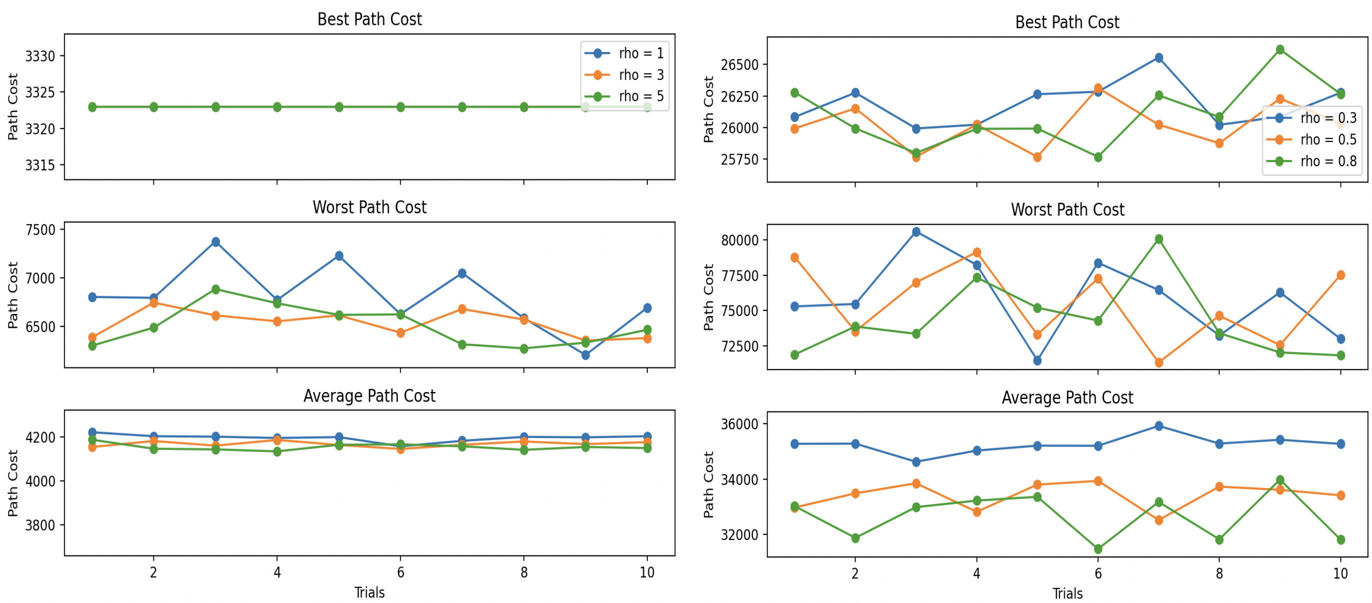
For our trials we will use the default values of:

