POLITECNICO DI MILANO

A Brief Review of Mono-dimensional Heat Transfer: Conduction and Convection

Ref: Y. A. Cengel, Heat Transfer, a practical approach

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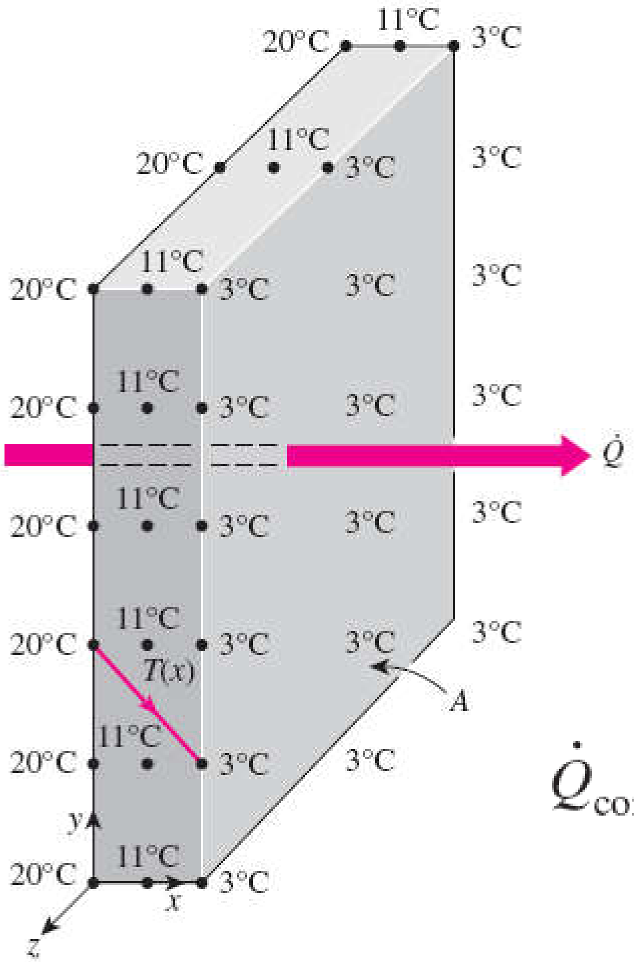
Steady State heat conduction in Plane wall

- Heat transfer through the wall of a house can be modeled as *steady* and *one-dimensional*.
- The temperature of the wall in this case depends on one direction only (say the *x*-direction) and can be expressed as $T(x)$.



$$\begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{into the wall}\end{pmatrix} - \begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{out of the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{of the wall}\end{pmatrix}$$

$$\dot{Q}_{in} - \dot{Q}_{out} = \frac{dE_{wall}}{dt}$$

for steady operation

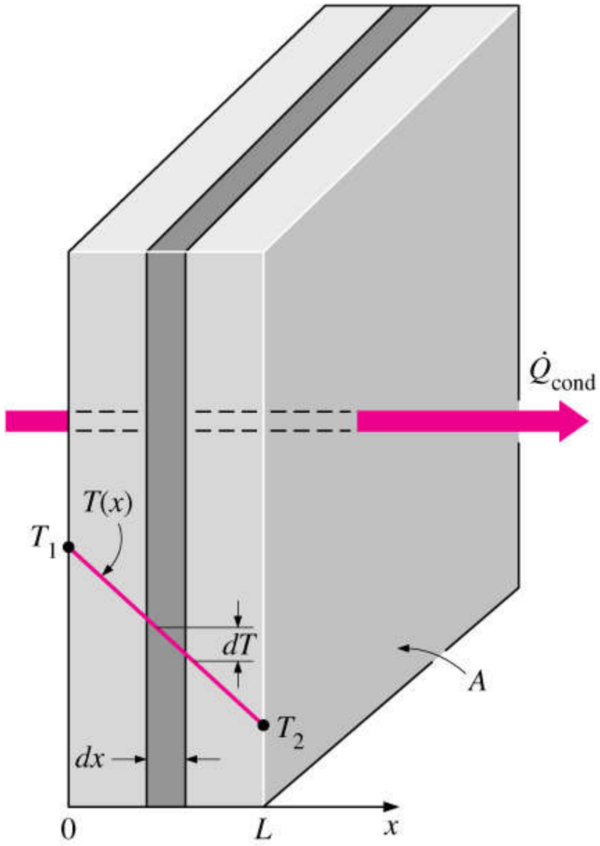
$$dE_{wall}/dt = 0$$

✓In steady operation, the rate of heat transfer through the wall is constant.

