ECE/CS/ME 539 – Fall 2024 — Activity 14

1.

(a) Given a logistic regression model with coefficients $\mathbf{w}$ and bias b, how does the model predict the probability of a sample with feature vector $\mathbf{x}$ to belong to the positive class, $P(Y = 1|\mathbf{x})$?

(b) Normally, a logistic regressor predicts the positive class if $P(Y = 1|\mathbf{x}) > 0.5$ or the negative class otherwise. Suppose that you are building a classifier for a problem, where the cost of a false negative is much larger than the cost of a false positive (e.g., a classifier to diagnose serious medical conditions). Should you increase or decrease the probability threshold p_{thr}?

(c) Show that, regardless of the probability threshold p_{thr}, the decision boundary $P(Y = 1|\mathbf{x}) = p_{thr}$ is linear; it defines a hyperplane in the feature space.

(d) Suppose each sample is represented by a 2D feature $\mathbf{x} = (x_1, x_2)$. Given a logistic regression model with coefficients $\mathbf{w}$ and bias b, write the equation for the decision boundary in the form $x_2 = mx_1 + c$, where the slope m and intercept c can be computed from $\mathbf{w}, b$, and p_{thr}.