

Problem 1: Matrix-Matrix Multiplication for Movie Recommendation

(a) Stacking the Vectors:

To construct matrix Q , we convert the three user preference vectors q_1, q_2 , and q_3 into row vectors and stack them vertically:

$$Q = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Similarly, for matrix K , we convert the movie attribute vectors k_1, k_2, k_3 , and k_4 into row vectors and stack them vertically:

$$K = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

(b) Matrix-Matrix Multiplication:

Compute the score matrix S by multiplying the query matrix Q with the transpose of the key matrix K :

$$S = Q \cdot K^T$$

First, compute the transpose of K :

$$K^T = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

Now, perform the matrix multiplication:

$$S = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

Calculating each element of S :

$$S = \begin{bmatrix} (2 \cdot 1 + 0 \cdot 0 + 1 \cdot 2) & (2 \cdot 0 + 0 \cdot 1 + 1 \cdot 1) & (2 \cdot 2 + 0 \cdot 1 + 1 \cdot 0) & (2 \cdot 1 + 0 \cdot 2 + 1 \cdot 1) \\ (1 \cdot 1 + 2 \cdot 0 + 0 \cdot 2) & (1 \cdot 0 + 2 \cdot 1 + 0 \cdot 1) & (1 \cdot 2 + 2 \cdot 1 + 0 \cdot 0) & (1 \cdot 1 + 2 \cdot 2 + 0 \cdot 1) \\ (0 \cdot 1 + 1 \cdot 0 + 3 \cdot 2) & (0 \cdot 0 + 1 \cdot 1 + 3 \cdot 1) & (0 \cdot 2 + 1 \cdot 1 + 3 \cdot 0) & (0 \cdot 1 + 1 \cdot 2 + 3 \cdot 1) \end{bmatrix}$$

Simplifying each element:

$$S = \begin{bmatrix} 4 & 1 & 4 & 3 \\ 1 & 2 & 4 & 5 \\ 6 & 4 & 1 & 5 \end{bmatrix}$$

(c) Interpretation:

The matrix S contains the predicted preference scores. Each entry S_{ij} represents the score that User i would give to Movie j . Higher scores indicate a stronger preference.

(d) Specific Entries:

1. S_{11} : The score User 1 would give to Movie 1 is 4. 2. S_{23} : The score User 2 would give to Movie 3 is 4. 3. S_{32} : The score User 3 would give to Movie 2 is 4. 4. S_{34} : The score User 3 would give to Movie 4 is 5.

(e) Update: When a user's preference vector is changed, it directly alters the corresponding row in the score matrix S . The new scores for all movies for that user will be recalculated based on their updated preferences.

If two users have identical preferences, their scores for all movies would be the same, leading to identical rows in the score matrix S . This would affect the recommendation by giving similar suggestions to users with similar preferences.