$$\dot{Q}_{cond,\ wall} = -kA\frac{dT}{dx} \qquad \text{(W)}$$

✓Fourier's law of heat conduction

Steady State heat conduction in Plane wall



$$\dot{Q}_{\text{cond, wall}} = -kA\frac{dT}{dx}$$

$$\int_{x=0}^{L} \dot{Q}_{\text{cond, wall}}\, dx = -\int_{T=T_1}^{T_2} kA\, dT$$

$$\dot{Q}_{\text{cond, wall}} = kA\frac{T_1 - T_2}{L} \qquad \text{(W)}$$

The rate of heat conduction through a plane wall:

✓ is proportional to the average thermal conductivity, the wall area, and the temperature difference

✓but is inversely proportional to the wall thickness.

✓Once the rate of heat conduction is available, the temperature $T(x)$ at any location x can be determined by replacing T_2 by T, and L by x.

✓Under steady conditions, the temperature distribution in a plane wall is a straight line: dT/dx = const.

Thermal Resistance Concept

$$\dot{Q}_{\text{cond, wall}} = kA\frac{T_1 - T_2}{L}$$

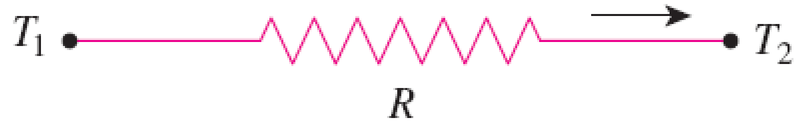
$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} \qquad \text{(W)}$$

$$R_{\text{wall}} = \frac{L}{kA} \qquad (\text{°C/W})$$

✓Conduction resistance of the wall: Thermal resistance of the wall against heat conduction.

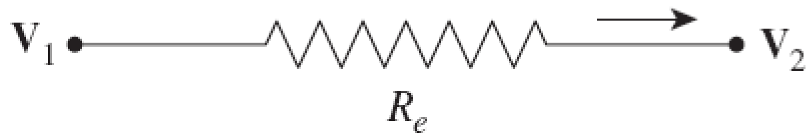
✓Thermal resistance of a medium depends on the geometry and the thermal properties of the medium.

$$\dot{Q} = \frac{T_1 - T_2}{R}$$



(a) Heat flow

$$I = \frac{\mathbf{V}_1 - \mathbf{V}_2}{R_e} \qquad R_e = L/\sigma_e A$$



(b) Electric current flow

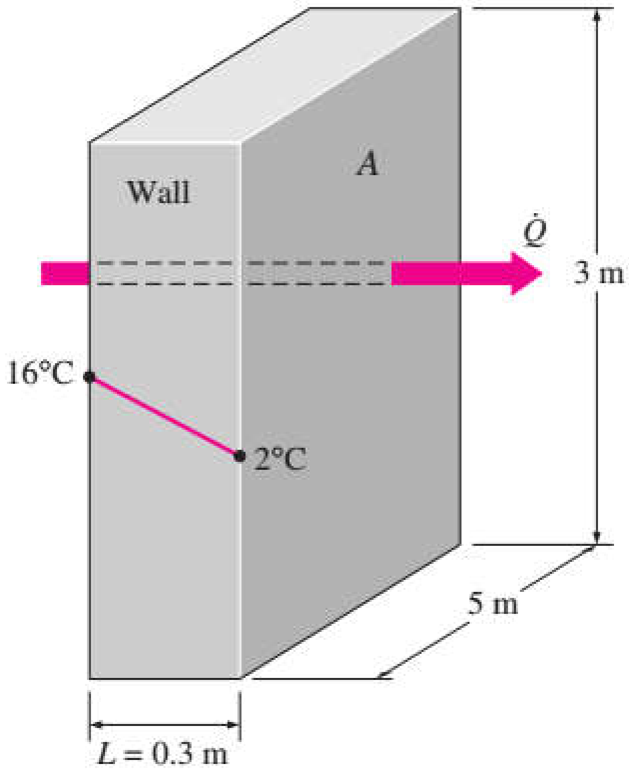
Analogy between thermal and electrical resistance concepts:

