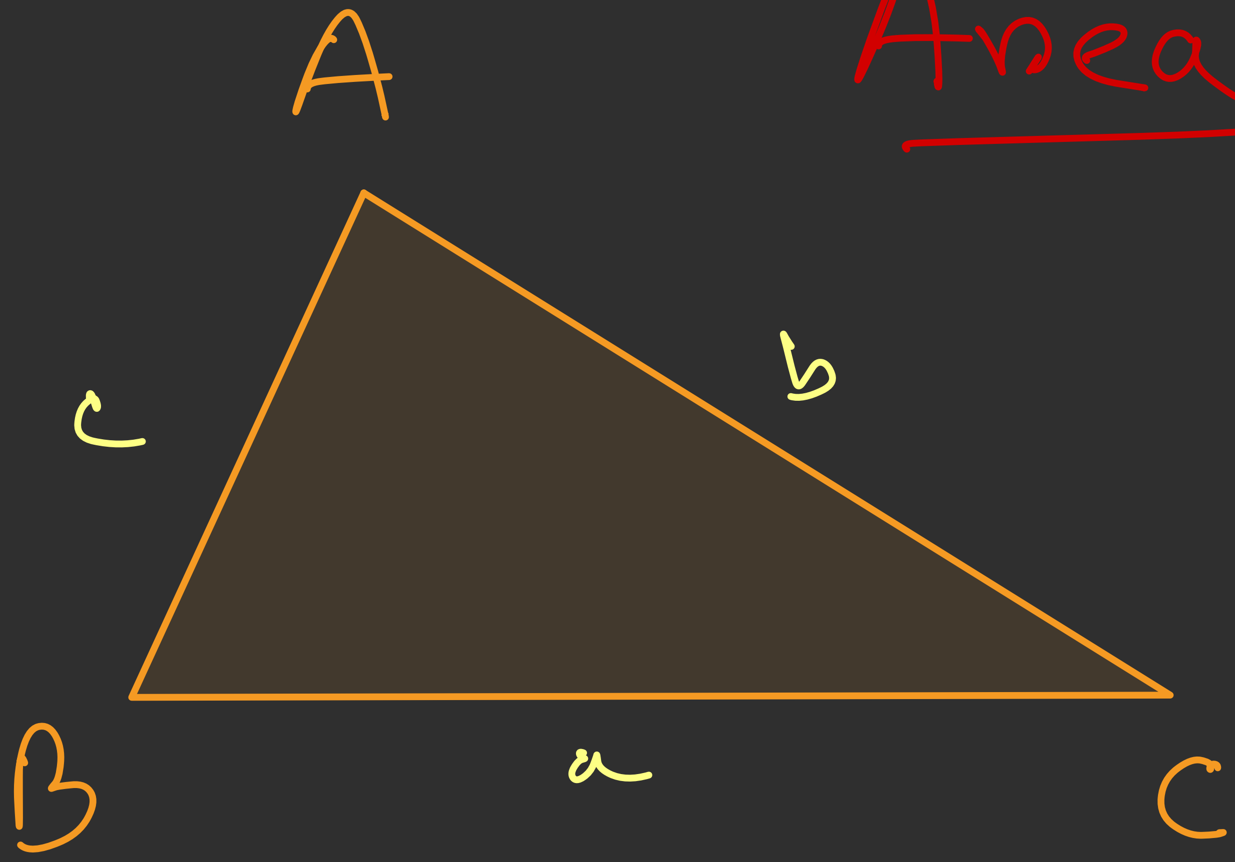
$\angle BOM = \angle A$

$\sin\angle A = \dfrac{\frac{a}{2}}{R} = \dfrac{a}{2R}$

$$\sin A = \frac{a}{2R}$$

$$\frac{a}{\sin A} = 2R$$

Area



$$(\Delta ABC) = \frac{abc}{4R}$$

$$= sr$$

$$= \frac{1}{2} bh = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \boxed{\frac{c}{\sin C} = 2R}$$

$$\boxed{\sin C = \frac{c}{2R}}$$

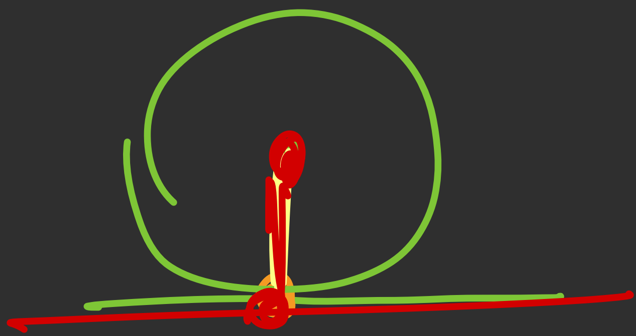
$$\Delta = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} ab \frac{c}{2R}$$

