RBFN

June 27, 2021

1 ECE 657 ASSIGNMENT 2: Problem 3

1.1 RBFN

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[1]:
```
# importing the libraries
import numpy as np
import pandas as pd
import seaborn as sns
from sklearn.cluster import KMeans
import matplotlib.pyplot as plt
from matplotlib.colors import ListedColormap
from sklearn.model_selection import train_test_split
from IPython.display import display
```

Defining the RBFN class

[2]:
```
class RBFN(object):
    """
    Class of a RBF Neural network with 1 input layer, 1 hidden layer and 1␣
 ↪output layer
    """
    def __init__(self, input_size = 2, hidden_size = 2, output_size = 1,
                 activation = "sigmoid", epochs = 1000, alpha = 0.001,
                 centroids = None, sigma = 0.5):
        """
        input_size = no of input features
        hidden_size = no of neurons in hidden layer
        output_size = no of classes for classification problem
        alpha = learning rate
        """

        self.centroids = centroids
        self.centroids_set = True

        if type(centroids) != np.ndarray:
```

```
            self.hiddenSize = hidden_size
            self.centroids_set = False
        else:
            self.hiddenSize = centroids.shape[0]
            self.centroids = centroids

        # setting the number on neurons in each layer
        self.inputSize = input_size
        self.outputSize = output_size
        self.epochs = epochs # no of training epochs
        self.sigma = sigma # width of rbf kernel
        self.alpha = alpha # learning rate

        # setting the default activation function
        self.activation = self.sigmoid
        self.activation_gradient = self.sigmoid_gradient

        # setting the activation function
        if activation == 'tanh':
            self.activation = self.tanh
            self.activation_gradient = self.tanh_gradient

        # Randomly initializing the weights (-1 to 1)

        # weight matrix from hidden to output layer
        np.random.seed(0)
        self.W2 = 2 * np.random.random((self.hiddenSize, self.outputSize)) - 1


    def forward(self, X):
        #forward propagation through the network
        self.z3 = np.dot(X, self.W2) # dot product of hidden layer output and␣
↪hidden - output weights
        o = self.activation(self.z3) # activation function to get predicted␣
↪output
        return o


    def backward(self, X, y, o):
        # backward propgation through the network
        self.o_error = y - o # error in output
        self.o_delta = self.o_error * self.activation_gradient(o) # applying␣
↪activation derivative to error

        # update only weights between hidden layer and output
        self.W2 += self.alpha * (X.T.dot(self.o_delta)) # updating (hidden -␣
↪output) weights
```

```
    def apply_rbf(self, X):
        new_X = []
        for centroid in self.centroids:
            new_X.append(self.rbf(X, centroid, self.sigma))

        new_X = np.array(np.mat(new_X).T)
        return new_X


    def fit(self, X, y):
        y = np.where(y < 0, 0, y)
        if self.centroids_set == False:
            self.centroids = X[:self.hiddenSize]

        X_rbf = self.apply_rbf(X)
        print("Training ", end="")
        for i in range(self.epochs): # trains the NN 'epochs' times
            if i % 10 == 0:
                print(".", end="")
            o = self.forward(X_rbf)
            self.backward(X_rbf, y, o)
        print()

        self.model_loss = self.calc_model_loss(X, y)


    def sigmoid(self, z):
        # sigmoid function
        return 1 / ( 1 + np.exp(-z))

    def rbf(self, x, c, s):
        return (np.exp(-1 / (2 * s**2) * np.sum((x-c)**2, axis=1))) / ((2 * np.pi * s**2) ** 0.5)


    def sigmoid_gradient(self, z):
        #derivative of sigmoid
        return z * (1 - z)


    def tanh(self, x):
        # tanh function
        return np.tanh(x)
```

```
    def tanh_gradient(self, x):
        # derivative of tanh function
        return 1.0 - x**2


    def get_model(self):
        # returns the learned weights of the model
        return [self.W1, self.W2]


    def predict_proba(self, x_new):
        # returns the probability of input belonging to the different classes

        X_rbf = self.apply_rbf(x_new)
        y_pred_proba = self.forward(X_rbf)

        # for multiclass outputs, sum of classes probability must be 1
        if self.outputSize != 1:
            scale = y_pred_proba.sum(axis = 1).reshape(-1,1)
            y_pred_proba = y_pred_proba / scale

        return y_pred_proba


    def predict(self, x_new):
        # returns the model's class prediction of given input
        y_pred = self.predict_proba(x_new)

        # for binary classification set threshold = 0.5
        if self.outputSize == 1:
            y_pred = (y_pred >= 0.5).astype(int)
            y_pred = np.where(y_pred == 0, -1, y_pred)

        # for multiclass classification, assign to class with highest␣
↪probability
        else:
            y_pred = (y_pred == y_pred.max(axis=1)[:,None]).astype(int)

        return y_pred


    def calc_model_loss(self, X, y):
        X_rbf = self.apply_rbf(X)
        # mean sum squared loss
        return np.mean(np.square(y - self.forward(X_rbf)))
```

```python
    def save_model(self, filename):
        # saves model weights in specified filename
        filename = filename + ".npy"
        model = {"sigma": self.sigma, "centroids":self.centroids, "weights":␣
↪self.W2}
        np.save(filename, model, allow_pickle=True)


    def load_model(self, model):
        # load weights into neural network
        self.W2 = model["weights"]
        self.sigma = model["sigma"]
        self.centroids = model["centroids"]
```

Defining function for calculating accuracy

[3]:
```python
def accuracy(y_true, y_pred):
    """ Function for calculating the accuracy """
    if not (len(y_true) == len(y_pred)):
        print('Size of predicted and true labels not equal.')
        return 0.0

    corr = 0
    for i in range(0,len(y_true)):
        corr += 1 if (y_true[i] == y_pred[i]).all() else 0

    return corr/len(y_true)
```

Function for running the tests

[4]:
```python
def run_test(X_train, y_train, X_test, y_test, centroids, sigmas):
    """ function for running experiment using constant widths for all RBF
    and varying the widths to see the effects of changing widths """

    train_accs = []
    test_accs = []
    model_losses = []
    models = []
    for sigma in sigmas:

        # Initializing the Neural network

        rbfn = RBFN(input_size = X_train.shape[1],
                    output_size = y_train.shape[1],
                    centroids = centroids,
                    epochs = 500,
                    sigma = sigma)
```

```
        # Fitting the network to the train set
        rbfn.fit(X_train, y_train)

        # Checking the performance of the model on training set and test set
        y_pred_train = rbfn.predict(X_train)
        y_pred = rbfn.predict(X_test)

        train_acc = accuracy(y_train,y_pred_train) * 100
        test_acc = accuracy(y_test,y_pred) * 100
        model_loss = rbfn.model_loss

        train_accs.append(train_acc)
        test_accs.append(test_acc)
        model_losses.append(model_loss)
        models.append(rbfn)

        print("Sigma (width): ", sigma)
        print("Training Accuracy: ", train_acc)
        print("Test Accuracy: ", test_acc)
        print("Model loss: ", model_loss)
        print()

    return train_accs, test_accs, model_losses, models
```

Helper Functions

[5]:
```
def accuracy_loss_plot(accuracies, model_losses, sigmas, legend_1 = ["Train",␣
 ↪"Test"], legend_2 = ["Model Loss"]):
    """ Function for visualizing the model performance """
    x_counts = [i for i in range(len(sigmas))]
    x_ticks = [str(sigma) for sigma in sigmas]

    if model_losses == None:
        fig, ax1 = plt.subplots(1, 1, figsize=(6.5, 4))
    else:
        fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(14, 4))

    for accuracy in accuracies:
        ax1.plot(accuracy)

    ax1.set_title('Model accuracy')
    ax1.set_ylabel('Accuracy')
    ax1.set_xlabel('Sigma')
    ax1.legend(legend_1, loc='lower right')
    ax1.set_xticks(x_counts)
    ax1.set_xticklabels(x_ticks)
```

```
    if model_losses == None:
        return

    for model_loss in model_losses:
        ax2.plot(model_loss)

    ax2.set_title('Model loss')
    ax2.set_ylabel('loss')
    ax2.set_xlabel('Sigma')
    ax2.legend(legend_2, loc='upper right')
    ax2.set_xticks(x_counts)
    ax2.set_xticklabels(x_ticks)
```

