ECE/CS/ME 539 – Fall 2024 — Homework 7

(6 pts) Problem 1

Consider a 2D feature space (x_1, x_2) as shown in 1. The decision region labeled with 1 is the triangle ABC shaded in yellow. The rest will be labeled with 0. Let the 2D coordinate of the point A be (x_{a1}, x_{a2}), the point B be (x_{b1}, x_{b2}), and the point C be (x_{c1}, x_{c2}).

Recall that the line equation between two point (a, b) and (c, d) can be found as follows:

First, we write down the line (hyperplane) equation: $w_0+w_1x_1+w_2x_2 = 0$. Then we substitute the coordinates of these two points into the equation: $w_0+a\cdot w_1+b\cdot w_2 = 0$, and $w_0+c\cdot w_1+d\cdot w_2 = 0$. Or in the matrix formulation, one has

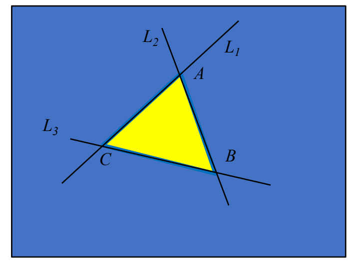

Figure 1: Triangle ABC with lines L_1, L_2, and L_3.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = -w_0 \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = -w_0 \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = -w_0 \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{-w_0}{ad-bc} \begin{bmatrix} d-b \\ a-c \end{bmatrix}$$

1. (2 pts) Given that the coordinates of point A: (5, 8), B: (6, 4), C: (3, 5), use above formula, derive the line equations for L_1, L_2, and L_3. To simplify the problem, let's assume $w_0 = -1$.

2. (2 pts) The interior of the triangle ABC is shaded in yellow. One may write a line equation $g(x_1, x_2) = w_0 + w_1x_1 + w_2x_2 = 0$

   The interior of the triangle is the intersection of 3 half-planes formed by these three lines. These half planes corresponding to an inequality $g(x_1, x_2) > 0$ or $g(x_1, x_2) < 0$. Determine the 3 half planes whose intersection form the ABC triangle. Hint: for L_1, substitute the coordinate of point B into the corresponding $g(x_1, x_2)$ and observe the sign. The same is true for other two lines.

3. (2 pts) Sketch a 2-layer MLP where each neuron has a threshold activation function. This MLP has 2 inputs x_1, x_2, three hidden units whose weights correspond to the 3 straight lines identified in part (2), and one output unit whose function is to AND the three hidden unit outputs and produce the final output.

(4 pts) Problem 2

Develop a MLP to classify the quality of white wine using the dataset winequality-white.csv. It has 7 class labels (last column): 3, 4, 5, 6, 7, 8, 9. You should convert the labels into one-hot encoding. Also, you should perform feature normalization before applying them to the MLP. Partition the data (Holdout, stratified) into 60/20/20 partitions where 60% are for training, 20% are for validation and the remaining 20% are for testing (and should never be used during training).

(4 pts) Problem 3

Let $\{x_m;\ 1 \le m \le M,\ x_m \in \{0,1\}\}$ be a set of m binary-valued inputs to a perceptron model that has a threshold activation function. Find an $M+1$ by 1 weight vector $\mathbf{w} = [w_0\ w_1\ \ldots\ w_M]$ to implement each of the following Boolean functions. Do not forget the bias input is a constant 1 and all $M+1$ entries of $\mathbf{w}$ need to be specified in each case. Note that since the answer is NOT unique, it helps to justify your answer.

1. $y = x_1 \wedge x_2 \wedge x_3 \wedge \cdots \wedge x_M$ where $\wedge$ is the logical AND operator.
   Hint: $y = 1$ only if **all** $x_k = 1$. Otherwise, $y = 0$.
2. $y = x_1 \vee x_2 \vee x_3 \vee \cdots \vee x_M$ where $\vee$ is the logical OR operator.
   Hint: $y = 1$ if **any** of the inputs $x_k = 1$. $y = 0$ only if **all** the inputs $x_k = 0$.
3. Let $M = 1$, $y = \neg x_1$.
   Hint: If $x_1 = 0$, then $y = 1$, and if $x_1 = 1$, then $y = 0$.
4. $y = 1$ if more than $M/2$ of x_m equal to 1 (assuming M is an odd number). Otherwise, $y = 0$. (This is a majority vote function.)

(4 pts) Problem 4

Consider the 2-layer perceptron shown in the figure 2, where the weights and biases are labeled. Assume all neurons use the sigmoid activation function. With the help of the provided starter code, implement this feedforward neural network.

1. Start by specifying the weight matrices and biases.
2. Implement both the hidden and output layer, in order to compute the model output for each datapoint specified in matrix X.
3. Visualize all three hidden neurons (after the sigmoid activation) as well as the outputs using a 3D surface plot. Note that the starter code defines the input X to be a grid in the 2D plane. The surface will show the output of each neuron at different points in the 2D plane. How do the directions of increase of the hidden neurons relate to the weight matrix?
4. After inspecting the visualization for the output neuron, we observe that the output for the datapoint $x = (1, -1)$ is low (compared to other datapoints). Propose a change of the model weights that would increase the model output at that point. Verify your proposal by running the model with the different weights.

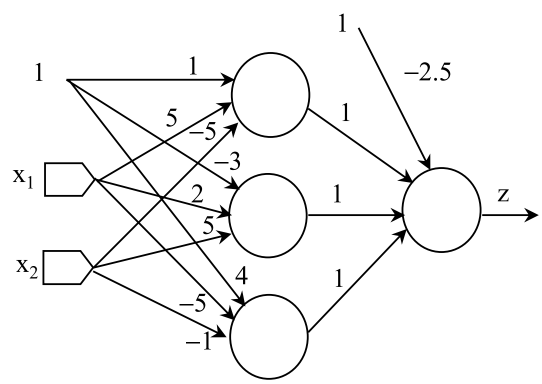

Figure 2: Problem 4