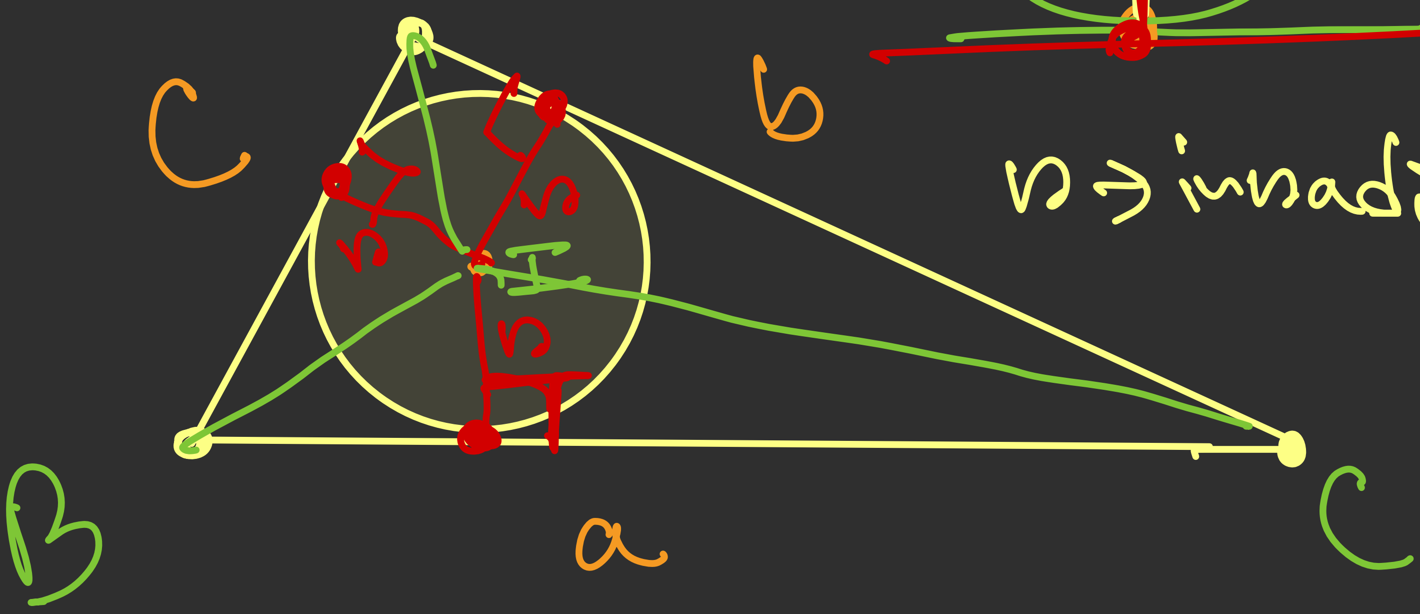
$$= \frac{abc}{4R}$$

$R \rightarrow$ অনুব্যাসার্ধ / Circum radius

$s \to$ semiperimeter



$r \to$ inradius



$$(\Delta ABC) = (\Delta AIB) + (\Delta BIC) + (CIB)$$

$$= \frac{cr}{2} + \frac{ar}{2} + \frac{br}{2}$$

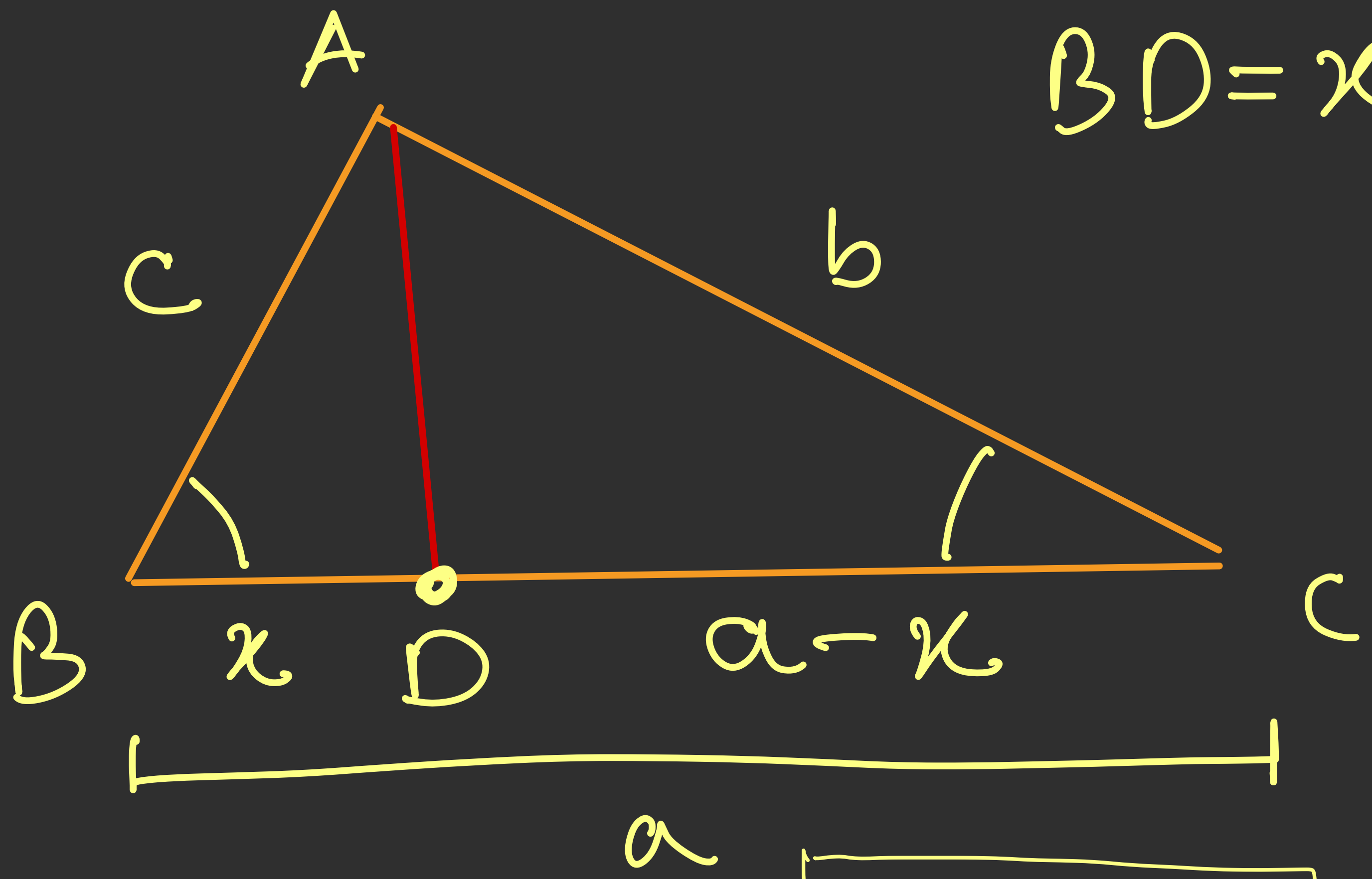
$$= \frac{r}{2}(a+b+c)$$

$$= r \cdot \left(\frac{a+b+c}{2}\right) = sr$$

Cos Law

$AD \perp BC$

$BD = x$



$$\boxed{\cos B = \frac{x}{c}}$$

$AD^2 = AB^2 - BD^2$

$AD^2 = AC^2 - CD^2$

$$AB^2 - BD^2 = AC^2 - CD^2$$

$$\Rightarrow c^2 - x^2 = b^2 - (a-x)^2$$

$$\Rightarrow c^2 - x^2 = b^2 - (a^2 - 2ax + x^2)$$

$$\Rightarrow c^2 \cancel{- x^2} = b^2 - a^2 + 2ax \cancel{- x^2}$$

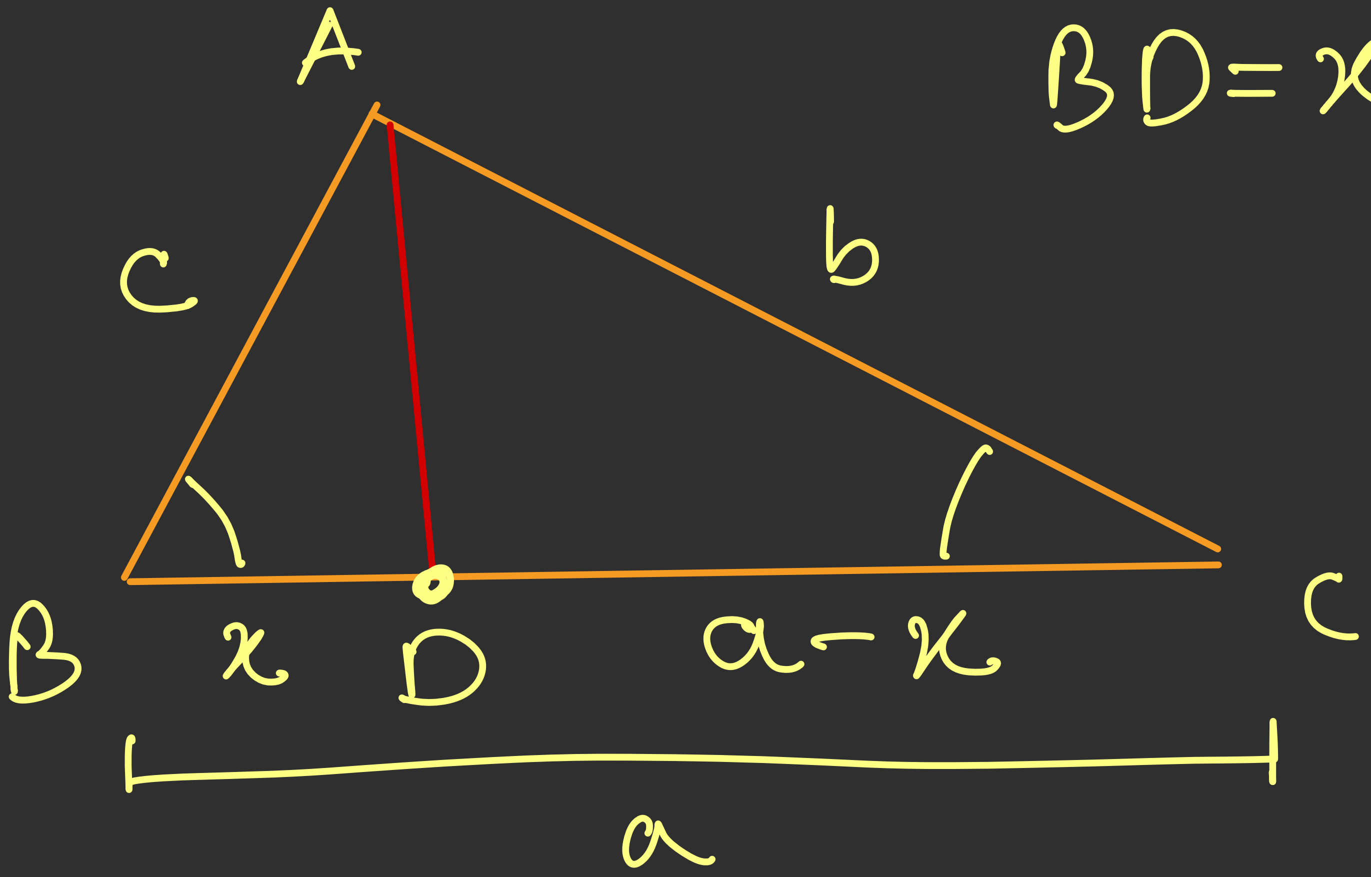
$$\Rightarrow a^2 + c^2 - b^2 = 2ax$$

$\therefore x = \dfrac{a^2+c^2-b^2}{2a}$ $\quad \therefore \cos B = \dfrac{a^2+c^2-b^2}{2ac}$