[6]:
```
def class_boundary(X_set, y_set, model, title):
    """ Function for visualizing model classification boundary """
    X1, X2 = np.meshgrid(np.arange(start = -2, stop = 2, step = 0.01),
                         np.arange(start = -2, stop = 2, step = 0.01))
    plt.contourf(X1, X2, model.predict(np.array([X1.ravel(), X2.ravel()]).T).
 ↪reshape(X1.shape),
                 alpha = 0.75, cmap = ListedColormap(('red', 'green')))
    plt.xlim(X1.min(), X1.max())
    plt.ylim(X2.min(), X2.max())
    for i, j in enumerate(np.unique(y_set)):
        plt.scatter(X_set[y_set[:,0] == j, 0], X_set[y_set[:,0] == j, 1],
                    color = ListedColormap(('red', 'green'))(i), label = j)
    plt.title(title)
    plt.xlabel('X1')
    plt.ylabel('X2')
    plt.legend(loc="lower right")
    plt.show()
```

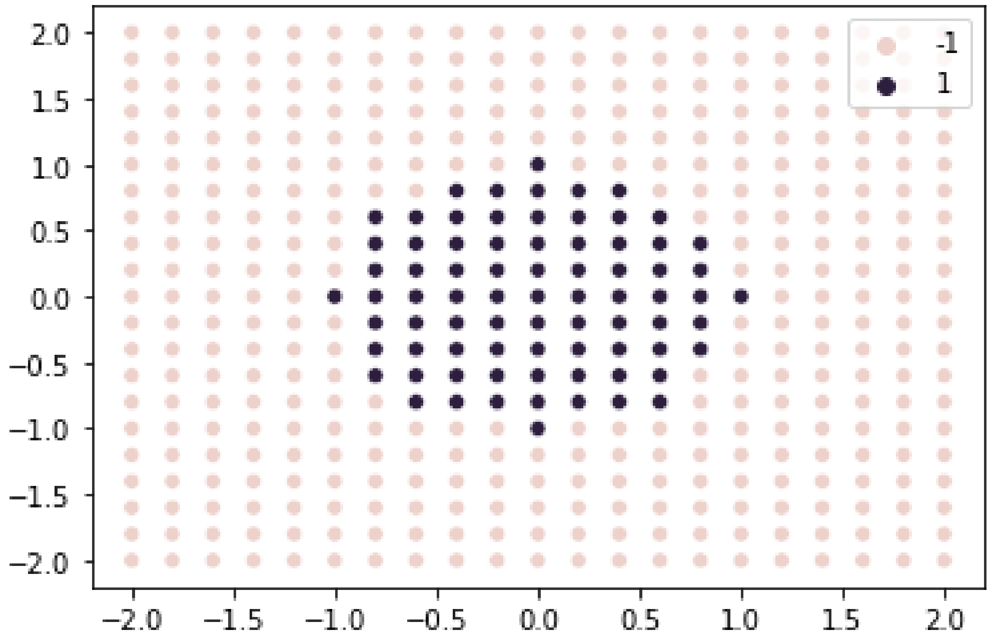
Creating the dataset

[7]:
```
""" Creating the dataset """
x = []
for i in range(21):
    for j in range(21):
        x.append([i,j])

x = np.array(x)

X = -2 + 0.2 * x

y = np.where((X[:,0] ** 2 + X[:,1] ** 2 <= 1), 1, -1)
```

Visualizing the dataset

[8]:
```
sns.scatterplot(x=X[:,0], y=X[:,1], hue = y)
```

[8]: <AxesSubplot:>



[9]:
```
""" splitting the data into training set and test set """
# training set and test set with 80:20 ratio
y = y.reshape((-1,1))
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2,␣
↪random_state=275)
```

1.2 Experiment 1

Experiment using all input points as centroids with constant widths for all RBF and varying the widths to see the effects of changing widths on model loss and accuracy of RBFN.

[10]:
```
sigmas = [0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 1, 2, 5, 10, 100]
```

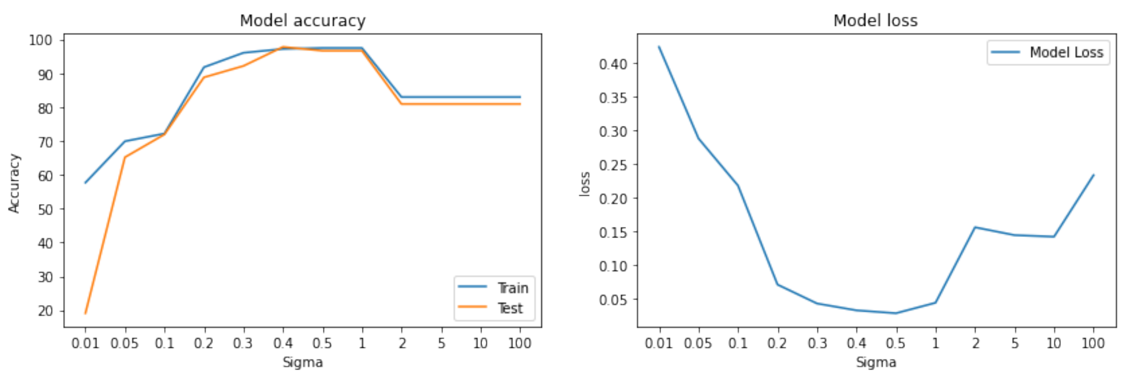
[11]:
```
%%time
centroids = X_train
train_acc_1, test_acc_1, model_losses_1, model_1 = run_test(X_train, y_train,␣
↪X_test, y_test, centroids, sigmas)
```

```
Training …
Sigma (width):  0.01
Training Accuracy:  57.67045454545454
Test Accuracy:  19.101123595505616
Model loss:  0.4231336031473347

Training …
```

```
Sigma (width):  0.05
Training Accuracy:  69.88636363636364
Test Accuracy:  65.1685393258427
Model loss:  0.28772297115062073

Training …
Sigma (width):  0.1
Training Accuracy:  72.1590909090909
Test Accuracy:  71.91011235955057
Model loss:  0.2180173848703248

Training …
Sigma (width):  0.2
Training Accuracy:  91.76136363636364
Test Accuracy:  88.76404494382022
Model loss:  0.0715807124130645

Training …
Sigma (width):  0.3
Training Accuracy:  96.02272727272727
Test Accuracy:  92.13483146067416
Model loss:  0.04361041193312453

Training …
Sigma (width):  0.4
Training Accuracy:  97.1590909090909
Test Accuracy:  97.75280898876404
Model loss:  0.03345460191662005

Training …
Sigma (width):  0.5
Training Accuracy:  97.44318181818183
Test Accuracy:  96.62921348314607
Model loss:  0.029230817761364714

Training …
Sigma (width):  1
Training Accuracy:  97.44318181818183
Test Accuracy:  96.62921348314607
Model loss:  0.04470002999110812

Training …
Sigma (width):  2
Training Accuracy:  82.95454545454545
Test Accuracy:  80.89887640449437
Model loss:  0.15633114148647856

Training …
```

```
Sigma (width):  5
Training Accuracy:  82.95454545454545
Test Accuracy:  80.89887640449437
Model loss:  0.1447270102776108

Training …
Sigma (width):  10
Training Accuracy:  82.95454545454545
Test Accuracy:  80.89887640449437
Model loss:  0.14241164201255466

Training …
Sigma (width):  100
Training Accuracy:  82.95454545454545
Test Accuracy:  80.89887640449437
Model loss:  0.2334901832069611

Wall time: 2.26 s
```

[12]:
```python
columns = ["Sigma", "Train Acc", "Test Acc", "Model Loss"]
results_1 = pd.DataFrame(np.array([sigmas, train_acc_1, test_acc_1,␣
↪model_losses_1]).T, columns = columns)
accuracy_loss_plot([train_acc_1, test_acc_1], [model_losses_1], sigmas)
```



1.2.1 Observations

In this experiment, all the training datapoints were used as centroids and the RBF width was varied through 0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 1, 2, 5, 10, 100 and the following results were gotten.

