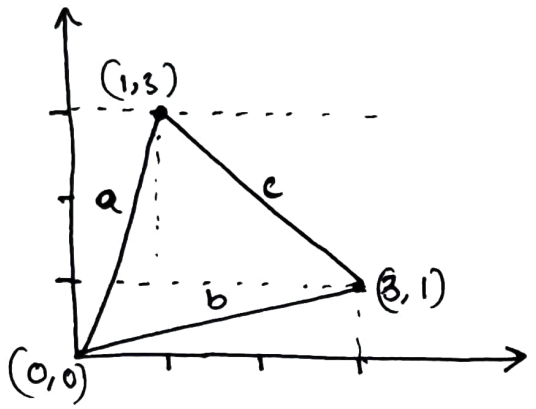
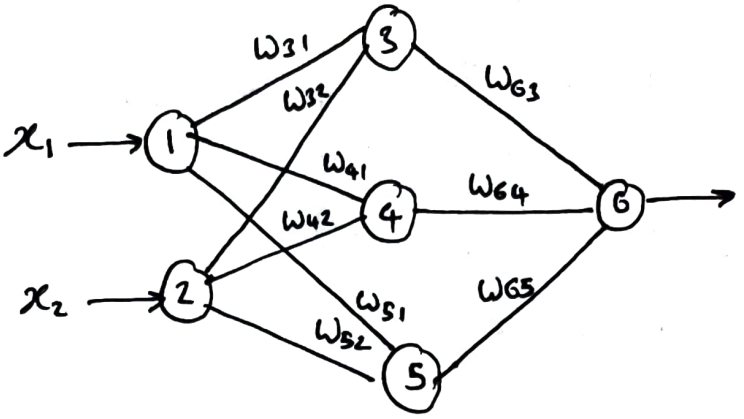
Given Vertices

(0,0), (1,3), (3,1)





* Equation of line a

$$y - y_1 = \left[\frac{y_2 - y_1}{x_2 - x_1}\right](x - x_1)$$

$$y - 0 = \left[\frac{3 - 0}{1 - 0}\right](x - 0)$$

$$y = 3x$$

for line a, $x_2 = 3x_1$

* Equation of line b

$$y - 0 = \left[\frac{1 - 0}{3 - 0}\right](x - 0)$$

$$y = \frac{1}{3}x$$

for line b $3x_2 = x_1$

* Equation of line c

$$y - 1 = \left[\frac{3-1}{1-3}\right](x-3)$$

$$y - 1 = -(x-3)$$

$$y = 4 - x$$

for line c, $x_2 = 4 - x_1$

* The equation of the lines represent the separators of the nodes in the hidden layer.

from line a

$$x_2 = 3x_1$$

$$3x_1 - x_2 = 0 \quad \text{(separator)}$$

for a point to satisfy this condition,

$$3x_1 - x_2 \geq 0$$

from line b

$$3x_2 = x_1$$

$$-x_1 + 3x_2 = 0$$

$$-x_1 + 3x_2 \geq 0 \quad \text{(satisfactory condition)}$$

from line c

$$x_2 = 4 - x_1$$

$$-x_1 - x_2 + 4 = 0$$

$$-x_1 - x_2 + 4 \geq 0 \quad \text{(satisfactory condition)}$$

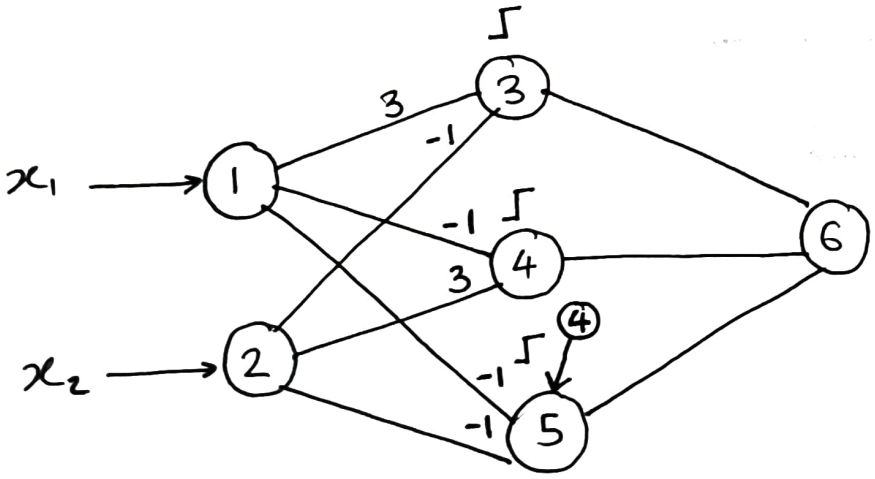
The coefficient of x_1 and x_2 in the equations give the weights of the corresponding nodes.

