

ECE/CS/ME 539 – Fall 2024 — Activity 2

ECE/CS/ME 539 Introduction to Artificial Neural Networks 9/6/24

Problem 1

- (a) Prove that the transpose of the transpose of any matrix is the matrix itself, that is:

$$(\mathbf{A}^\top)^\top = \mathbf{A}.$$

- (b) Given two matrices \mathbf{A} and \mathbf{B} , demonstrate that the transpose of their sum is equal to the sum of their transposes:

$$(\mathbf{A} + \mathbf{B})^\top = \mathbf{A}^\top + \mathbf{B}^\top.$$

- (c) For any square matrix \mathbf{A} , determine whether the matrix $\mathbf{A} + \mathbf{A}^\top$ is always symmetric. Can you establish this result by using only the conclusions from the previous two exercises?
- (d) Consider any square matrix \mathbf{D} . Show that $\mathbf{D}\mathbf{D}^\top$ is always symmetric, regardless of whether \mathbf{D} itself is symmetric.
- (e) Given three square matrices \mathbf{A} , \mathbf{B} , and \mathbf{C} , prove that if $\mathbf{A} + \mathbf{B} = \mathbf{C}$ and \mathbf{A} and \mathbf{B} are symmetric, then \mathbf{C} must also be symmetric.

Problem 2

Consider the figure 1. The line H can be described by a hyperplane equation:

$$H : \{\mathbf{x} | g(\mathbf{x}) = \mathbf{w}^T \mathbf{x} + b = 0\}$$

Suppose H intersects with the two axes at $(5, 0)$ and $(0, 4)$ respectively.

- (a) Find w , b . Note the solution is not unique. So find a solution such that $\|\mathbf{w}\| = 1$.
- (b) Recall that the distance from any point \mathbf{x} to the hyperplane H is given by

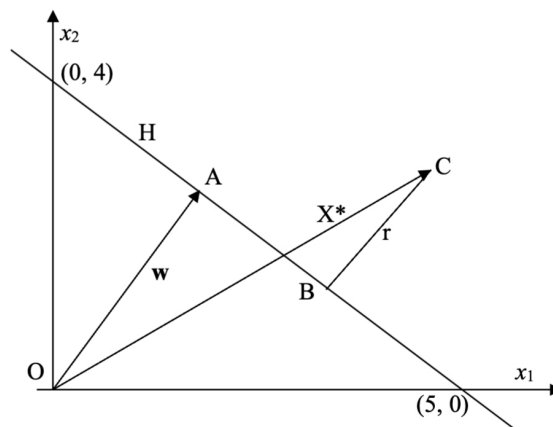


Figure 1: Problem 2

$$r = \frac{|\mathbf{w}^T \mathbf{x} + b|}{\|\mathbf{w}\|}$$

Prove this fact.

- (c) Suppose we restrict \mathbf{w} to be a unit vector ($\|\mathbf{w}\| = 1$). Using the equation in part b, show that the distance of the hyperplane to the origin (the length of the line segment OA) equals $|b|$.
- (d) Now, let point $C = (4.5, 3)$. Find r . The value of r may be positive or negative. What does the sign of r mean?