[13]:
```python
display(results_1)
```

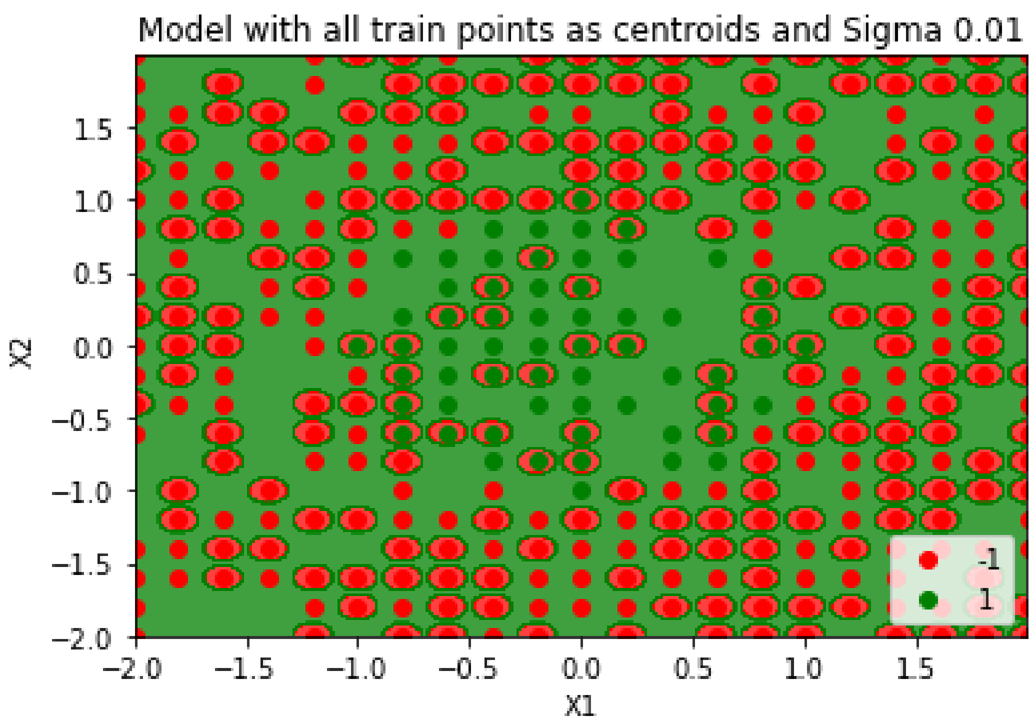
	Sigma	Train Acc	Test Acc	Model Loss
0	0.01	57.670455	19.101124	0.423134
1	0.05	69.886364	65.168539	0.287723
2	0.10	72.159091	71.910112	0.218017

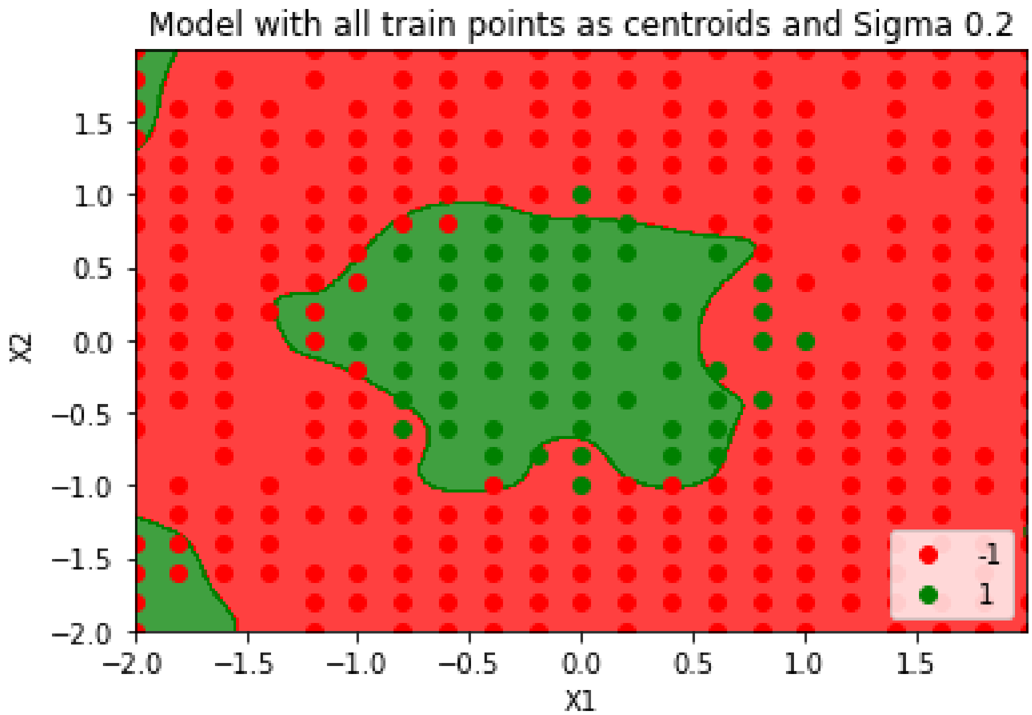
3	0.20	91.761364	88.764045	0.071581
4	0.30	96.022727	92.134831	0.043610
5	0.40	97.159091	97.752809	0.033455
6	0.50	97.443182	96.629213	0.029231
7	1.00	97.443182	96.629213	0.044700
8	2.00	82.954545	80.898876	0.156331
9	5.00	82.954545	80.898876	0.144727
10	10.00	82.954545	80.898876	0.142412
11	100.00	82.954545	80.898876	0.233490

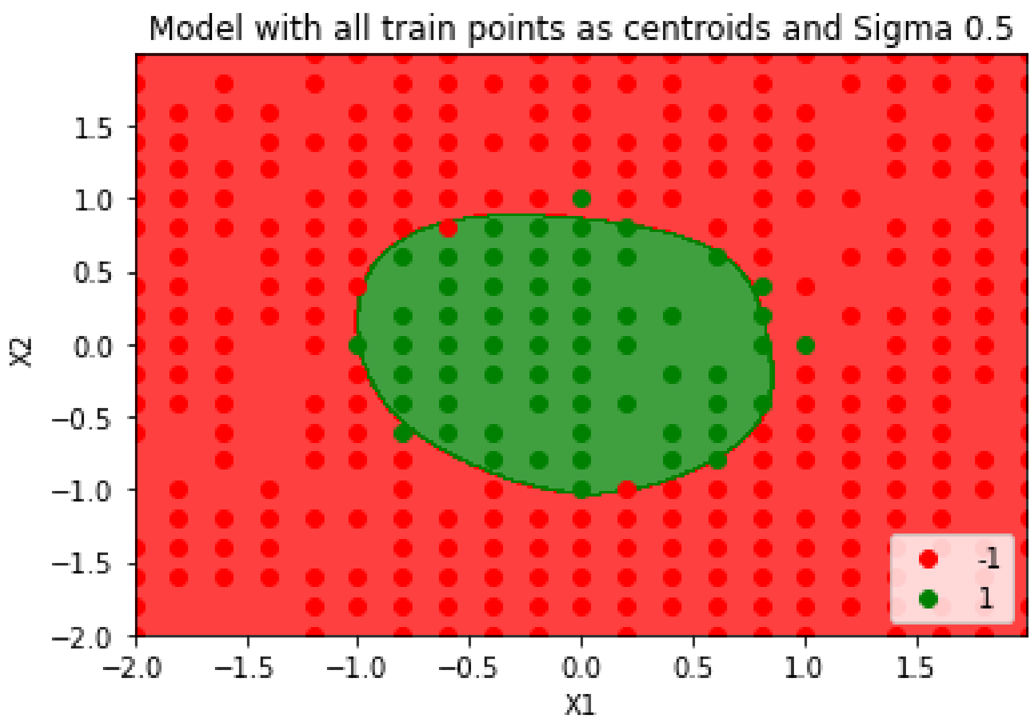
From the table of results and the accuracy and loss plots it can be observed that the model perform poorly in terms of training accuracy, testing accuracy and model loss for very low values of sigma (widths) and as the width increases, the performance of the model improves until it reaches a peak and starts to drop again. The model performed best with maximum accuracy and minimum loss with sigma (width) between 0.4 and 0.5.