Cos Law

$AD \perp BC$

$BD = x$

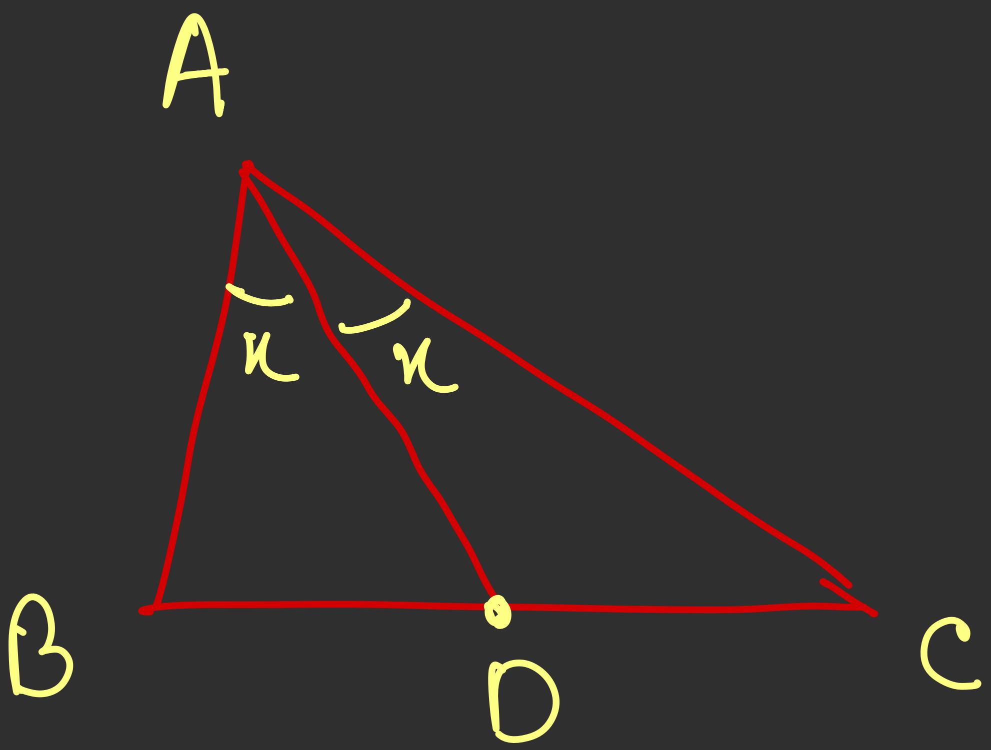


$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Angle Bisector Th.

