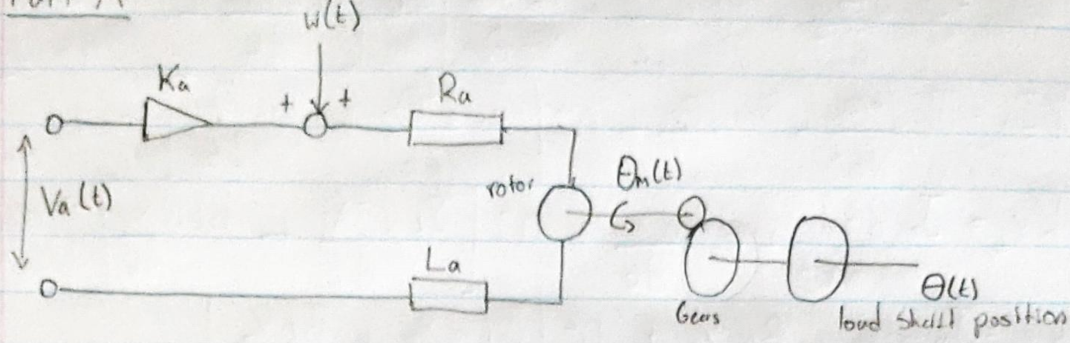
Lab 3: Preparatory Labwork

Part A

1.



x_1 = Angular position of the motor shaft
x_2 = " Velocity of the motor shaft
x_3 = Armature current (I_a)
u = V_a
y = Angular position of the load shaft

Apply KVL to the equation,

$$L_a \frac{dI_a}{dt} + R_a I_a + E_b = E_a$$ → We know E_a and E_b

$$L_a \frac{dI_a}{dt} + R_a I_a + \theta_m K_b = V_a(t) \cdot K_a$$ → put it in terms of x_3

$$L_a \frac{dx_3}{dt} + R_a x_3 + x_2 K_b = U \cdot K_a$$

$$\frac{dx_3}{dt} = -\frac{R_a x_3}{L_a} - \frac{x_2 K_b}{L_a} + \frac{U \cdot K_a}{L_a}$$ → solved in terms of x_3

By apply newtons law of motion,

$$T_m = J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} + T_1$$ → We know $T_2 = \frac{\theta_m}{\theta_l} T_1$

$$T_m = J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} + \frac{\theta_l}{\theta_m} T_2$$ → We know T_2

$$T_m = J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} + \frac{\theta_l}{\theta_m}\left[J_l \frac{d^2\theta_l}{dt^2} + B_l \frac{d\theta_l}{dt}\right]$$ → We know T_m and θ_l

$$KI_a = J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} + \frac{n\,\theta_m}{\theta_m}\left[n\left[J_L \frac{d^2\theta_m}{dt} + B_L \frac{d\theta_m}{dt}\right]\right]$$

$$KI_a = J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} + n^2\left[J_L \frac{d^2\theta_m}{dt} + B_L \frac{d\theta_m}{dt}\right] \rightarrow \text{put in terms of states}$$

$$Kx_3 = J_m \frac{d^2x_1}{dt^2} + B_m \frac{dx_1}{dt} + n^2\left[J_L \frac{d^2x_1}{dt} + B_L \frac{dx_1}{dt}\right] \rightarrow \frac{dx_1}{dt} = x_2$$

$$Kx_3 = J_m \frac{dx_2}{dt} + B_m x_2 + n^2\left[J_L \frac{dx_2}{dt} + B_L x_2\right]$$

$$Kx_3 = J_m \frac{dx_2}{dt} + B_m x_2 + n^2 J_L \frac{dx_2}{dt} + n^2 B_L x_2$$

$$\frac{dx_2}{xt} = \frac{K}{J_{eq}} x_3 - \frac{B_{eq}}{J_{eq}} x_2 \rightarrow \text{Simplified equation}$$

Since the derivative of velocity is position, we know that

$$\frac{dx_1}{dt} = x_2$$

$\therefore$ From the equations derived, we can put the values into state system form

$\rightarrow$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & -B_{eq}/J_{eq} & K/J_{eq} \\ 0 & -K/L & -R/L \end{bmatrix}}_{A} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix}}_{B} u$$

$$y = \underbrace{\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

2.

Using a sampling frequency of 150 Hz.

sysc = ss(A, B, C, D) -> file attached
sysd = c2d(sysc, 1/150) <- sampling period

By solving for the equivalent discrete-time ss model, the result is:

$$X_{k+1} = \Phi x_k + \Gamma u_k$$
$$y_k = C x_k + D u_k$$

$$\Phi = \begin{bmatrix} 1 & 0.005875 & 0.005595 \\ 0 & 0.7706 & 0.7402 \\ 0 & -0.001567 & -0.001506 \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} 0.1288 \\ 37.3 \\ 0.2581 \end{bmatrix}$$

$\rightarrow$

To check if the system is stable, we need to find the poles of the system:

System poles = eigenvalues of Φ

Using MATLAB;

$$\text{eig } \Phi = \begin{cases} 1.00 \\ 0.7691 \\ 0 \end{cases}$$

Not all the poles are inside the unit circle therefore the system is not stable.