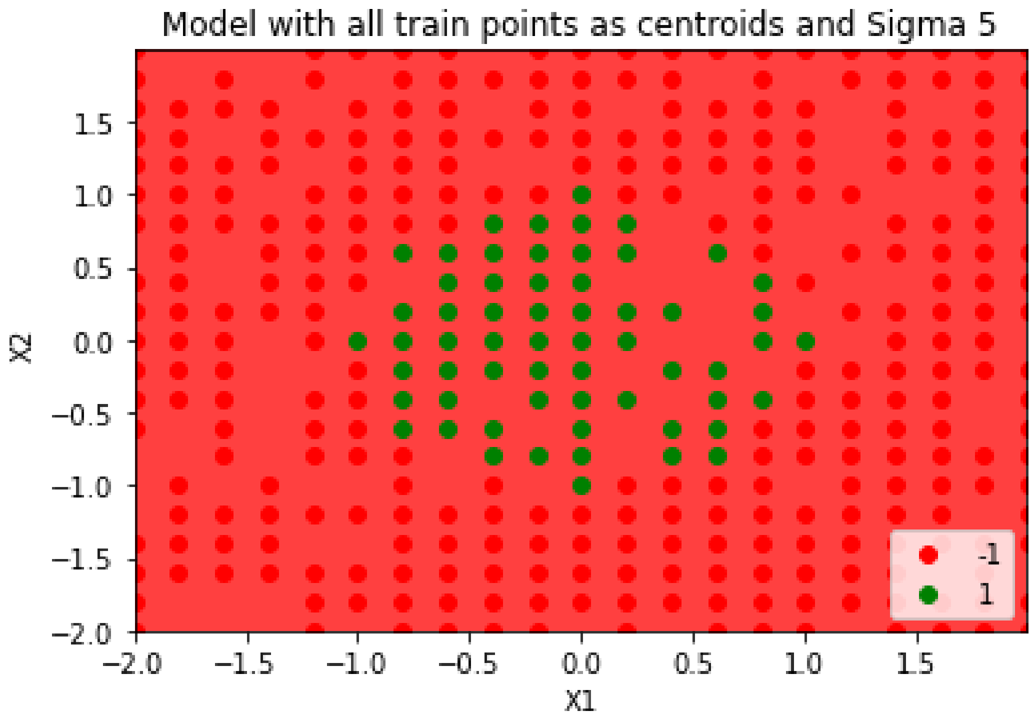
1.2.2 Further investigation using classification boundary

[14]:
```
for i in range(len(model_1)):
    if i % 3 == 0:
        title = "Model with all train points as centroids and Sigma " +␣
↪str(sigmas[i])
        class_boundary(X_train, y_train, model_1[i], title)
```




Model with all train points as centroids and Sigma 0.2
X2
X1
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
-1
1


Model with all train points as centroids and Sigma 0.5
X2
X1
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
-1
1



From the plots showing the classification boundaries, it can be seen that for very low values of sigma, the model tends to memorize the centroid locations and since the training points were used as centroid locations, it tends to memorize a particualr class of the training data thus performing poorly on the other class and not generalizing well.

For optimal values of sigma, the model performs great and learns an appropriate ovoidal boundary at the center of the plot showing that the model has learnt an appropriate approximate function for mapping the inputs to the classes. The model also generalizes well as it performs well on new observations (test set).

For large values of sigma, the model again performs poorly as it only classifies one class. It assumes all inputs belong to one class and is unable to classify samples belonging to the other class.

1.3 Experiment 2a

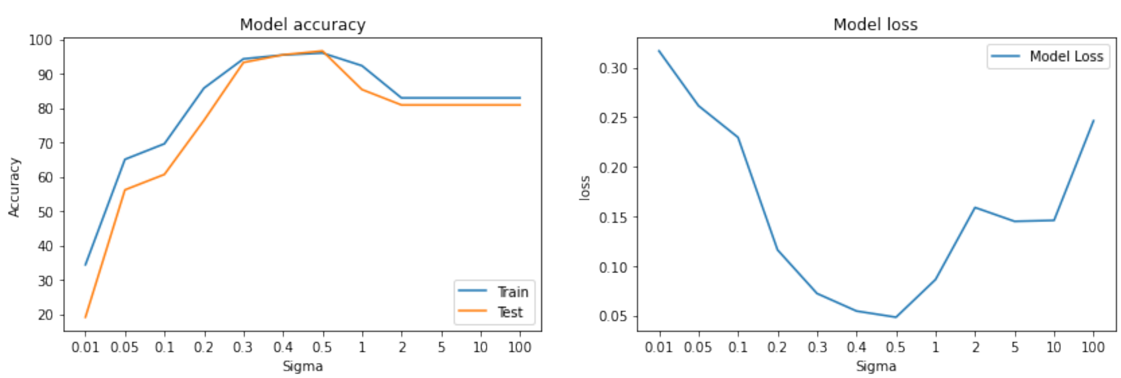
Experiment selecting 150 random samples from the training data without replacement as centroids with constant widths for all RBF and varying the widths to see the effects of changing widths on model loss and accuracy of RBFN.

[15]:
```
%%time
idx = np.random.choice(X_train.shape[0], size=150, replace=False)
centroids = X_train[idx]
train_acc_2, test_acc_2, model_losses_2, model_2  = run_test(X_train, y_train,␣
 ↪X_test, y_test, centroids, sigmas)
```

```
Training …
Sigma (width):  0.01
Training Accuracy:  34.375
Test Accuracy:  19.101123595505616
Model loss:  0.31677466069564725

Training …
Sigma (width):  0.05
Training Accuracy:  65.05681818181817
Test Accuracy:  56.17977528089888
Model loss:  0.2614622820594803

Training …
Sigma (width):  0.1
Training Accuracy:  69.60227272727273
Test Accuracy:  60.67415730337079
Model loss:  0.22958887082276147

Training …
Sigma (width):  0.2
Training Accuracy:  85.79545454545455
Test Accuracy:  76.40449438202246
Model loss:  0.11616569908886198

Training …
Sigma (width):  0.3
Training Accuracy:  94.31818181818183
Test Accuracy:  93.25842696629213
Model loss:  0.0721640784003872

Training …
Sigma (width):  0.4
Training Accuracy:  95.45454545454545
Test Accuracy:  95.50561797752809
Model loss:  0.05445130294687071

Training …
Sigma (width):  0.5
Training Accuracy:  96.02272727272727
Test Accuracy:  96.62921348314607
Model loss:  0.04822992790369985

Training …
Sigma (width):  1
Training Accuracy:  92.32954545454545
Test Accuracy:  85.39325842696628
Model loss:  0.08623358563461897
```

```
Training …
Sigma (width):  2
Training Accuracy:  82.95454545454545
Test Accuracy:  80.89887640449437
Model loss:  0.15888789919458313

Training …
Sigma (width):  5
Training Accuracy:  82.95454545454545
Test Accuracy:  80.89887640449437
Model loss:  0.14492804320430894

Training …
Sigma (width):  10
Training Accuracy:  82.95454545454545
Test Accuracy:  80.89887640449437
Model loss:  0.14590682554053058

Training …
Sigma (width):  100
Training Accuracy:  82.95454545454545
Test Accuracy:  80.89887640449437
Model loss:  0.24637916695991013

Wall time: 1.58 s
```

[16]:
```
results_2 = pd.DataFrame(np.array([sigmas, train_acc_2, test_acc_2,␣
 ↪model_losses_2]).T, columns = columns)
accuracy_loss_plot([train_acc_2, test_acc_2], [model_losses_2], sigmas)
```



1.3.1 Observations

In this experiment, all the training datapoints were used as centroids and the RBF width was varied through 0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 1, 2, 5, 10, 100 and the following results were gotten.