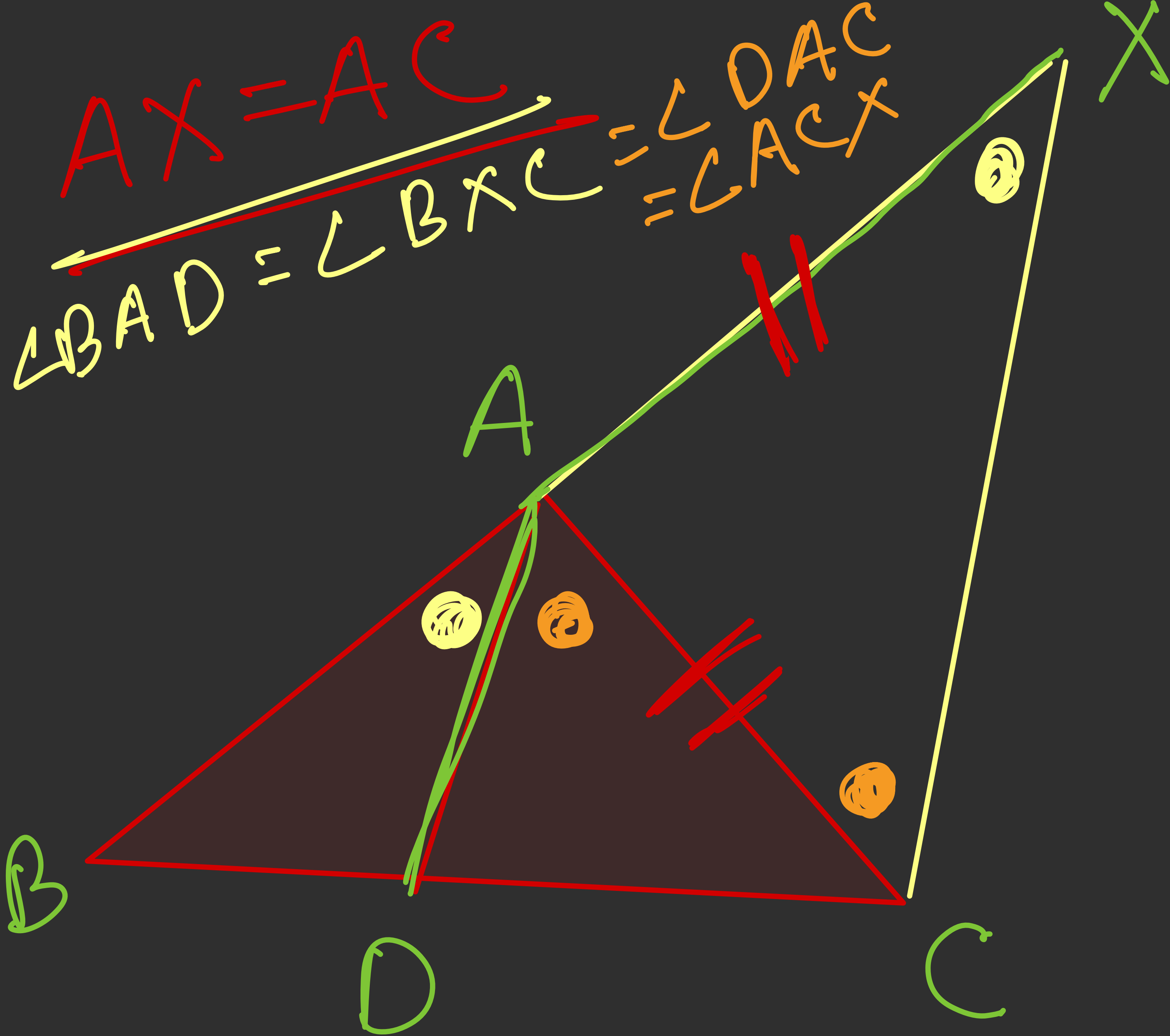


$$\frac{AB}{AC} = \frac{BD}{CD}$$

$AX = AC$

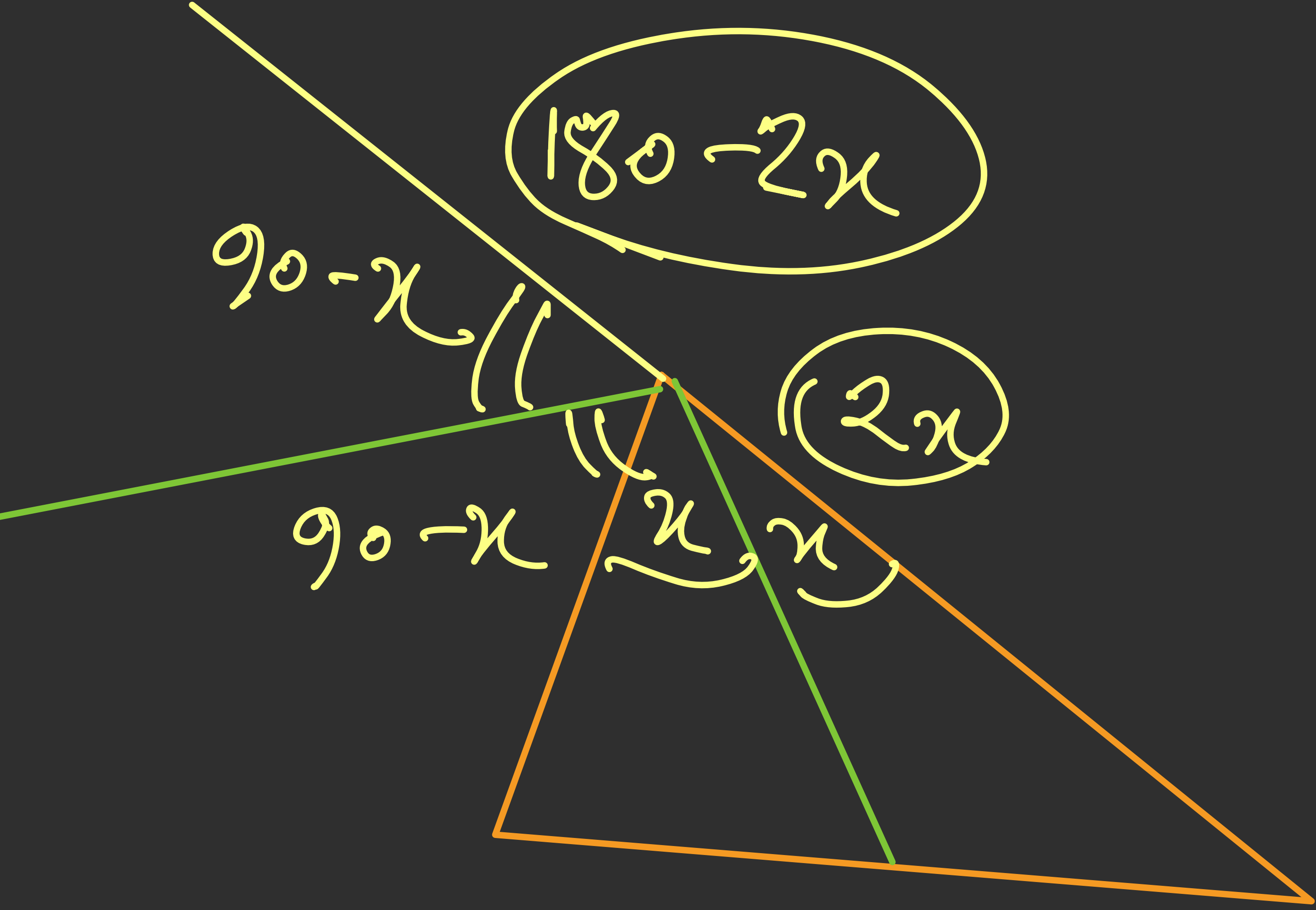
$\angle BAD = \angle BAC = \angle DAC$

$= \angle ACX$

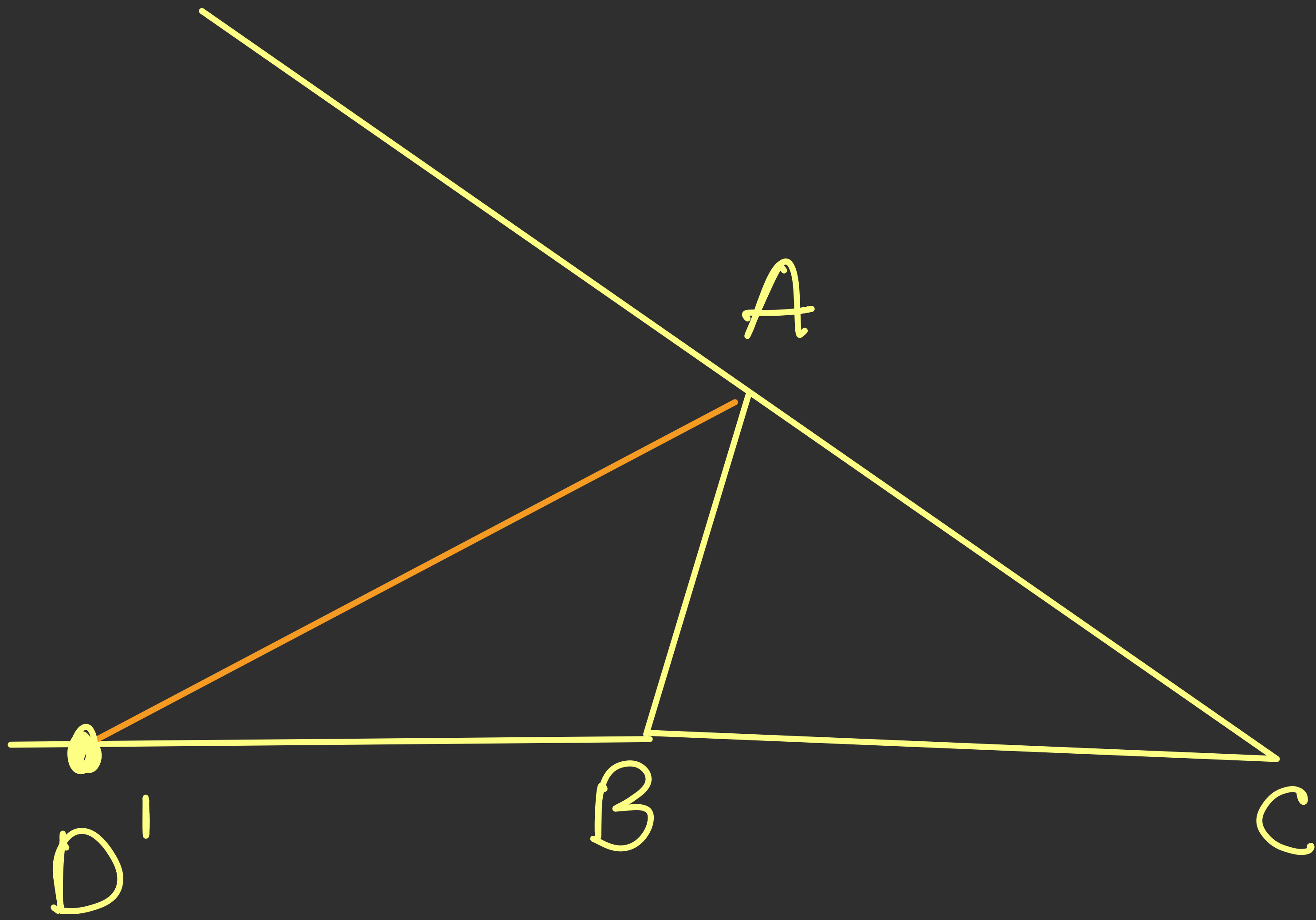