An activation function which outputs 1 when a point passes any of the conditions above and 0 when a point fails any of the conditions is needed.

The Heaviside function was chosen.

$$H(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Applying the weights and Heaviside activation function to the hidden layer



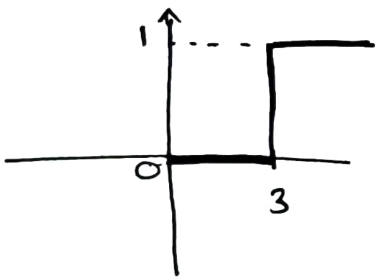
~~* for a point to be~~

* for a point to be classified as 1, it must pass all the conditions ie output of all hidden nodes must be 1

So we set the weights W_{63}, W_{64} and W_{65} to 1 and a point that is in class 1 will have a value of 3 sent to the output layer.

So the activation function of the output layer must output 1 if the input is 3 and 0 if it is less than 3

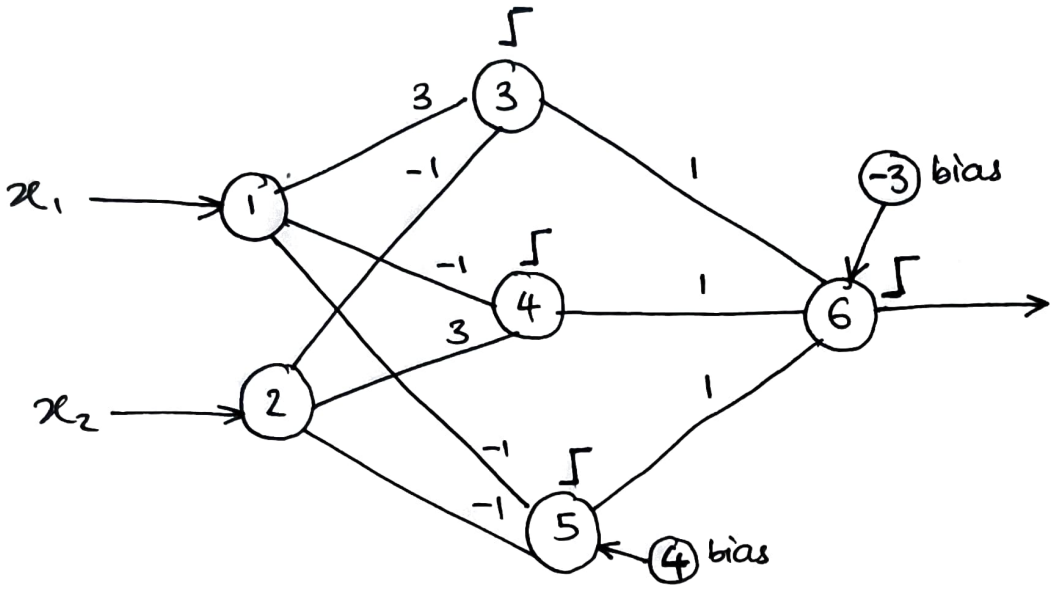
We can achieve this with the Heaviside function with an offset



to cater for the offset, we add a bias weight of -3 to the output node.

This makes a class 1 point have a value of 0 which will be classified by the activation function as Class 1 and all other points will have -ve values which will be Classified as 0.

Final network



$w_{31} = 3$ $w_{32} = -1$ $w_{63} = 1$

$w_{41} = -1$ $w_{42} = 3$ $w_{64} = 1$

$w_{51} = -1$ $w_{52} = -1$ $w_{65} = 1$