ECE/CS/ME 539 – Fall 2024 — Activity Solution 22

Problem 1 Solution

Given:

$$A = \begin{bmatrix} 1 & 5 & -2 & 4 \\ -2 & 2 & 4 & -1 \\ 1 & -2 & 3 & -3 \\ 6 & 3 & 6 & 1 \end{bmatrix}, \quad K = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

(a) Convolution of A with K (Stride = 1, No Padding)

1. Position (1,1): $(1 \cdot 1) + (5 \cdot 0) + (-2 \cdot 0) + (2 \cdot -1) = 1 - 2 = -1$
2. Position (1,2): $(5 \cdot 1) + (-2 \cdot 0) + (2 \cdot 0) + (4 \cdot -1) = 5 - 4 = 1$
3. Position (1,3): $(-2 \cdot 1) + (4 \cdot 0) + (4 \cdot 0) + (-1 \cdot -1) = -2 + 1 = -1$
4. Position (2,1): $(-2 \cdot 1) + (2 \cdot 0) + (1 \cdot 0) + (-2 \cdot -1) = -2 + 2 = 0$
5. Position (2,2): $(2 \cdot 1) + (4 \cdot 0) + (-2 \cdot 0) + (3 \cdot -1) = 2 - 3 = -1$
6. Position (2,3): $(4 \cdot 1) + (-1 \cdot 0) + (3 \cdot 0) + (-3 \cdot -1) = 4 + 3 = 7$
7. Position (3,1): $(1 \cdot 1) + (-2 \cdot 0) + (6 \cdot 0) + (3 \cdot -1) = 1 - 3 = -2$
8. Position (3,2): $(-2 \cdot 1) + (3 \cdot 0) + (3 \cdot 0) + (6 \cdot -1) = -2 - 6 = -8$
9. Position (3,3): $(3 \cdot 1) + (-3 \cdot 0) + (6 \cdot 0) + (1 \cdot -1) = 3 - 1 = 2$

Thus, the resulting matrix $A * K$ is:

$$A * K = \begin{bmatrix} -1 & 1 & -1 \\ 0 & -1 & 7 \\ -2 & -8 & 2 \end{bmatrix}$$

(b) Convolution of A with K (Stride = 2, No Padding)

1. Position (1,1): $(1 \cdot 1) + (5 \cdot 0) + (-2 \cdot 0) + (2 \cdot -1) = 1 - 2 = -1$
2. Position (1,2): $(-2 \cdot 1) + (4 \cdot 0) + (4 \cdot 0) + (-1 \cdot -1) = -2 + 1 = -1$
3. Position (2,1): $(1 \cdot 1) + (-2 \cdot 0) + (6 \cdot 0) + (3 \cdot -1) = 1 - 3 = -2$
4. Position (2,2): $(3 \cdot 1) + (-3 \cdot 0) + (6 \cdot 0) + (1 \cdot -1) = 3 - 1 = 2$

Thus, the resulting matrix $A * K$ with stride 2 is:

$$A * K = \begin{bmatrix} -1 & -1 \\ -2 & 2 \end{bmatrix}$$

Problem 2 Solution

Given:

$$W = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 10 & 10 & 0 & 0 & 10 & 10 & 10 & 0 \\ 0 & 10 & 0 & 10 & 0 & 0 & 10 & 0 & 10 & 0 \\ 0 & 10 & 10 & 10 & 0 & 0 & 10 & 10 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 0 \\ 0 & 0 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 0 \\ 0 & 0 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 0 \\ 0 & 0 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The Sobel operators are:

$$S_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad S_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

(a) Sobel X and Sobel Y Convolution Results

The convolution results with Sobel X (G_x) and Sobel Y (G_y) are given by:

$$G_x = \begin{bmatrix} 20 & 0 & -20 & -30 & 30 & 20 & 0 & -20 \\ 20 & 0 & -20 & -40 & 40 & 20 & 0 & -20 \\ 20 & 0 & -20 & -30 & 30 & 20 & 0 & -20 \\ 20 & 10 & -10 & -10 & 10 & 10 & 0 & -20 \\ 30 & 30 & 0 & 0 & 0 & 0 & 0 & -30 \\ 40 & 40 & 0 & 0 & 0 & 0 & 0 & -40 \\ 40 & 40 & 0 & 0 & 0 & 0 & 0 & -40 \\ 30 & 30 & 0 & 0 & 0 & 0 & 0 & -30 \end{bmatrix}$$

$$G_y = \begin{bmatrix} -20 & -20 & -20 & -10 & -10 & -20 & -20 & -20 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 20 & 20 & 20 & 10 & 10 & 20 & 20 & 20 \\ 20 & 10 & -10 & -30 & -30 & -10 & 0 & 0 \\ -10 & -30 & -40 & -40 & -40 & -40 & -40 & -30 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 30 & 40 & 40 & 40 & 40 & 40 & 30 \end{bmatrix}$$

(b) Analyzing Results

After calculating for the central and edge areas, we observe that high values in S_x and S_y convolutions will appear along regions with sharp intensity changes.

(c) Gradient Magnitude Calculation

The gradient magnitude at each location is calculated as:

$$|\nabla I| = \sqrt{G_x^2 + G_y^2}$$

The computed gradient magnitudes for each cell in W are as follows:

$$|\nabla I| = \begin{bmatrix} 28.28 & 20.00 & 28.28 & 31.62 & 31.62 & 28.28 & 20.00 & 28.28 \\ 20.00 & 0.00 & 20.00 & 40.00 & 40.00 & 20.00 & 0.00 & 20.00 \\ 28.28 & 20.00 & 28.28 & 31.62 & 31.62 & 28.28 & 20.00 & 28.28 \\ 28.28 & 14.14 & 14.14 & 31.6228 & 31.6228 & 14.1421 & 0.00 & 20.00 \\ 31.62 & 42.4264 & 40.00 & 40.0000 & 40.0000 & 40.0000 & 40.0000 & 42.4264 \\ 40.00 & 40.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 40.00 \\ 42.4264 & 42.4264 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 40.00 \\ 31.6228 & 42.4264 & 40.00 & 40.00 & 40.00 & 40.00 & 40.00 & 42.4264 \end{bmatrix}$$