rate of heat transfer → electric current

thermal resistance → electrical resistance

temperature difference → voltage difference

Example A: Steady State heat conduction



- *Find the rate of heat loss through the wall*

✓K=0.9 W/m C

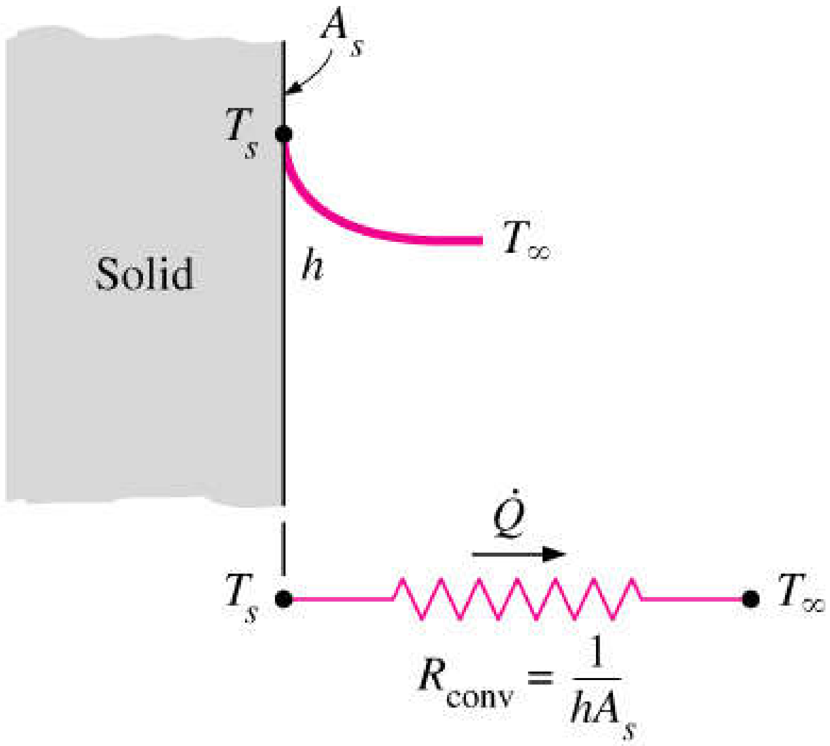
Convection: Newton's law of cooling

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$$

$$\dot{Q}_{conv} = \frac{T_s - T_\infty}{R_{conv}} \qquad \text{(W)}$$

$$R_{conv} = \frac{1}{hA_s} \qquad (°C/W)$$

Convection resistance of the surface: Thermal resistance of the surface against heat convection.



✓When the convection heat transfer coefficient is very large ($h \to \infty$), the convection resistance becomes *zero* and $T_s \approx T$.

✓That is, the surface offers *no resistance to convection*, and thus it does not slow down the heat transfer process.

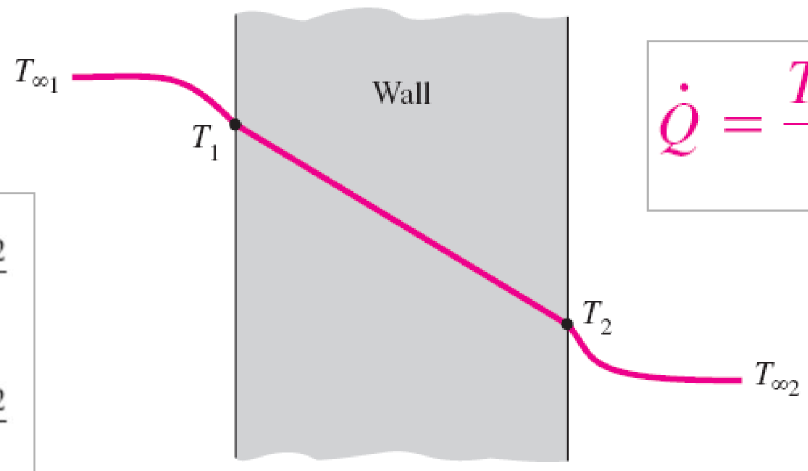
✓This situation is approached in practice at surfaces where boiling and condensation occur.