To check if the system is controllable, we need to solve for the controllability matrix. The controllability matrix is:

$$W_c = \begin{bmatrix} \Gamma & \Phi\Gamma & \Phi^2\Gamma \end{bmatrix}$$

In this case, $\text{rank}(W_c) = 3$, but we can verify if the system is controllable if the $\det(W_c) \neq 0$. With MATLAB, $\det(W_c) = -2.4190$ $\therefore$ the system is controllable.

To check if the system is observable, we need to solve for the observability matrix. The observability matrix is:

$$W_o = \begin{bmatrix} C \\ C\Phi \\ C\Phi^2 \end{bmatrix}$$

$\rightarrow$

In this case, the rank $(W_o) = 3$ and the det $(W_o) = 2.17 \times 10^{-7}$
$\therefore$ Since the det$(W_o) \neq 0$, the system is therefore observable.

PART B

Discrete Time Model:

$$x_{k+1} = \Phi x_k + \Gamma u_k \qquad \text{where } x_k = [x_{1k}, x_{2k}, x_{3k}]^T$$
$$y_k = C x_k + D u_k$$

Design a feedback controller:

$$u_k = \bar{N} r_k - K x_k$$

w/ $PO \leq 15\%$ and Settling time: $T_s(1\%) \leq 0.25s$.

$$\zeta = \frac{-\ln(PO/100)}{\sqrt{\pi^2 + \ln^2(PO/100)}} = \frac{-\ln(15/100)}{\sqrt{\pi^2 + \ln^2(15/100)}} = 0.5169$$

$$T_s(1\%) \leq 0.25 \rightarrow \frac{4.6}{\zeta \omega_n} = 0.25 \rightarrow \frac{4.6}{(0.5169)\omega_n} = 0.25 \rightarrow \omega_n = 35.60$$

$\therefore$ The dominant CL poles are:

$$s_d = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = 18.4 \pm j30.465$$

$\longrightarrow$

However, we can use the Bessel transfer function table to solve for the pole locations

$\phi_{CL}(s)$ for 3rd order function is:

$\phi_{CL}(s) = (s+0.9420)(s+0.7455 \pm 0.7112j) \rightarrow$ for $\omega_n = 1$

$\hookrightarrow$ Put in terms of $\omega_n = 35.6$

$$
\begin{aligned}
s_d &= \omega_n \times (s+0.9420)(s+0.7455 \pm 0.7112j) \\
&= 35.6 \times (s+0.9420)(s+0.7455 \pm 0.7112j) \\
&= (s+33.5352)(s+26.5398 \pm 25.3187j)
\end{aligned}
$$

$$z_d = e^{s_d T} = (0.7997+0j)(0.8259 \pm 0.1407j)$$

$$\phi^d_{CL}(z) = [1.00 \quad -2.4515 \quad 2.0229 \quad -0.5613]$$

$$K_1 = [0.0401 \quad -0.0033 \quad -2.1844]$$

$$\bar{N} = 0.0401$$

$\longrightarrow$

Part C

The servo will have a non-zero steady state error due to the presence of a dc offset disturbance, w_k.

$\therefore \quad u_k = \bar{N} r_k - K\hat{x}_k - \hat{w}_k$

→ Since the disturbance is a dc offset, assume the disturbance to be constant. In this case, the OL system becomes the plant + integration and 4CL poles are required.

Therefore, the Bessel poles in Part B and an extra pole will be used at $-4\zeta\omega_n \rightarrow s_e = -52.9306. \rightarrow z_e = 0.7027$

(OL) System model = Plant + Integrator

$$\begin{bmatrix} x_{1,k+1} \\ x_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & -C \\ \bar{0} & \Phi \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_k \end{bmatrix} + \begin{bmatrix} 0 \\ \Gamma \end{bmatrix} u_k + \begin{bmatrix} 0 \\ \Gamma \end{bmatrix} w_k + \begin{bmatrix} 1 \\ \bar{0} \end{bmatrix} r_k$$

$$y_k = \begin{bmatrix} 0 & C \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_k \end{bmatrix}$$

$$\therefore \quad x^A_{k+1} = \Phi^A_k x^A_k + \Gamma^A u_k + \Gamma^A w_k + \Gamma^B r_k$$
$$y_k = C^A x^A_k$$

<u>Controller:</u>

$$u_k = \bar{N} r_k - K_A x^A_k \qquad K^A = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} (W_c^A)^{-1} \phi^d_{CL}(\Phi^A)$$

$$K^A = -0.015 \quad 0.2022 \quad 0.006 \quad -1.3349$$

$$K^A = [\underbrace{K_I}_{1\times1} \quad \underbrace{K}_{1\times3}]$$

→

There are a total of 3 possibilities for $\bar{N}$:

a) $\bar{N} = N_U + K N_x = 0.2022$

b) $\bar{N} = \emptyset$

c) $\bar{N} = \frac{K_I}{Z_e - 1} = 0.0401$