

## ECE/CS/ME 539 – Fall 2024 — Activity 13

1. Assume that we have some data  $x_1, \dots, x_n \in \mathbb{R}$ . Our goal is to find a constant  $b$  such that  $\sum_i (x_i - b)^2$  is minimized.
  - (a) Find an analytic solution for the optimal value of  $b$ .
  - (b) How does this problem and its solution relate to the normal distribution?
  - (c) What if we change the loss from  $\sum_i (x_i - b)^2$  to  $\sum_i |x_i - b|$ ? Can you find the optimal solution for  $b$ ?
2. Prove that the affine functions that can be expressed by  $\mathbf{x}^\top \mathbf{w} + b$  are equivalent to linear functions on  $(\mathbf{x}, 1)$ .
3. Recall that one of the conditions for the linear regression problem to be solvable was that the design matrix  $\mathbf{X}^\top \mathbf{X}$  has full rank.
  - (a) What happens if this is not the case?
  - (b) How could you fix it? What happens if you add a small amount of coordinate-wise independent Gaussian noise to all entries of  $\mathbf{X}$ ?
  - (c) What is the expected value of the design matrix  $\mathbf{X}^\top \mathbf{X}$  in this case?