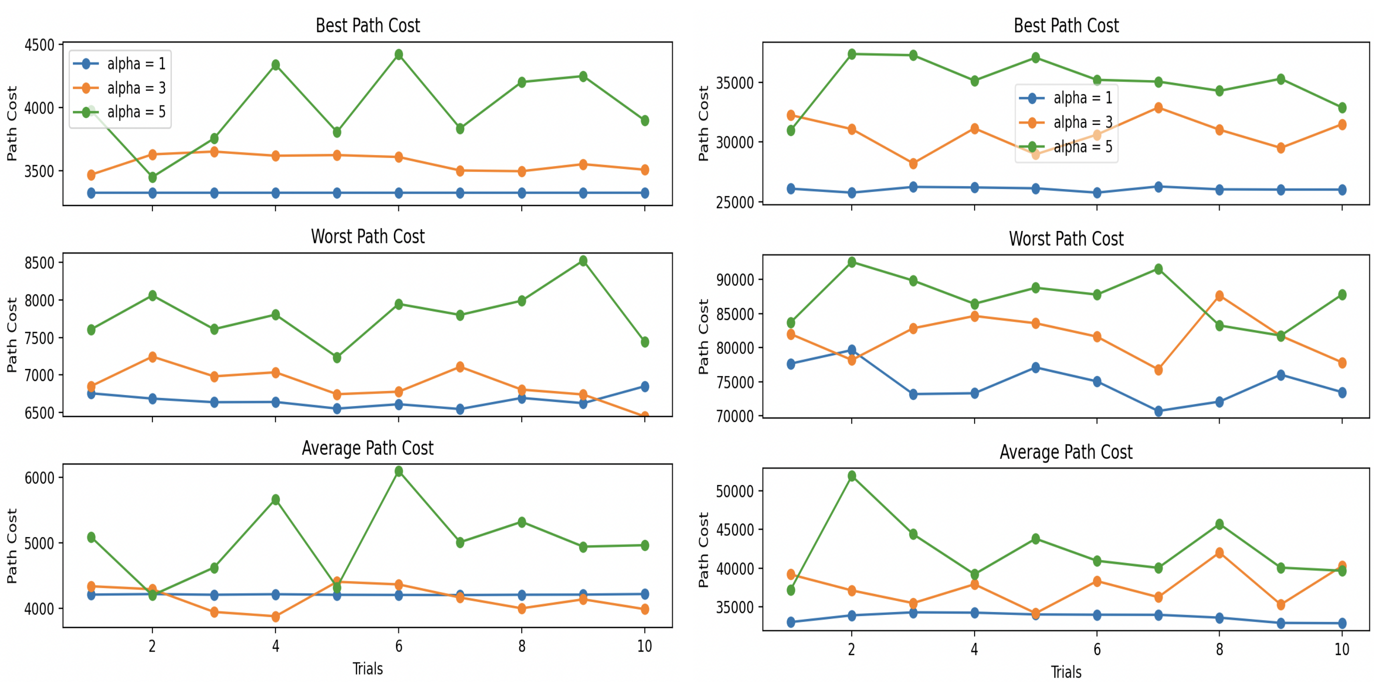
$$\rho = 0.5,\ \alpha = 1,\ \beta = 1,\ \text{k} = 50,\ \text{iterations} = 200,$$




(a) Burma results

(b) Brazil Results

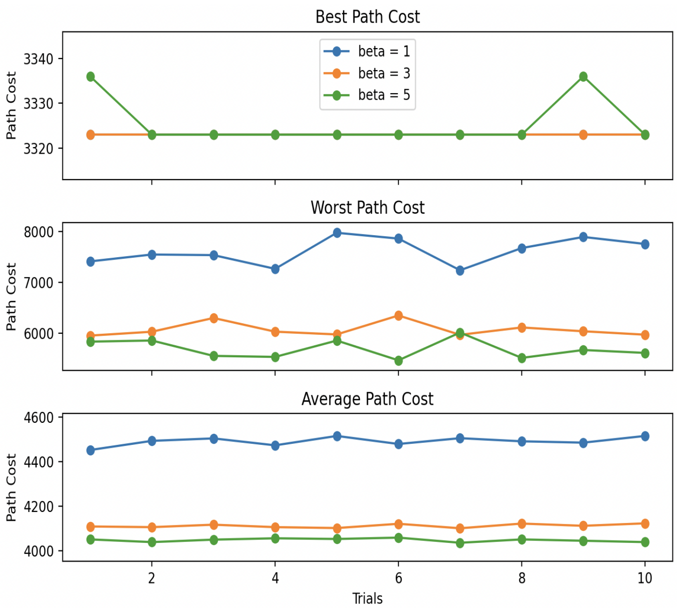
Figure 1: Results with our default values and adjusting the $\rho$ parameter



