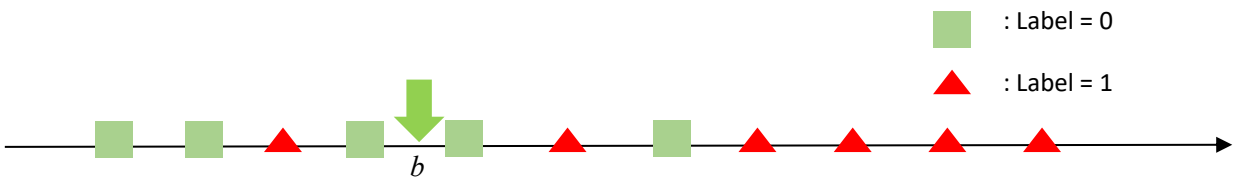
ECE/CS/ME 539 – Fall 2024 — Activity Solution 7

1 Performance Metrics

Consider the same set of posterior probabilities estimated by a classifier for a 2-class pattern classification problem. Assume now the threshold b is at a different value. Any data point to the left of b is predicted to have label 0, while any point to the right is predicted to have label 1. Manually evaluate the following quantities (represent fractional values in the form of fractions, not decimal numbers):



TP = 5 FN = 1 FP = 2 TN = 3

Sensitivity = 5/6 Specificity = 3/5

Pr. Miss = 1/6 Pr. False Alarm = 2/5

Precision = 5/7 Recall = 5/6

2 Calculating AUROC by Hand

In this exercise, you will calculate the Area Under the Receiver Operating Characteristic Curve (AUROC) for a simple binary classification model. AUROC is a common metric used to evaluate the performance of a classifier, which measures the ability of the model to distinguish between positive and negative classes.

Step 1: Compute the Classifier Scores

For each point, compute the classifier score $f(x_1, x_2)$:

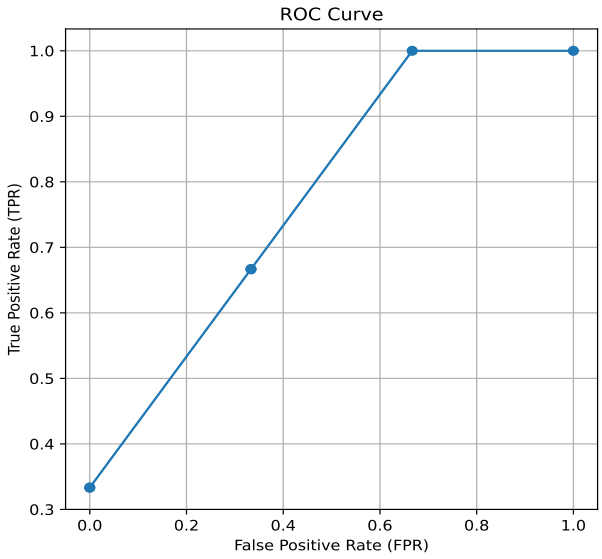
Point	x_1	x_2	True Label	$f(x_1, x_2)$
1	0.5	0.5	Negative	1.0
2	1.0	0.5	Negative	1.5
3	1.0	2.0	Negative	3.0
4	1.0	1.0	Positive	2.0
5	2.5	1.0	Positive	3.5
6	3.0	2.0	Positive	5.0

Step 2: Determine the TPR and FPR at Different Thresholds

Using the classifier scores, calculate the TPR and FPR for the following thresholds: $5.0, 4.0, 3.0, 2.5, 1.5, 1.0$.

Threshold	TPR (Sensitivity)	FPR
5.0	1/3	0.0
4.0	1/3	0.0
3.0	2/3	1/3
2.5	2/3	1/3
1.5	1.0	2/3
1.0	1.0	1.0

Step 3: Plot the ROC Curve



Step 4: Calculate the AUROC by Hand

$$\text{AUROC} = 1.0 - \frac{1}{2} \times \frac{2}{3} \times \frac{2}{3} = 0.778$$

Step 5: Interpolation

This linear interpolation is justified because we consider that a classifier can output a continuous probability score for each instance instead of a hard classification.

A randomized classifier can operate on a point on the straight line: A classifier can be designed to operate at any point on the straight line by employing a randomized strategy. Suppose a classifier at threshold T_1 corresponds to point A on the ROC curve, and a classifier at threshold T_2 corresponds to point B . By randomly choosing between these two thresholds with some probability p (such that the classifier uses threshold T_1 with probability p and threshold T_2 with probability $1 - p$), the classifier's operating point will lie on the straight line connecting points A and B .

Conclusion

The AUROC is a measure of how well your classifier distinguishes between positive and negative classes. A higher AUROC indicates better performance.