Thermal Resistance Network

$$\begin{pmatrix}\text{Rate of}\\ \textit{heat convection}\\ \text{into the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \textit{heat conduction}\\ \text{through the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \textit{heat convection}\\ \text{from the wall}\end{pmatrix}$$

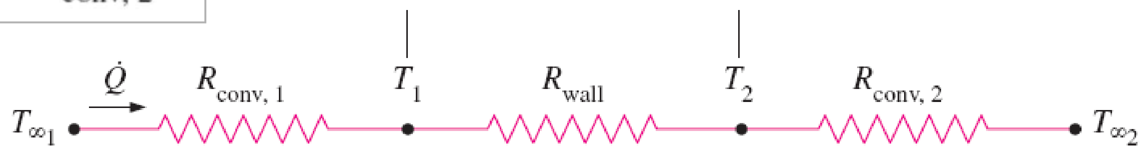
$$\dot{Q} = \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}$$

$$= \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} = \frac{T_1 - T_2}{R_{\text{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv, 2}}}$$



$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}}}$$



Thermal network

$$I = \frac{V_1 - V_2}{R_{e,1} + R_{e,2} + R_{e,3}}$$

Electrical analogy

The thermal resistance network for heat transfer through a plane wall subjected to convection on both sides, and the electrical analogy.