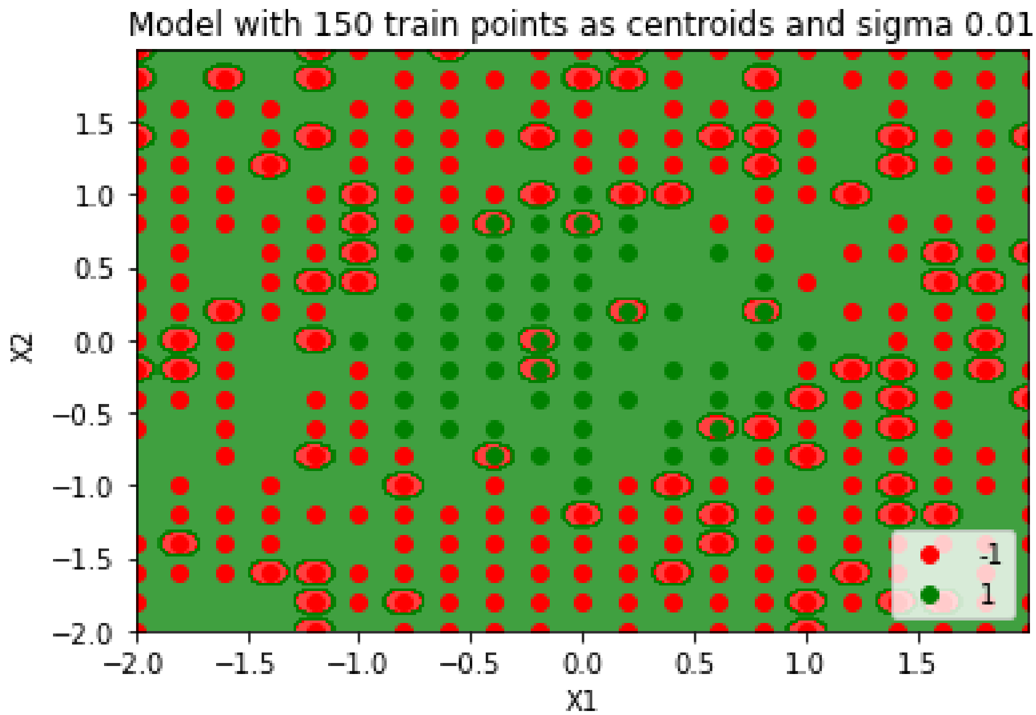
[17]:
```
display(results_2)
```

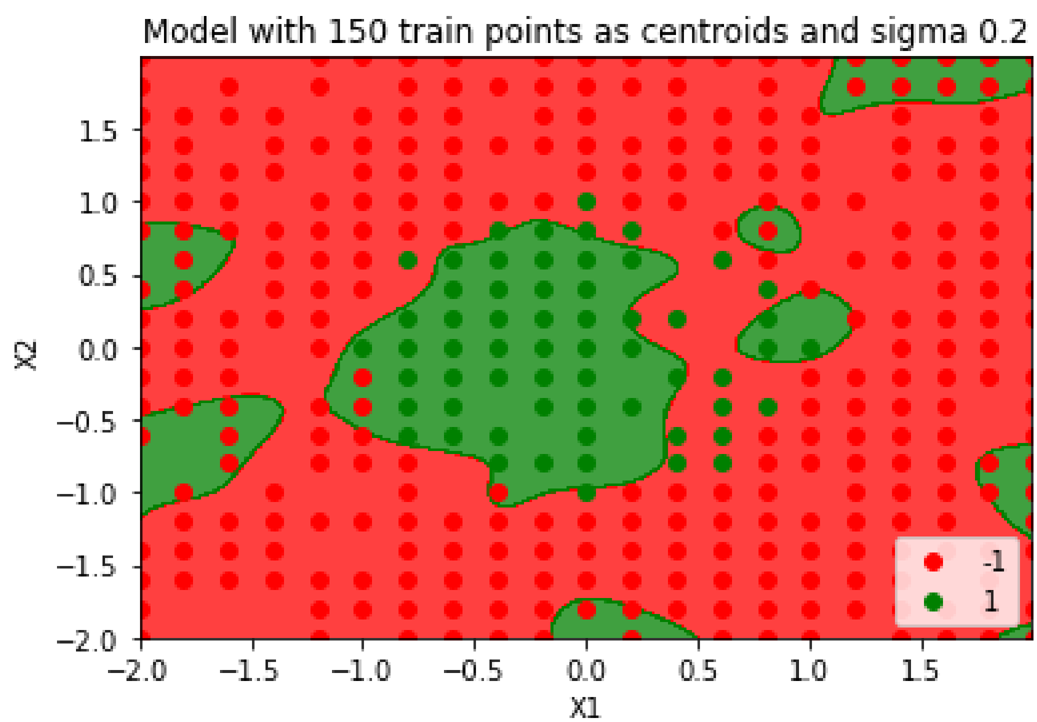
	Sigma	Train Acc	Test Acc	Model Loss
0	0.01	34.375000	19.101124	0.316775
1	0.05	65.056818	56.179775	0.261462
2	0.10	69.602273	60.674157	0.229589
3	0.20	85.795455	76.404494	0.116166
4	0.30	94.318182	93.258427	0.072164
5	0.40	95.454545	95.505618	0.054451
6	0.50	96.022727	96.629213	0.048230
7	1.00	92.329545	85.393258	0.086234
8	2.00	82.954545	80.898876	0.158888
9	5.00	82.954545	80.898876	0.144928
10	10.00	82.954545	80.898876	0.145907
11	100.00	82.954545	80.898876	0.246379

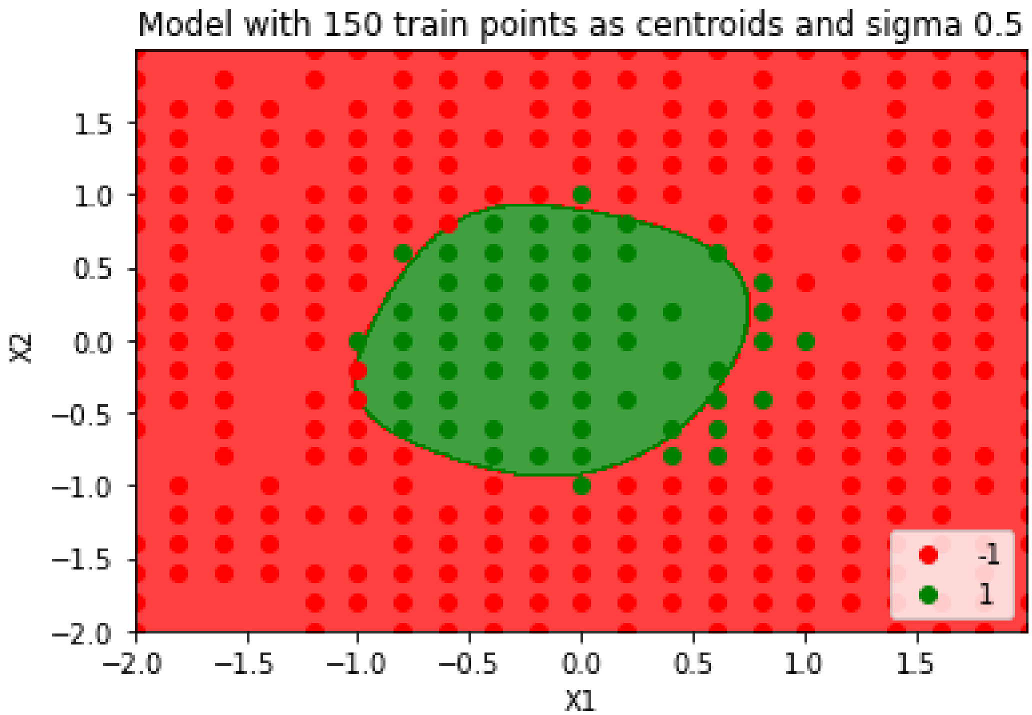
From the table of results and the accuracy and loss plots it can be observed that the model perform poorly in terms of training accuracy, testing accuracy and model loss for very low values of sigma (widths) and as the width increases, the performance of the model improves until it reaches a peak and starts to drop again. The model performed best with maximum accuracy and minimum loss with sigma (width) between 0.4 and 0.5.

