ECE/CS/ME 539 – Fall 2024 — Activity 3

1.

Outcomes of tossing a coin with a Head (H) side and a Tail (T) side leads to a Bernoulli distribution: Let p be the probability of showing a Head (outcome $x = 1$).

$$P_X(x) = \begin{cases} p & \text{if } x = 1, \\ 1 - p & \text{if } x = 0. \end{cases}$$

If one repeatedly tosses the same coin N times, and denote the outcomes to be $x_1, x_2, \ldots, x_N$, where $x_i = 1$ if outcome = Head, and $x_i = 0$ if outcome = Tail. Thus, each x_i is a random variable whose value (0 or 1) is a function of the outcome (Head or Tail).

(a) Compute the expectation of x_i. Hint: It should be a function of p.

(b) The probability p may be estimated empirically as a function of $\{x_i\}$. This estimate can be expressed as:

$$\hat{p}(x_1, \ldots, x_N) = \frac{1}{N} \sum_{i=1}^{N} x_i$$

Show that this is an unbiased estimate. Namely,

$$E[\hat{p}(x_1, \ldots, x_N)] = p$$

(c) Define an event A as all the experiments where at least one of the 20 trials yields a Head. Find an expression of $p(A)$ in terms of p.

2.

Consider a Random Variable X with a Uniform PDF:

$$f(x) = \begin{cases} \frac{1}{b-a}, & \text{if } a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}$$

Find the mean $\mu_X = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$ and variance $\sigma_X^2 = \int_{-\infty}^{\infty} (x - \mu_X)^2 \cdot f(x)\, dx$

3.

Consider a Regular Deck of 52 Playing Cards. A regular deck contains 4 different suits of cards (spades, clubs, hearts, and diamonds), and 13 different cards within each suit (2 through 10, jack, queen, king, and ace).

(a) If you draw a card and it's a red suit (hearts or diamonds), what is the probability that it's a king?

(b) If you draw a card and it's a black suit (spades or clubs), what is the probability that it's an ace?

(c) Are the events "card is red" and "card is a king" statistically independent? Justify your answer.