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \qquad (^\circ\text{C/W})$$

Temperature Drop

$$\Delta T = \dot{Q}R \qquad (°C)$$

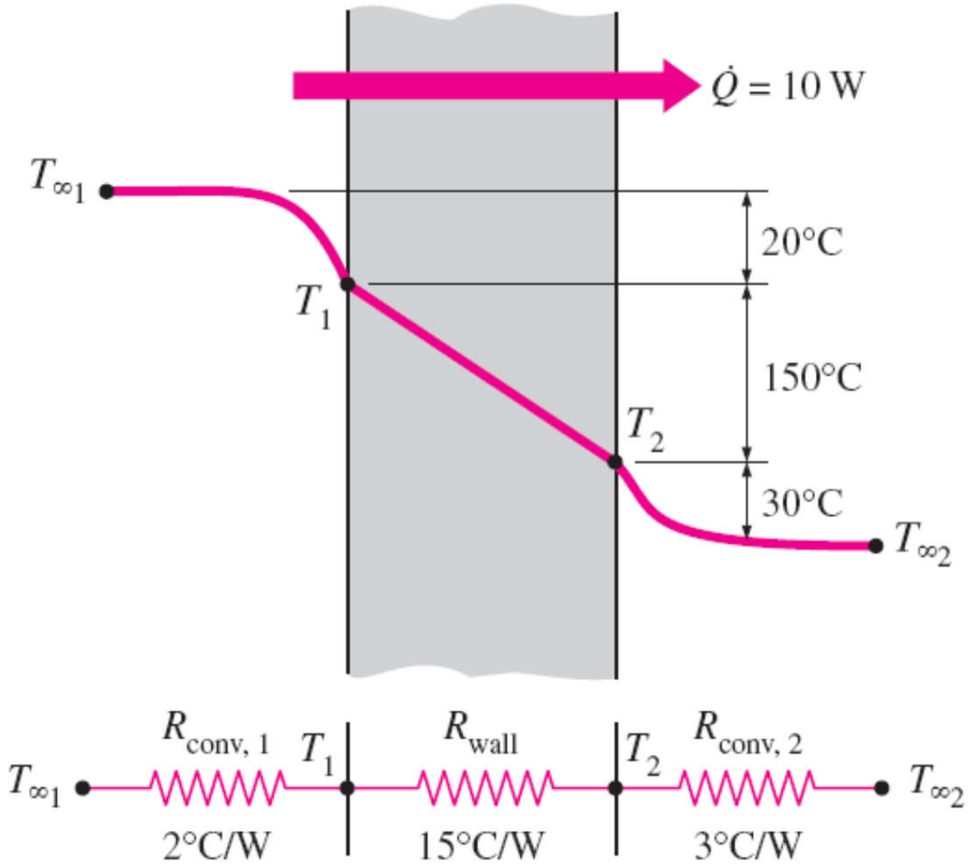
$$\dot{Q} = UA\,\Delta T \qquad (W)$$

$$UA = \frac{1}{R_{total}} \qquad (°C/K)$$

U: overall heat transfer coefficient

Once Q is evaluated, the surface temperature T_1 can be determined from

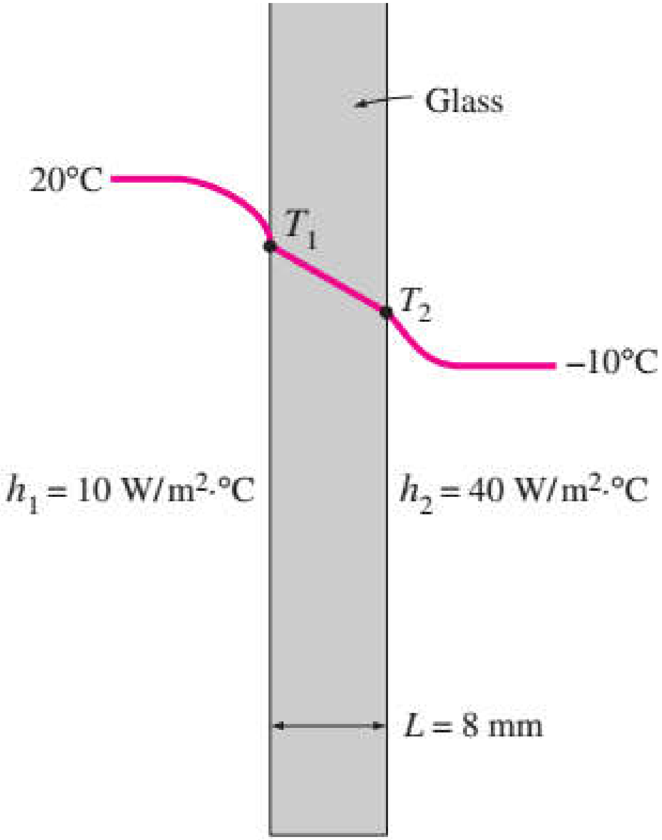
$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,\,1}} = \frac{T_{\infty 1} - T_1}{1/h_1 A}$$



$$\Delta T = \dot{Q}R$$

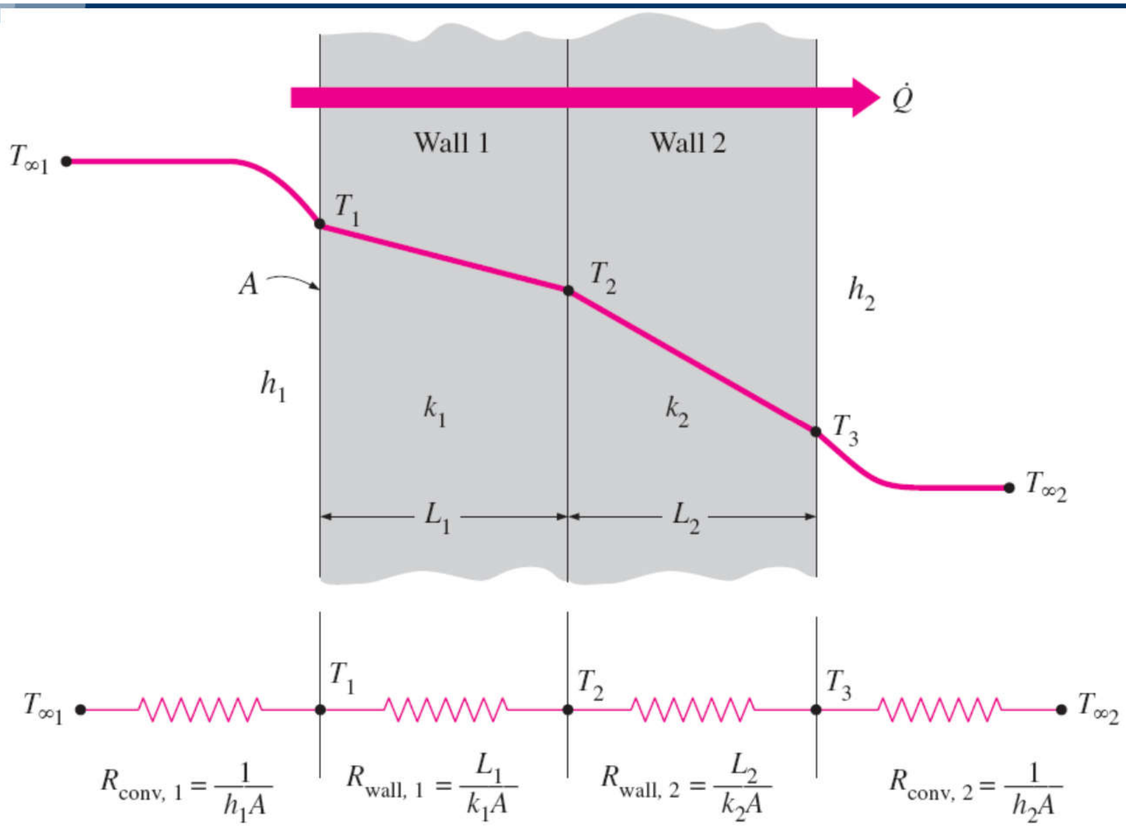
The temperature drop across a layer is proportional to its thermal resistance.