$\frac{BD}{DC} = \frac{BA}{AX}$ $[\because AD \parallel CX]$

$\frac{BD}{DC} = \frac{BA}{AC}$ ✓✓



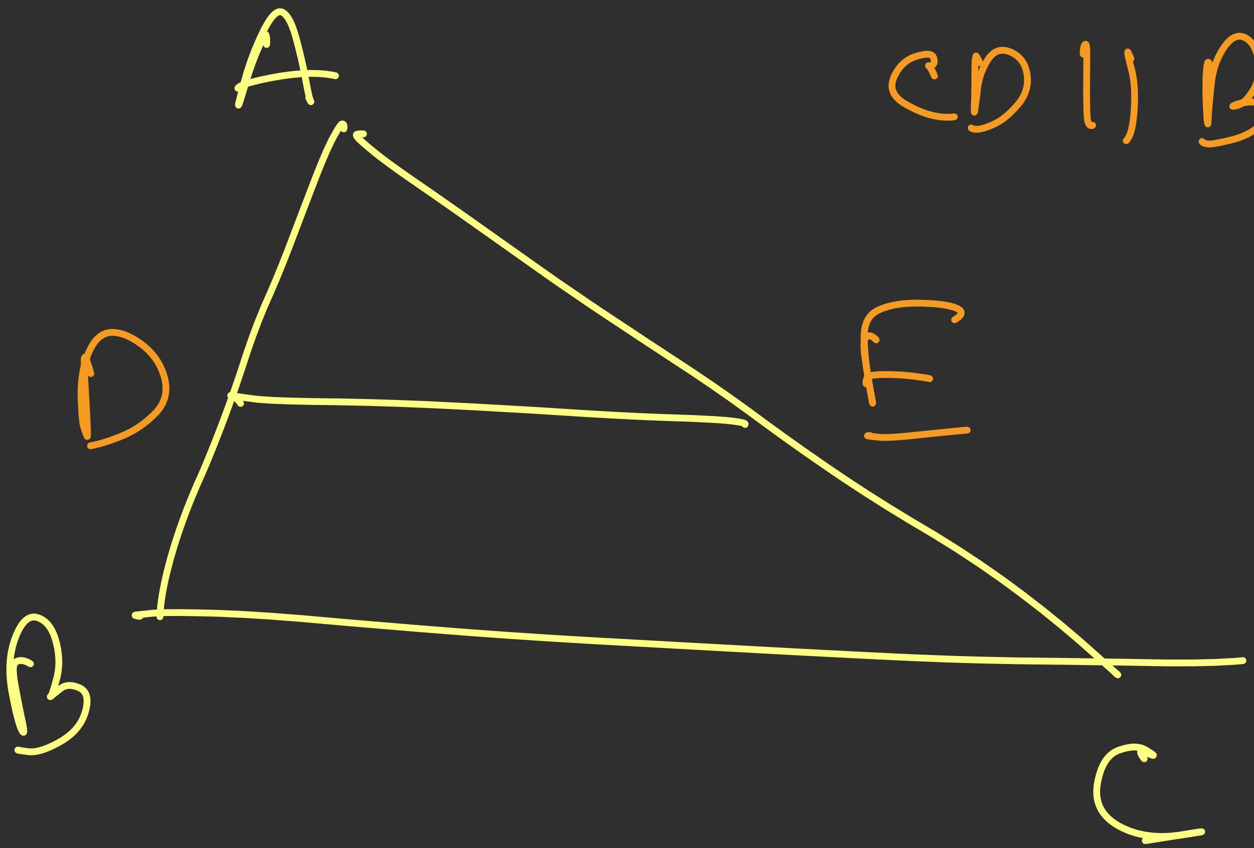
$$90^\circ - x + x = 90^\circ$$



$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{AB}{AC} = \frac{BD'}{D'C}$$