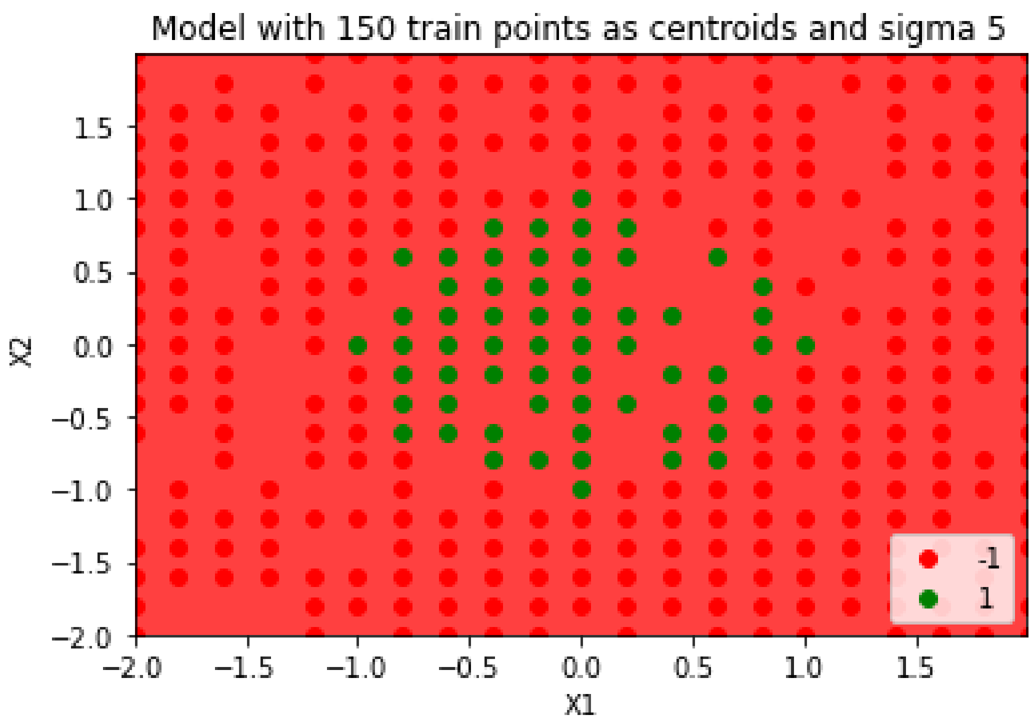
1.3.2 Further investigation using classification boundary

[18]:
```
for i in range(len(model_2)):
    if i % 3 == 0:
        title = "Model with 150 train points as centroids and sigma " +␣
↪str(sigmas[i])
        class_boundary(X_train, y_train, model_2[i], title)
```


Model with 150 train points as centroids and sigma 0.01
X2
X1
-1
1


Model with 150 train points as centroids and sigma 0.2
X2
X1
-1
1


Model with 150 train points as centroids and sigma 0.5
X2
X1
-1
1


Model with 150 train points as centroids and sigma 5
X2
X1
-1
1

From the plots showing the classification boundaries, it can be seen that for very low values of sigma, the model tends to memorize the centroid locations and since the training points were used as centroid locations, it tends to memorize a particualr class of the training data thus performing poorly on the other class and not generalizing well.

For optimal values of sigma, the model performs great and learns an appropriate ovoidal boundary at the center of the plot showing that the model has learnt an appropriate approximate function for mapping the inputs to the classes. The model also generalizes well as it performs well on new observations (test set).

For large values of sigma, the model again performs poorly as it only classifies one class. It assumes all inputs belong to one class and is unable to classify samples belonging to the other class.

1.4 Experiment 2b

Experiment using KMeans clustering to find 150 centroid points with constant widths for all RBF and varying the widths to see the effects of changing widths on model loss and accuracy of RBFN.

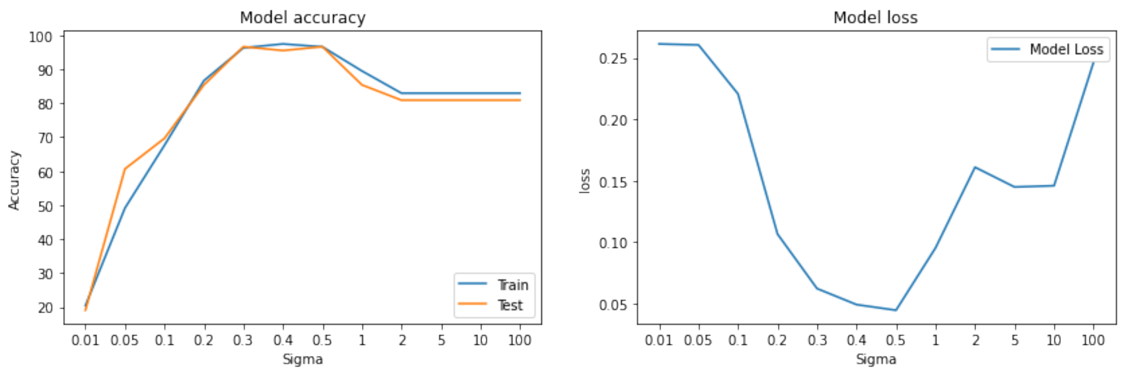
[19]:
```
%%time
# using KNN to find the centroids
kmeans = KMeans(n_clusters = 150)
kmeans.fit(X_train)

centroids = kmeans.cluster_centers_
train_acc_3, test_acc_3, model_losses_3, model_3  = run_test(X_train, y_train,␣
↪X_test, y_test, centroids, sigmas)
```

```
Training …
Sigma (width):  0.01
Training Accuracy:  20.454545454545457
Test Accuracy:  19.101123595505616
Model loss:  0.26137137879208877

Training …
Sigma (width):  0.05
Training Accuracy:  49.14772727272727
Test Accuracy:  60.67415730337079
Model loss:  0.260569272113858

Training …
Sigma (width):  0.1
Training Accuracy:  67.61363636363636
Test Accuracy:  69.66292134831461
Model loss:  0.22065808127896885

Training …
Sigma (width):  0.2
Training Accuracy:  86.64772727272727
Test Accuracy:  85.39325842696628
```

```
Model loss:  0.10651291594067548

Training …
Sigma (width):  0.3
Training Accuracy:  96.30681818181817
Test Accuracy:  96.62921348314607
Model loss:  0.062126582465003306

Training …
Sigma (width):  0.4
Training Accuracy:  97.44318181818183
Test Accuracy:  95.50561797752809
Model loss:  0.04905218306367384

Training …
Sigma (width):  0.5
Training Accuracy:  96.5909090909091
Test Accuracy:  96.62921348314607
Model loss:  0.04447392434576394

Training …
Sigma (width):  1
Training Accuracy:  89.48863636363636
Test Accuracy:  85.39325842696628
Model loss:  0.09525777322257238

Training …
Sigma (width):  2
Training Accuracy:  82.95454545454545
Test Accuracy:  80.89887640449437
Model loss:  0.1609492977993744

Training …
Sigma (width):  5
Training Accuracy:  82.95454545454545
Test Accuracy:  80.89887640449437
Model loss:  0.1448481354567191

Training …
Sigma (width):  10
Training Accuracy:  82.95454545454545
Test Accuracy:  80.89887640449437
Model loss:  0.1458768257963121

Training …
Sigma (width):  100
Training Accuracy:  82.95454545454545
Test Accuracy:  80.89887640449437
```

```
Model loss:  0.24637878208976052

Wall time: 2.51 s
```

[20]:
```python
results_3 = pd.DataFrame(np.array([sigmas, train_acc_3, test_acc_3,␣
 ↪model_losses_3]).T, columns = columns)
accuracy_loss_plot([train_acc_3, test_acc_3], [model_losses_3], sigmas)
```



1.4.1 Observations

In this experiment, all the training datapoints were used as centroids and the RBF width was varied through 0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 1, 2, 5, 10, 100 and the following results were gotten.

