ECE/CS/ME 539 – Fall 2024 — Homework 1

1. Exercises from D2L:

- **(3 points total)**: In D2L section 2.3.13 Exercises, do problems 4, 5, and 6.

2. Matrix and Vector Operations:

- **(4 points total)**

  (a) **(1 point)**: Let $a = [1, -2, 3, 2]^T$ and $b = [2, -3, 1, -1]^T$ be two column vectors. Write a Python program to compute:

  $$c = ab^T \quad \text{and} \quad d = a^T b$$

  and provide their corresponding dimensions.

  (b) **(1 point)**: Given matrices:

  * $A = [a_{ij}]$ is a 3×2 matrix,
  * $D = [d_{ij}]$ is a 2×2 diagonal matrix,
  * $B = [b_{ij}]$ is a 2×4 matrix.
  * $E =$ ADB

  Show that

  $$E = \sum_{i=1}^{2} d_{ii} a_i b_i^T,$$

  where a_i is the i-th column of matrix A, and b_i^T is the i-th row of matrix B.

  (c) **(1 point)**: Let $a = [0, 1, 2, \ldots, 19]$ be a 1×20 row vector. Write a Python program to reshape this vector into a 5×4 matrix A.

  (d) **(1 point)**: Using matrix A from part (c), write a Python program to compute the Hadamard product $C = A \odot A$.

3. Tensor Operations:

- **(3 points total)**

  (a) **(1 point)**: Given a row vector $b = [0, 1, 2, \ldots, 23]$, write a Python program to reshape it into a $2 \times 3 \times 4$ tensor, denoted as B.

  (b) **(1 point)**: Use Python to compute the sum of all elements in the tensor B from part (a).

  (c) **(1 point)**: Since B is a $2 \times 3 \times 4$ tensor, it consists of two 3×4 matrices C and D. Write a Python program to find matrices C and D.

4. Matrix properties:

Consider a matrix A as shown below. Write a program to answer the problems.

$$A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \\ 3 & -1 \end{pmatrix}$$

- **(3 points total)** Write a code to evaluate:
  (a) **(1 point)**: The rank of A.
  (b) **(1 point)**: The singular value decomposition of A such that $A = U\Sigma V^T$. Find the value of K, and corresponding $\{\sigma_i, u_i, v_i; 1 \leq i \leq K\}$. What are the dimensions of matrices U, Σ, and V?
  (c) **(1 point)**: Now compute $B = AA^T$, and the eigenvalue decomposition of $\mathbf{B} = \mathbf{W}\Lambda\mathbf{W}^T = \sum_{i=1}^{M} \lambda_i \mathbf{w}_i \mathbf{w}_i^T$. Find M, $\{\lambda_i, w_i; 1 \leq i \leq M\}$.