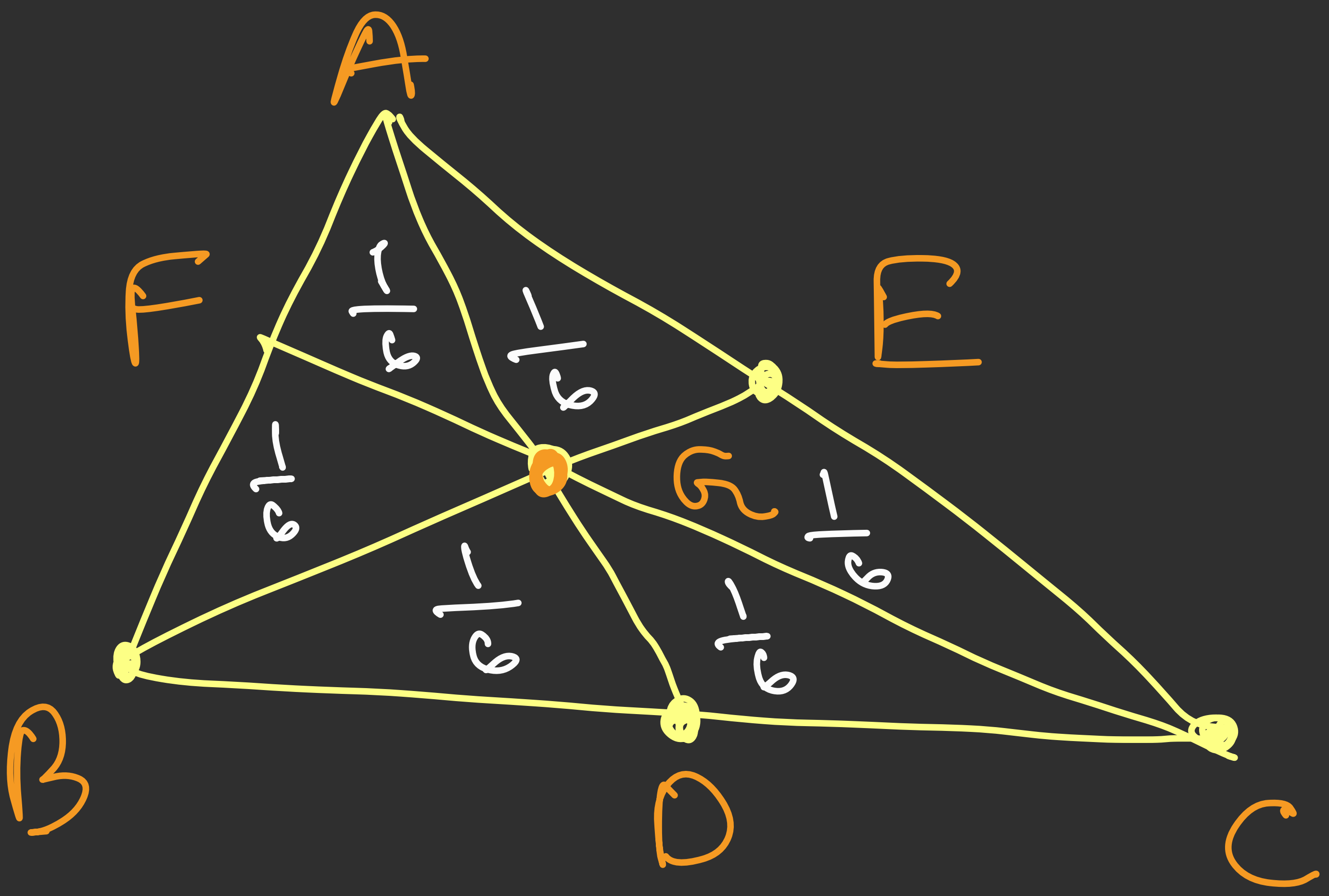
$CD \parallel BC$



$$\frac{AD}{BD} = \frac{AE}{EC} \quad *$$

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

$$\frac{BD}{AB} = \frac{EC}{AC}$$

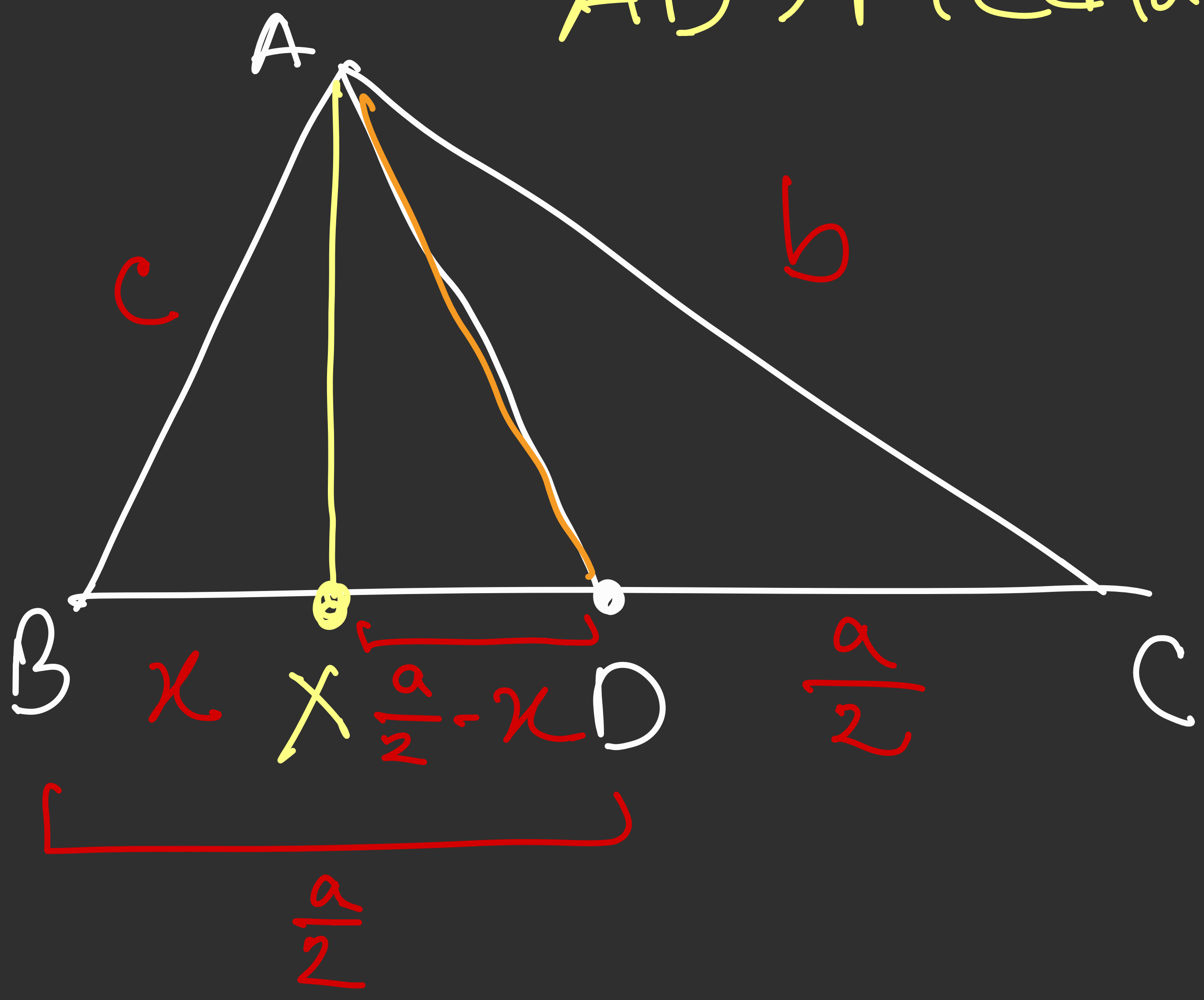


$$\frac{AF}{FB} \times \frac{BD}{CD} \times \frac{CE}{EA} = 1$$

✓

$$AG : GD = 2 : 1$$

$AD \rightarrow$ Median



$$AD^2 = AX^2 + XD^2$$

$$= c^2 - x^2 + \left(\frac{a}{2} - x\right)^2$$

$$= c^2 - \cancel{x^2} + \left(\frac{a}{2}\right)^2 - ax + \cancel{x^2}$$

$$AD^2 = c^2 + \frac{a^2}{4} - ax$$

$$= c^2 + \frac{a^2}{4} - \cancel{a} \cdot \frac{a^2 + c^2 - b^2}{2\cancel{a}}$$

$$= \frac{4c^2 + a^2 - 2a^2 - 2c^2 + 2b^2}{4}$$

$$= \frac{2b^2 + 2c^2 - a^2}{4}$$

$$\therefore AD = \frac{\sqrt{2b^2 + 2c^2 - a^2}}{2}$$

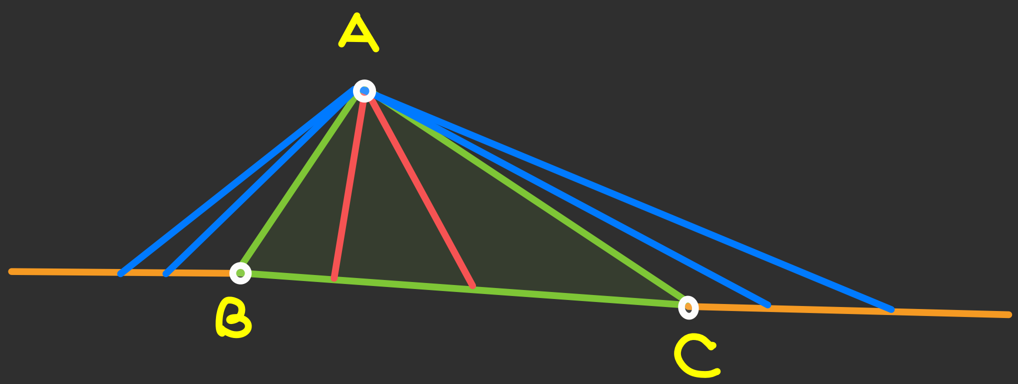
$$BE = \frac{\sqrt{2a^2 + 2c^2 - b^2}}{2}$$

$$CF = \frac{\sqrt{2a^2 + b^2 - c^2}}{2}$$

Cheva
&
Menalaus

-Jubain

Chevian

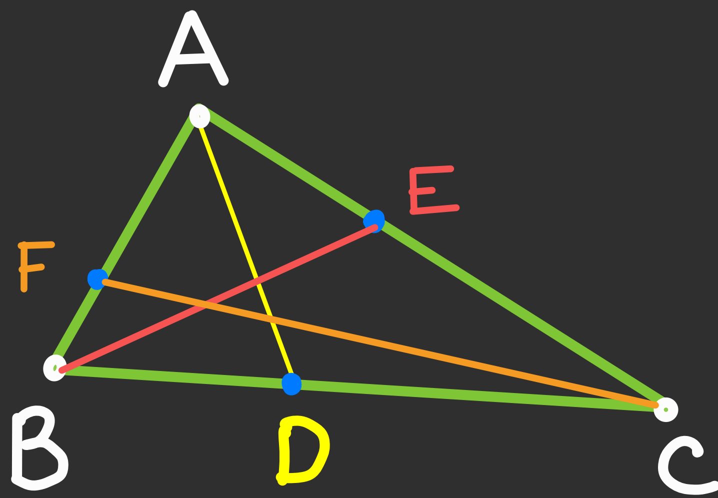


ABC is a triangle.

Red lines are <u>internal</u> A-chevian.

Blue lines are <u>external</u> A-chevian.