Example B: Heat loss through a single pane window



❖Consider a 0.8 m high and 1.5 m wide glass window, shown above with a thermal conductivity of k =0.78 W/m·°C. Determine the steady rate of heat transfer through this glass window and the temperature of its inner surface

Multi-layer Plane walls

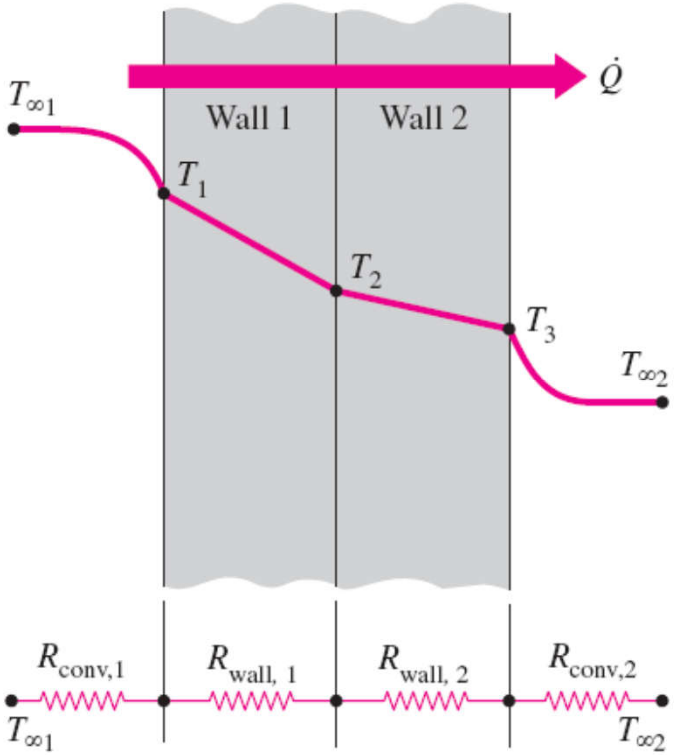


$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall, 1}} + R_{\text{wall, 2}} + R_{\text{conv, 2}}$$
$$= \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A}$$

$$\dot{Q} = \frac{T_i - T_j}{R_{\text{total},\, i-j}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv},\,1} + R_{\text{wall},\,1}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1 A} + \dfrac{L_1}{k_1 A}}$$



