# ECE/CS/ME 539 – Fall 2024 — Activity 22

## Problem 1

Please solve the problem by hand.

Consider the $4 \times 4$ matrix

$$A = \begin{bmatrix} 1 & 5 & -2 & 4 \\ -2 & 2 & 4 & -1 \\ 1 & -2 & 3 & -3 \\ 6 & 3 & 6 & 1 \end{bmatrix}$$

and the $2 \times 2$ kernel

$$K = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

(a) Perform the convolution operation $A * K$ without padding and using a stride of 1.

(b) Perform the convolution operation $A * K$ with a stride of 2 (without padding).

## Exercise 2

Sobel operators are convolutional kernels used in image processing to detect edges by calculating the gradient magnitude of pixel intensity in horizontal and vertical directions. There are two Sobel operators, typically referred to as the Sobel X and Sobel Y operators:

$$S_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad S_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

Now, consider the $10 \times 10$ matrix $W$ as the image:

$$W = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 10 & 10 & 0 & 0 & 10 & 10 & 10 & 0 \\ 0 & 10 & 0 & 10 & 0 & 0 & 10 & 0 & 10 & 0 \\ 0 & 10 & 10 & 10 & 0 & 0 & 10 & 10 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 0 \\ 0 & 0 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 0 \\ 0 & 0 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 0 \\ 0 & 0 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Manually apply the Sobel X and Sobel Y operators to $W$. Calculate each convolution result by hand.

(b) Analyze the resulting matrices. There are some high values appear in both outputs. Can you explain why?

(c) Note that Sobel filters $S_x$ and $S_y$ approximate the gradient of the image intensities $I$ along the $x$- and $y$-directions. Compute the gradient magnitude $\|\nabla I\|_2 = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2}$ at all locations.