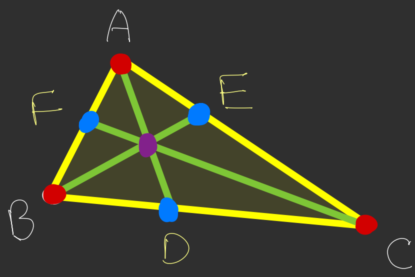
Rules for theorem



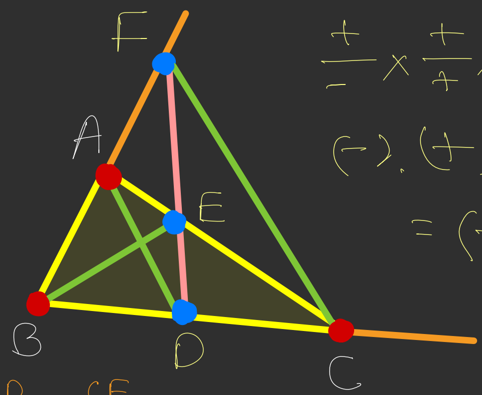
A- chevian meets BC at D

B - chevian meets AC at E

C - chevian meets AB at F



3 internal chevian
0 external chevian
followed chevas theorem



$$\frac{+}{-}\times\frac{+}{+}\times\frac{+}{+}$$

$$(-)\cdot(+)\cdot(+)$$

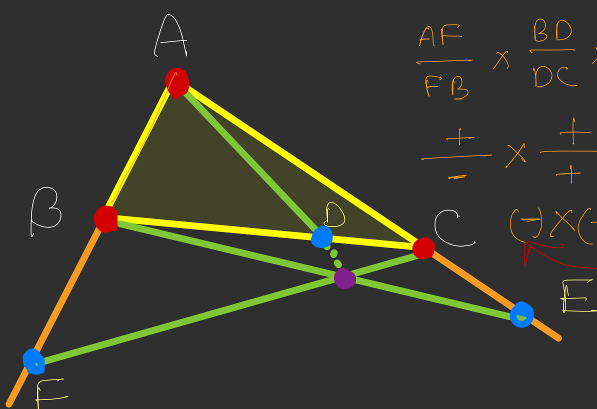
$$=(-)1$$

2 internal chevian
1 external chevian
followed menalaus theorem

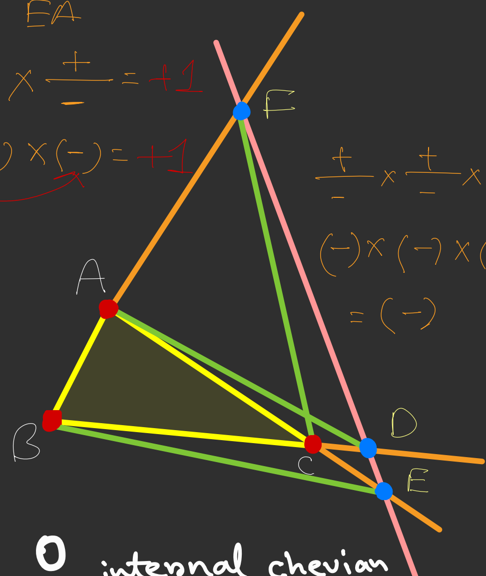
$$\frac{AF}{FB}\times\frac{BD}{DC}\times\frac{CE}{EA}$$

$$\frac{+}{-}\times\frac{+}{+}\times\frac{+}{-}=+1$$

$$(-)\times(+)\times(-)=+1$$



1 internal chevian
2 external chevian
followed chevas theorem



$$\frac{+}{-}\times\frac{+}{-}\times\frac{+}{-}$$

$$(-)\times(-)\times(-)$$

$$=(-)$$

0 internal chevian
3 external chevian
followed menalaus theorem

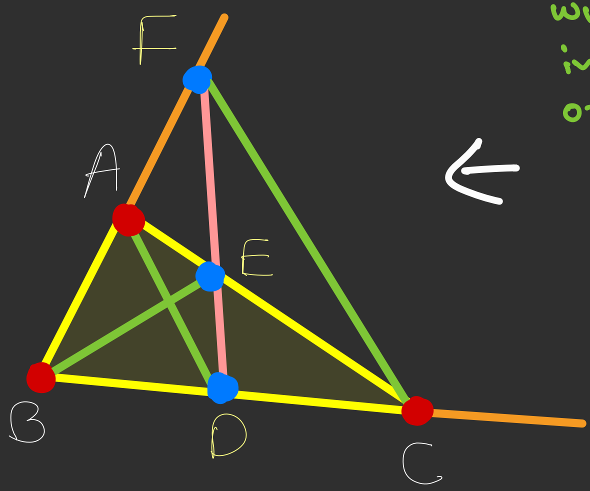
Every combination for cheva & menalaus theorem

For every picture,

$$\frac{AF}{FB}\times\frac{BD}{DC}\times\frac{CE}{EA}=1$$

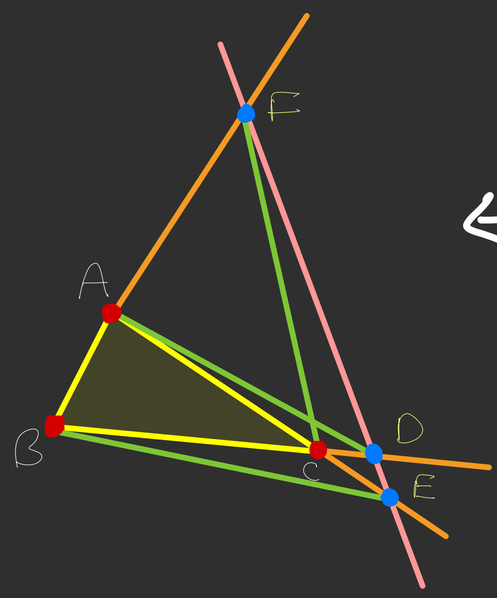
Either green lines (chevian) are concurrent or,
blue points are collinear

Why 2 combination will not work for cheva's T.?

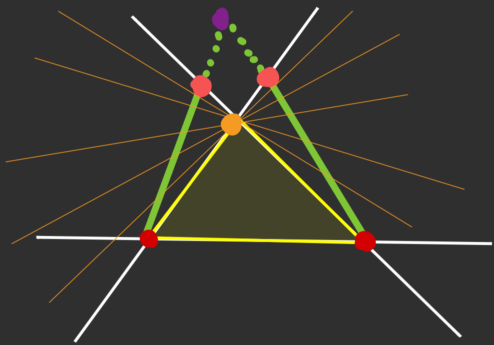


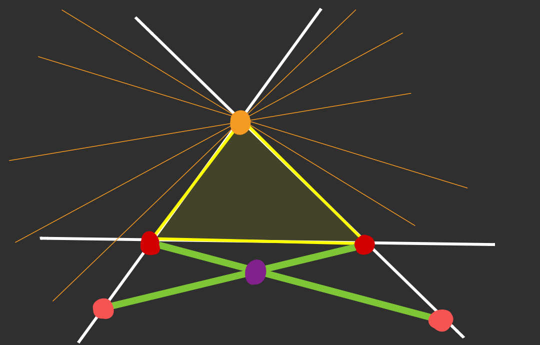
2 internal chevian will meet internally in the inner side of the triangle

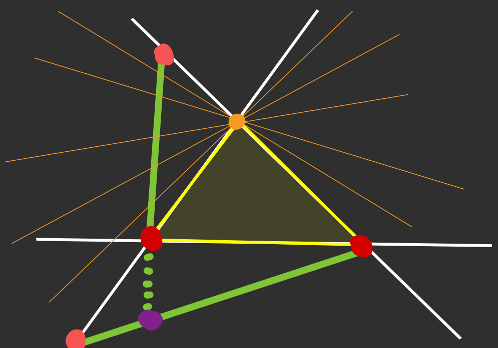
So the 3rd external chevian can't not meet that common point.

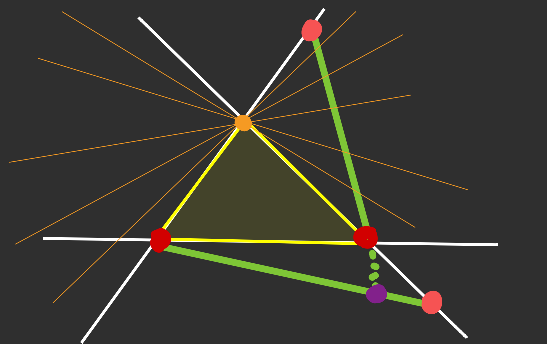


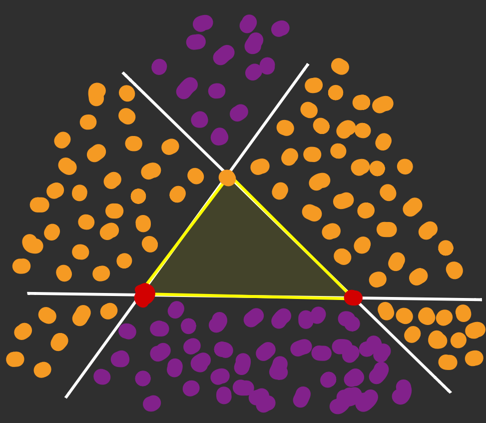
Details on next page











orange and violet areas are different. That's why no possibility to meet

Basic Ratio Operations

1) if $\frac{a}{b} = \frac{c}{d}$ then

i) * $\frac{a \pm b}{b} = \frac{c \pm d}{d}$

$$\boxed{\begin{array}{l} \frac{a}{b} \pm 1 = \frac{c}{d} \pm 1 \\ \Rightarrow \frac{a \pm b}{b} = \frac{c \pm d}{d} \end{array}}$$