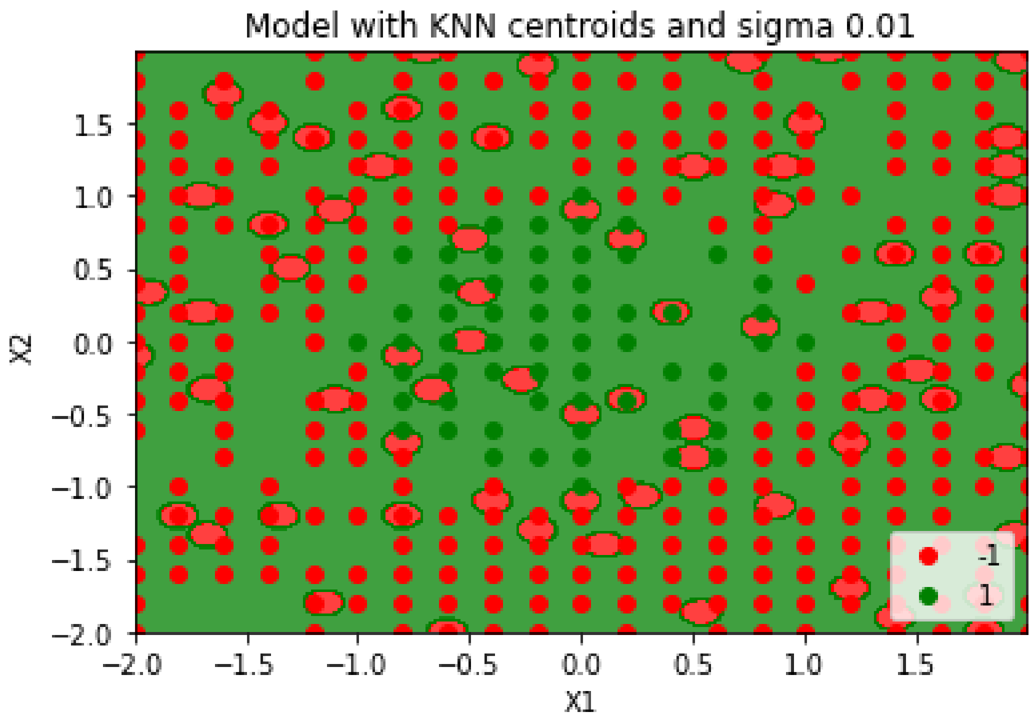
[21]:
```python
display(results_3)
```

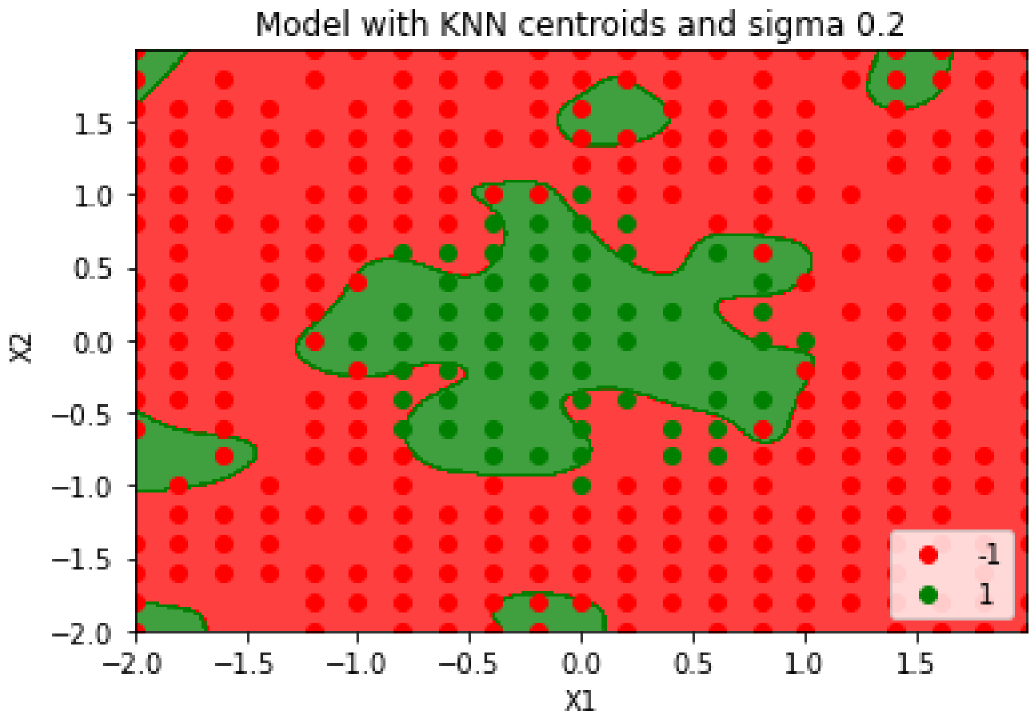
	Sigma	Train Acc	Test Acc	Model Loss
0	0.01	20.454545	19.101124	0.261371
1	0.05	49.147727	60.674157	0.260569
2	0.10	67.613636	69.662921	0.220658
3	0.20	86.647727	85.393258	0.106513
4	0.30	96.306818	96.629213	0.062127
5	0.40	97.443182	95.505618	0.049052
6	0.50	96.590909	96.629213	0.044474
7	1.00	89.488636	85.393258	0.095258
8	2.00	82.954545	80.898876	0.160949
9	5.00	82.954545	80.898876	0.144848
10	10.00	82.954545	80.898876	0.145877
11	100.00	82.954545	80.898876	0.246379

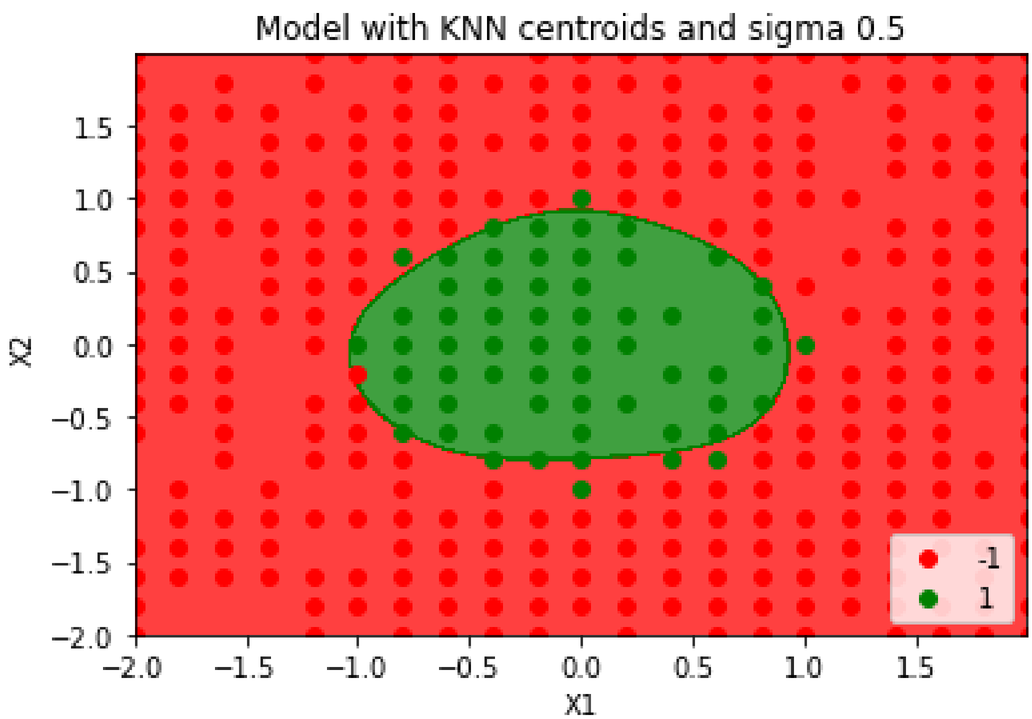
From the table of results and the accuracy and loss plots it can be observed that the model perform poorly in terms of training accuracy, testing accuracy and model loss for very low values of sigma (widths) and as the width increases, the performance of the model improves until it reaches a peak and starts to drop again. The model performed best with maximum accuracy and minimum loss with sigma (width) between 0.4 and 0.5.