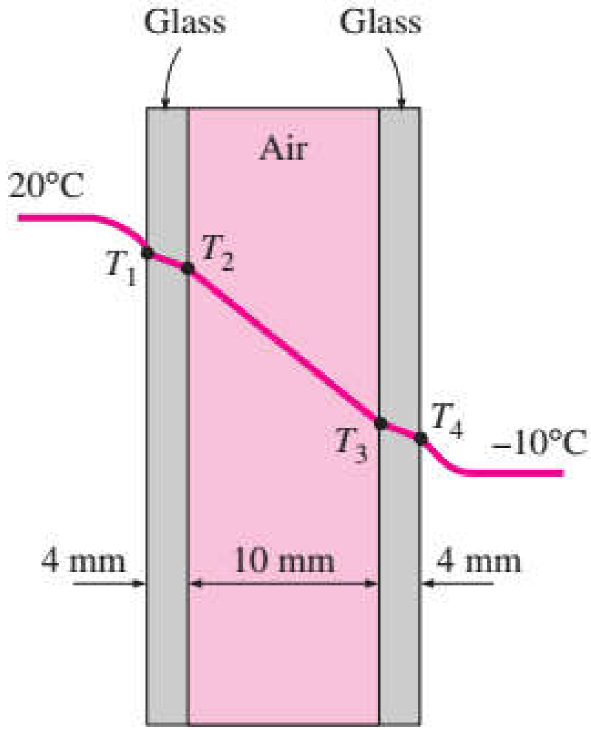
The evaluation of the surface and interface temperatures when $T_{\infty 1}$ and $T_{\infty 2}$ are given and $\dot{Q}$ is calculated.

To find T_1: $\dot{Q} = \dfrac{T_{\infty 1} - T_1}{R_{\text{conv},1}}$

To find T_2: $\dot{Q} = \dfrac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_{\text{wall},\,1}}$

To find T_3: $\dot{Q} = \dfrac{T_3 - T_{\infty 2}}{R_{\text{conv},2}}$

Example C: Heat loss through a double pane window



❖ Consider a 0.8-m-high and 1.5-m-wide double-pane window consisting of two 4-mm-thick layers of glass (k= 0.78 W/m.°C) separated by a 10-mm-wide stagnant air space (k= 0.026 W/m.°C). Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surface.

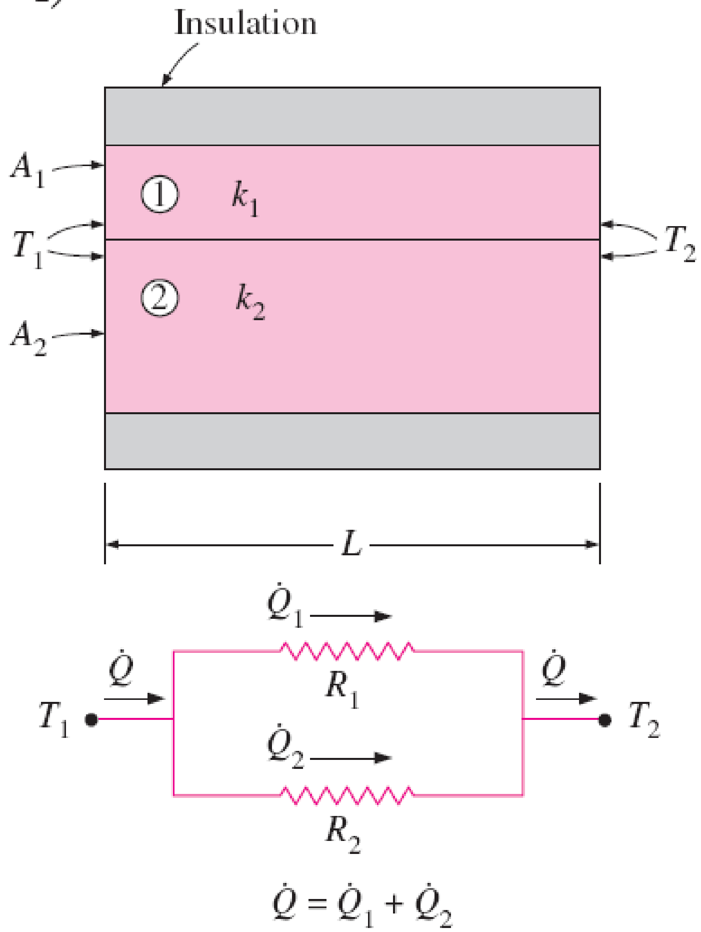
✓Take the convection heat transfer coefficients on the inner and outer surfaces of the window to be $h1$= 10 W/m2 · °C and $h2$=40 W/m2 · °C, which includes the effects of radiation.

