Problem 3

$$\Delta w^{(K)} = \eta\left(t^{(K)} - w^{(K)} x^{(K)}\right) \frac{x^{(K)}}{\|x^{(K)}\|^2} \quad \text{—— } (1)$$

Since same input vector is presented at iteration $(K+1)$, $x^{(K+1)} = x^{(K)}$

Since $\boxed{x^{(K+1)} = x^{(K)}}$, then $\boxed{t^{(K+1)} = t^{(K)}}$

$$\Delta w^{(K+1)} = \eta\left(t^{(K+1)} - w^{(K+1)} x^{(K+1)}\right) \frac{x^{(K+1)}}{\|x^{(K+1)}\|^2}$$

$$\Delta w^{(K+1)} = \eta\left(t^{(K)} - w^{(K+1)} x^{(K)}\right) \frac{x^{(K)}}{\|x^{(K)}\|^2} \quad \left[\text{Substituting } x^{(K+1)} = x^{(K)} \text{ and } t^{(K+1)} = t^{(K)}\right]$$

Since, $\Delta w^{(K)} = w^{(K+1)} - w^{(K)}$, we have, $\boxed{w^{(K+1)} = \Delta w^{(K)} + w^{(K)}}$ —— (2)

$$\Delta w^{(K+1)} = \eta\left(t^{(K)} - (\Delta w^{(K)} + w^{(K)}) x^{(K)}\right) \frac{x^{(K)}}{\|x^{(K)}\|^2} \quad \left[\text{From } (2)\right]$$

$$\Delta w^{(K+1)} = \eta\left(t^{(K)} - w^{(K)} x^{(K)} - \Delta w^{(K)} x^{(K)}\right) \frac{x^{(K)}}{\|x^{(K)}\|^2}$$

$$\Delta w^{(K+1)} = \eta\left(t^{(K)} - w^{(K)} x^{(K)}\right) \frac{x^{(K)}}{\|x^{(K)}\|^2} - \eta\left(\Delta w^{(K)} x^{(K)}\right) \frac{x^{(K)}}{\|x^{(K)}\|^2}$$

$$\Delta w^{(K+1)} = \Delta w^{(K)} - \eta\left(\Delta w^{(K)} x^{(K)}\right) \frac{x^{(K)}}{\|x^{(K)}\|^2} \quad \left[\text{From } (1)\right]$$

$$\Delta w^{(K+1)} = \Delta w^{(K)} - \eta\, \Delta w^{(K)} \cdot \frac{\|x^{(K)}\|^2}{\|x^{(K)}\|^2}$$

$$\Delta w^{(K+1)} = \Delta w^{(K)} - \eta\, \Delta w^{(K)}$$

$$\boxed{\Delta w^{(K+1)} = (1 - \eta)\, \Delta w^{K}.}$$

So, if the same input vector is presented at iteration $(K+1)$, then the weight vector decreases by factor $(1-\eta)$ going from iteration (K) to iteration $(K+1)$.