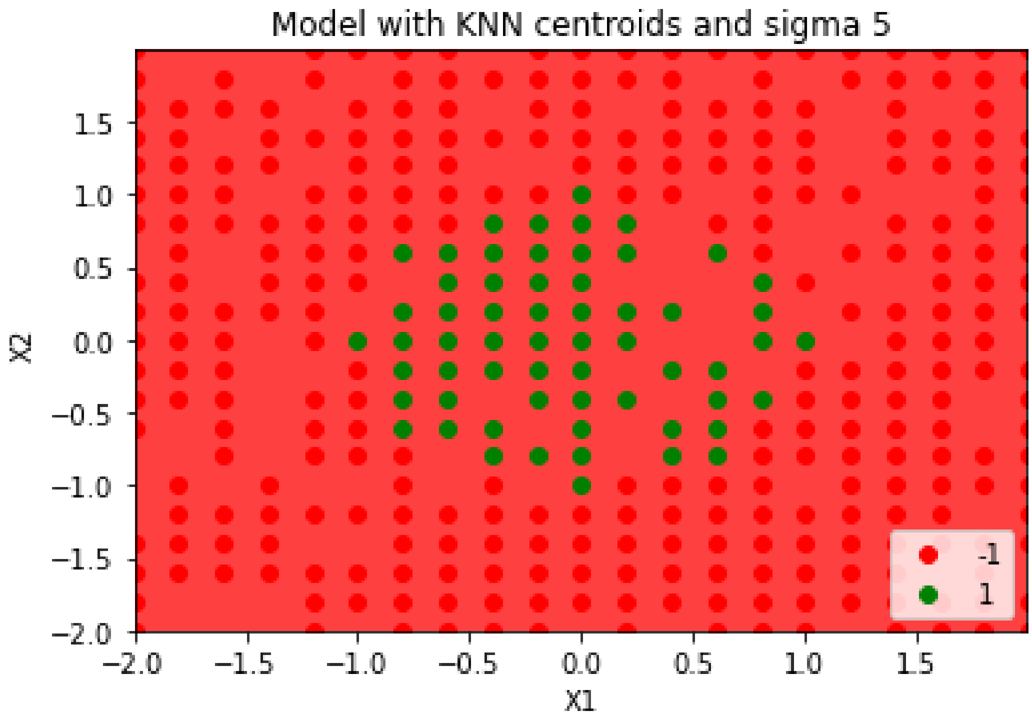
1.4.2 Further investigation using classification boundary

[22]:
```
for i in range(len(model_3)):
    if i % 3 == 0:
        title = "Model with KNN centroids and sigma " + str(sigmas[i])
        class_boundary(X_train, y_train, model_3[i], title)
```




Model with KNN centroids and sigma 0.2
X2
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
X1
-1
1


Model with KNN centroids and sigma 0.5
X2
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
X1
-1
1



From the plots showing the classification boundaries, it can be seen that for very low values of sigma, the model tends to memorize the centroid locations and since the training points were used as centroid locations, it tends to memorize a particualr class of the training data thus performing poorly on the other class and not generalizing well.

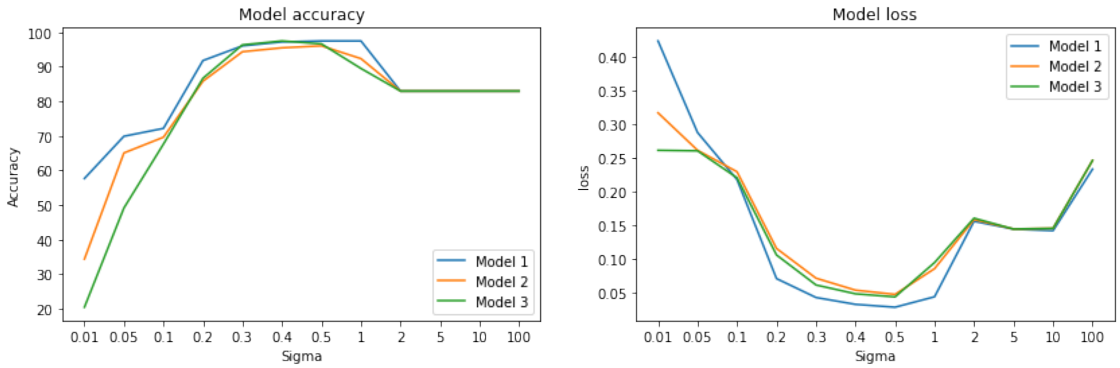
For optimal values of sigma, the model performs great and learns an appropriate ovoidal boundary at the center of the plot showing that the model has learnt an appropriate approximate function for mapping the inputs to the classes. The model also generalizes well as it performs well on new observations (test set).

For large values of sigma, the model again performs poorly as it only classifies one class. It assumes all inputs belong to one class and is unable to classify samples belonging to the other class.

1.5 Comparing the different centroid initialization methods

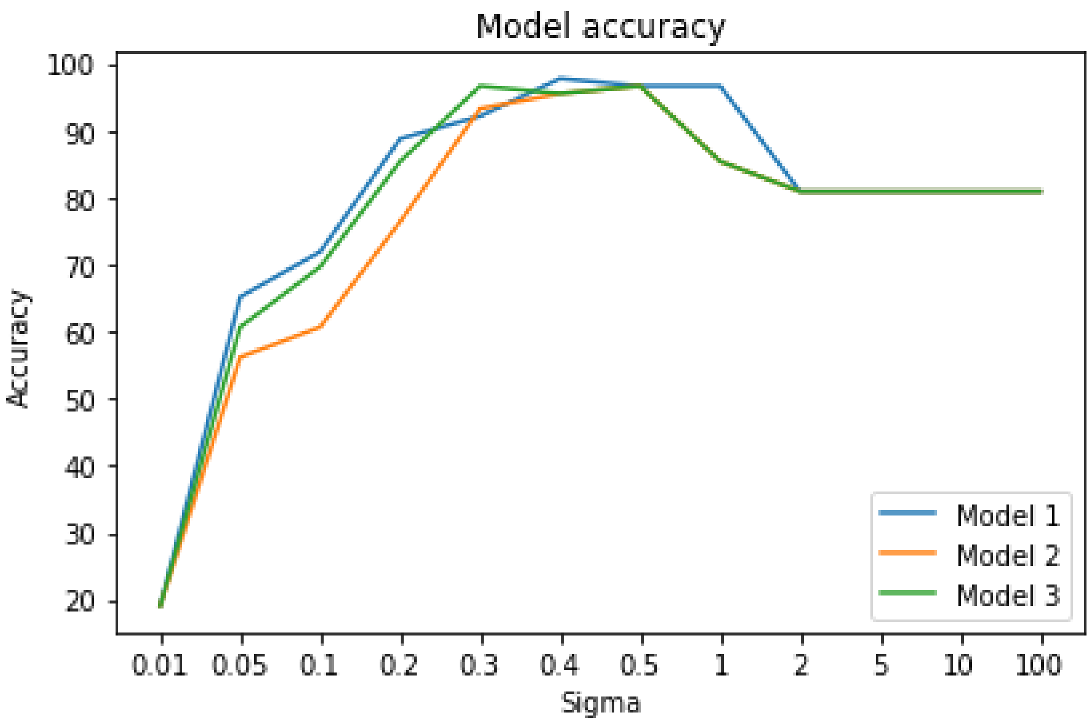
Training accuracy and model loss

[23]:
```
accuracy_loss_plot([train_acc_1, train_acc_2, train_acc_3],
                   [model_losses_1, model_losses_2, model_losses_3],
                   sigmas,
                   legend_1 = ["Model 1", "Model 2", "Model 3"],
                   legend_2 = ["Model 1", "Model 2", "Model 3"])
```



Test accuracy

```
[24]: accuracy_loss_plot([test_acc_1, test_acc_2, test_acc_3],
                         None,
                         sigmas,
                         legend_1 = ["Model 1", "Model 2", "Model 3"])
```



1.5.1 Observations

Model 1: Model with all training datapoints as centroid locations
Model 2: Model with 150 random training datapoints as centroid locations

Model 3: Model with 150 centroids gotten using KMeans clustering

Overall, The models with the 3 different RBF centroid initializations performed comparably the same with the optimal sigma value (0.4 - 0.5). They all had training accuracies > 96, test accuracies > 96 and model loss <= 0.05. Model 1 was observed to show the minimum model loss among the three models for this sigma range.

For very small RBF widths, the Model 1 had the highest training accuracy but had the highest model loss and generalization error with is indicative of the fact the the model tends to memorize a class rather than learning an approximate function to distinguish the two classes.

For very large RBF widths (sigma > 2), all the models had similar performance. Their performance was lower than that of the optimal sigma value and from the classification boundary plots previously shown, it is indicative of the model classifyin all points as the -1 class.

[ ]:

```

```