Generalized Thermal Resistence Networks

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{total}}}$$

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} \longrightarrow R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2}$$



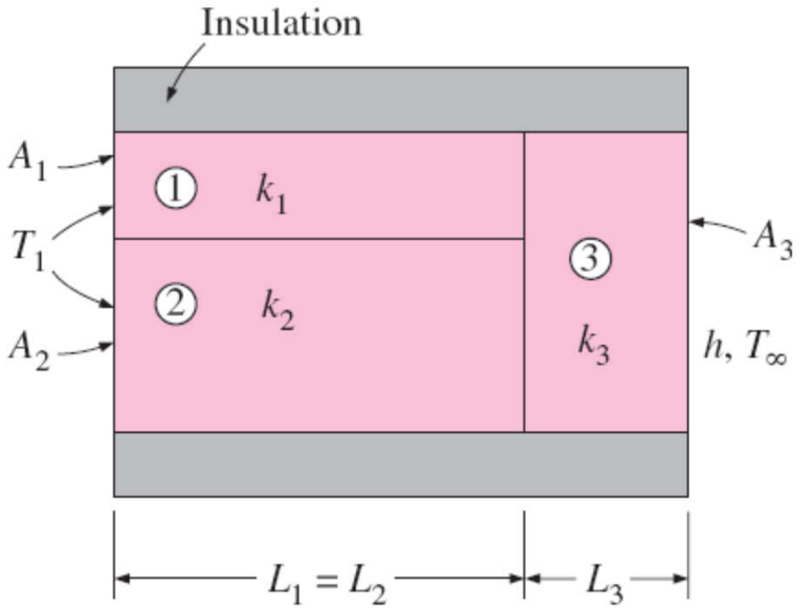
Thermal resistance network for two parallel layers.

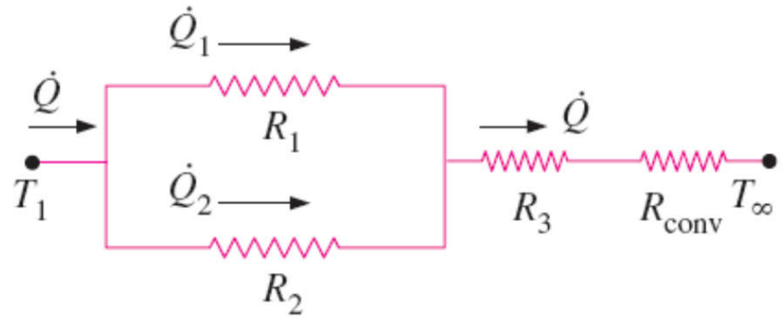
Generalized Thermal Resistence Networks

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}} \qquad R_1 = \frac{L_1}{k_1 A_1} \qquad R_2 = \frac{L_2}{k_2 A_2}$$

$$R_3 = \frac{L_3}{k_3 A_3} \qquad R_{conv} = \frac{1}{hA_3}$$

$$R_{total} = R_{12} + R_3 + R_{conv} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{conv}$$





Thermal resistance network for combined series-parallel arrangement.

- Two possible assumptions in solving complex multidimensional heat transfer problems by treating them as one-dimensional using the thermal resistance network are :

- ✓ any plane wall normal to the x-axis is *isothermal* (i.e., to assume the temperature to vary in the x-direction only)
- ✓ any plane parallel to the x-axis is *adiabatic* (i.e., to assume heat transfer to occur in the x-direction only)

Example D: Heat loss through a composite wall

- ✓ A 3 m high and 5 m wide wall consists of long 16 cm 22 cm cross section horizontal bricks (k =0.72 W/m · °C) separated by 3 cm thick plaster layers (k =0.22 W/m · °C).
- ✓ There are also 2 cm thick plaster layers on each side of the brick and a 3-cm-thick rigid foam (k 0.026 W/m · °C) on the inner side of the wall
- ✓ The indoor and the outdoor temperatures are 20°C and 10°C, and the convection heat transfer coefficients on the inner and the outer sides are $h1$=10 W/m2 · °C and $h2$ =25 W/m2 · °C, respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.