* $\frac{a}{a \pm b} = \frac{c}{c \pm d}$

* $\frac{a}{b \pm a} = \frac{c}{d \pm c}$

* $\frac{b \pm a}{b} = \frac{d \pm c}{d}$

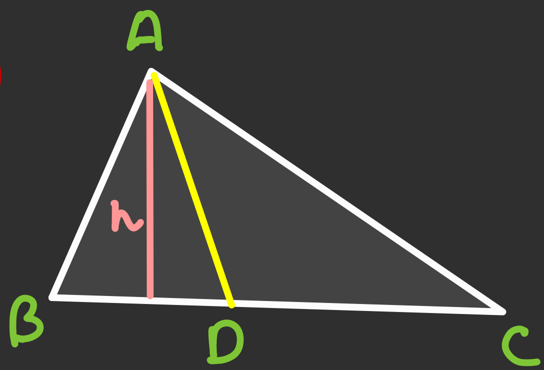
iii) $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

iii) if $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots$

then $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots = \frac{a+c+e}{b+d+f}$

Basic Geometry

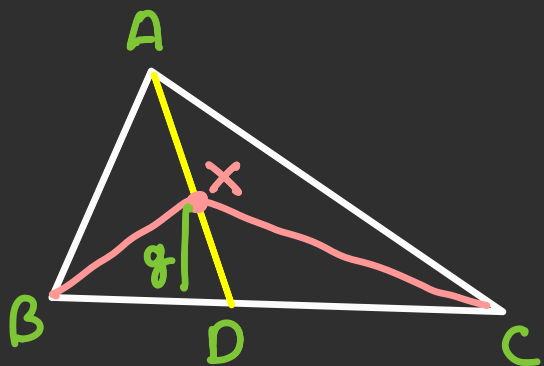
1)



$$\frac{BD}{CD} = \frac{\frac{1}{2} \times BD \times h}{\frac{1}{2} \times CD \times h}$$

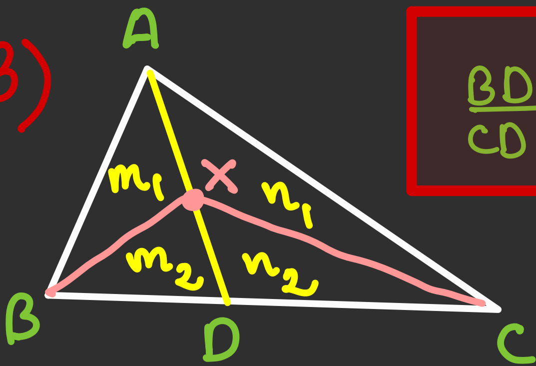
$$= \frac{(\Delta ABD)}{(\Delta ACD)}$$

2)



$$\frac{BD}{CD} = \frac{(\Delta ABD)}{(\Delta ACD)} = \frac{(\Delta XBD)}{(\Delta XCD)} = \frac{\frac{1}{2} BD \times y}{\frac{1}{2} \times CD \times y}$$

3)



$$\boxed{\frac{BD}{CD} = \frac{(\Delta AB\cancel{X})}{(\Delta AC\cancel{X})} = \frac{m_1}{m_2}}$$

$$\frac{BD}{CD} = \frac{(\Delta ABD)}{(\Delta ACD)} = \frac{(\Delta XBD)}{(\Delta XCD)}$$

$$\Rightarrow \frac{m_1+m_2}{n_1+n_2} = \frac{m_2}{n_2}$$

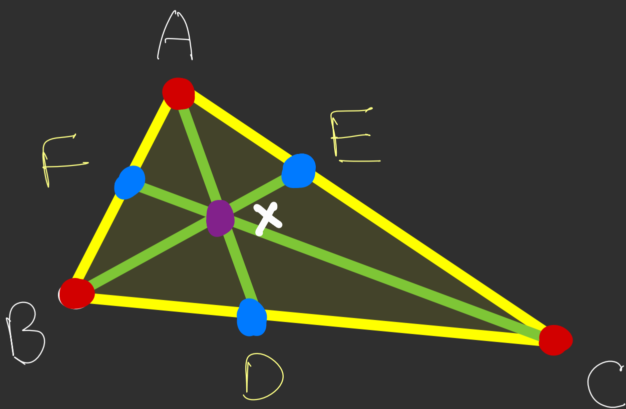
$$\Rightarrow \frac{m_1+m_2}{m_2} = \frac{n_1+n_2}{n_2}$$

$$\Rightarrow \frac{m_1}{m_2} + 1 = \frac{n_1}{n_2} + 1$$

$$\therefore \frac{m_2}{n_2} = \frac{m_1}{n_1}$$

$$\boxed{\frac{BD}{CD} = \frac{m_1+m_2}{n_1+n_2} = \frac{m_2}{n_2} = \frac{m_1}{n_1}}$$

Prove of Cheva's Theorem



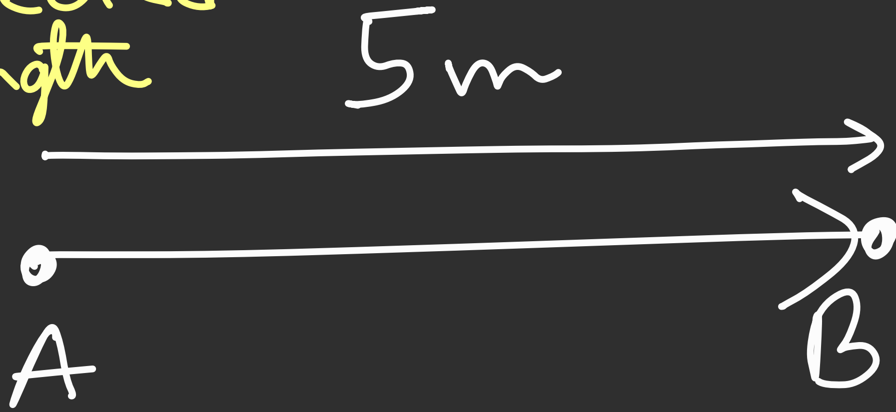
If AD, BE and CF are concurent, then

$$\frac{AF}{FB} \times \frac{BD}{DC} \times \frac{CE}{EA} = 1$$

$$\Rightarrow \frac{\cancel{(\Delta AXC)}}{\cancel{(\Delta BXC)}} \times \frac{\cancel{(\Delta AXB)}}{\cancel{(\Delta AXC)}} \times \frac{\cancel{(\Delta BXC)}}{\cancel{(\Delta AXB)}}$$

$$= 1$$

Directed Length

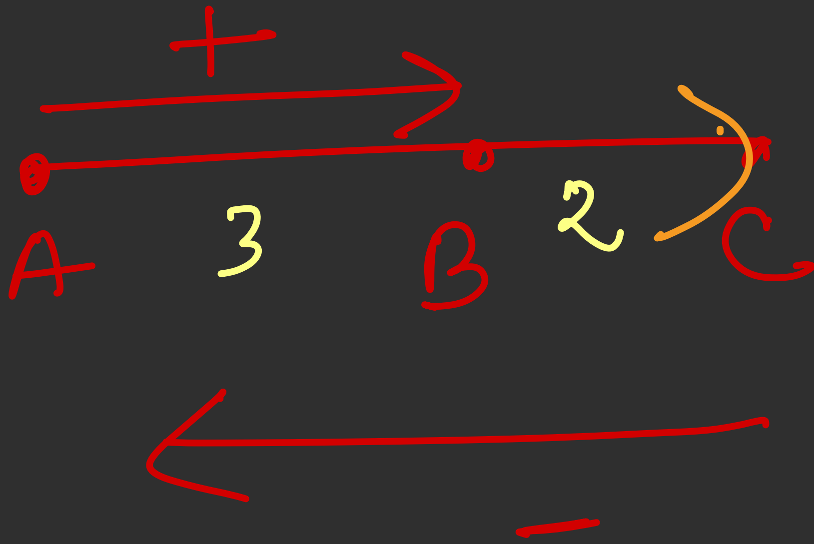


$$\overrightarrow{AB} = +5$$

$$\overrightarrow{BA} = -5$$

$$\overrightarrow{AB} = -\overrightarrow{BA}$$

$$AB = BA$$



$$\frac{\overrightarrow{AB}}{\overrightarrow{BC}} = \frac{+3}{+2} = +1.5$$

$$\frac{\overrightarrow{AB}}{\overrightarrow{CB}} = \frac{+3}{-2} = -1.5$$