ECE/CS/ME 539 – Fall 2024 — Activity 1

1. Matrix-matrix multiplication for movie recommendation

In this exercise, you will explore how matrix-matrix multiplication can be used in a movie recommendation system. Each user has certain preferences based on movie attributes, and each movie has its own set of attributes. By computing the inner product (i.e., alignment) between a user's preferences and a movie's attributes, we can predict how much the user will like a particular movie.

Imagine you have three users, each with their own set of preferences for various movie attributes such as "romantic", "adventuruous", and "mysterious". Additionally, there are four movies, each described by these same attributes.

The users' preferences (i.e., query vectors) and the movies' attributes (i.e., key vectors) are given as follows:

$$\text{User 1's preference.:} \quad \mathbf{q_1} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \quad \text{U2:} \quad \mathbf{q_2} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \text{U3:} \quad \mathbf{q_3} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

$$\text{Movie 1's attributes:} \quad \mathbf{k_1} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \text{M2:} \quad \mathbf{k_2} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad \text{M3:} \quad \mathbf{k_3} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \quad \text{M4:} \quad \mathbf{k_4} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

(a) **Stacking the Vectors:**

   (a) Construct the matrix $\mathbf{Q}$ by converting the three user preference vectors $\mathbf{q_1}$, $\mathbf{q_2}$, and $\mathbf{q_3}$ into row vectors and then stacking them vertically.

   (b) Do the same for $\mathbf{K}$.

(b) **Matrix-Matrix Multiplication:** Compute the score matrix $\mathbf{S}$ by multiplying the query matrix $\mathbf{Q}$ with the transpose of the key matrix $\mathbf{K}$. That is, compute:

$$\mathbf{S} = \mathbf{Q} \cdot \mathbf{K}^\top$$

Show all your steps.

(c) **Interpretation:** The matrix $\mathbf{S}$ contains the predicted preference scores. Each entry S_{ij} represents the score that User i would give to Movie j. Describe the meaning of each entry S_{ij} in this matrix.

(d) **Specific Entries:** Compute the following specific scores:

   (a) S_{11}: The score User 1 would give to Movie 1.

(b) S_{23}: The score User 2 would give to Movie 3.

(c) S_{32}: The score User 3 would give to Movie 2.

(d) S_{34}: The score User 3 would give to Movie 4.

(e) **Update:** Discuss how changing a user's preferences or a movie's attributes would impact the score matrix $\mathbf{S}$. What happens if two users have identical preferences? How does this affect the recommendation?