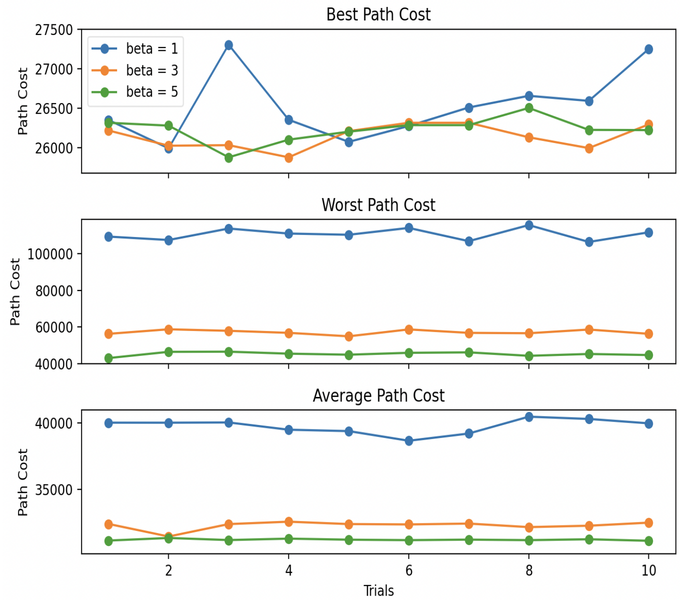

(a) Burma results

(b) Brazil Results

Figure 2: Results with our default values and adjusting the $\alpha$ parameter

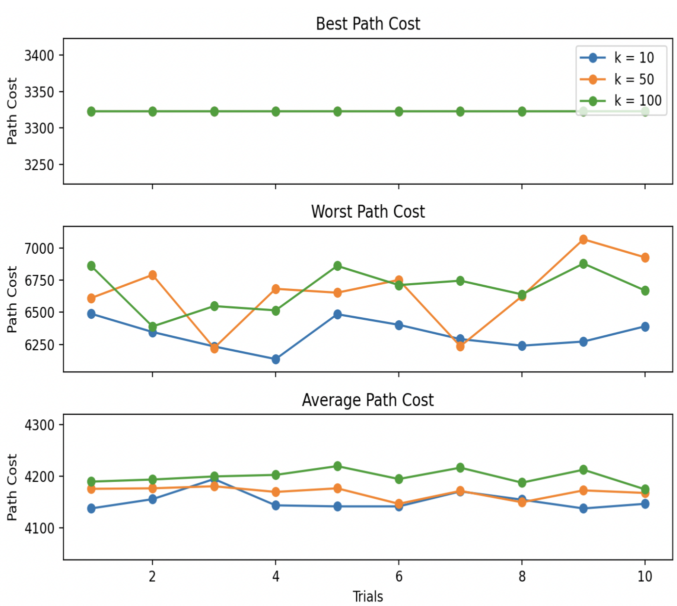



(a) Burma results

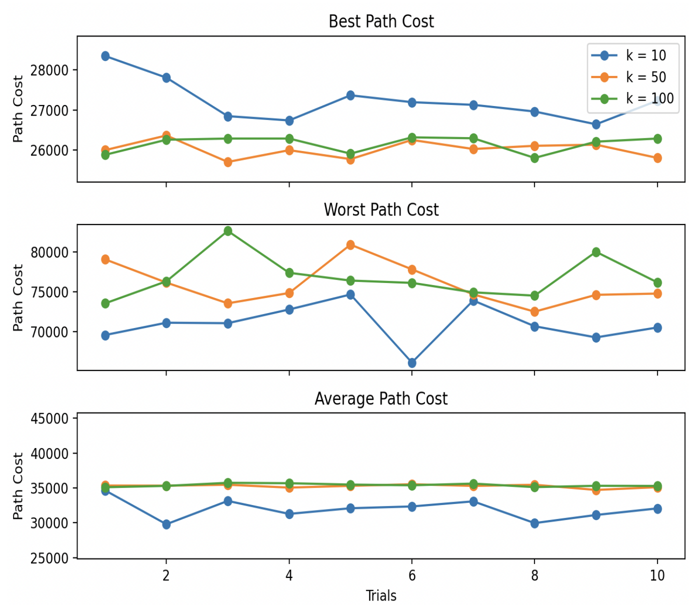



(b) Brazil Results

Figure 3: Results with our default values and adjusting the $\beta$ parameter

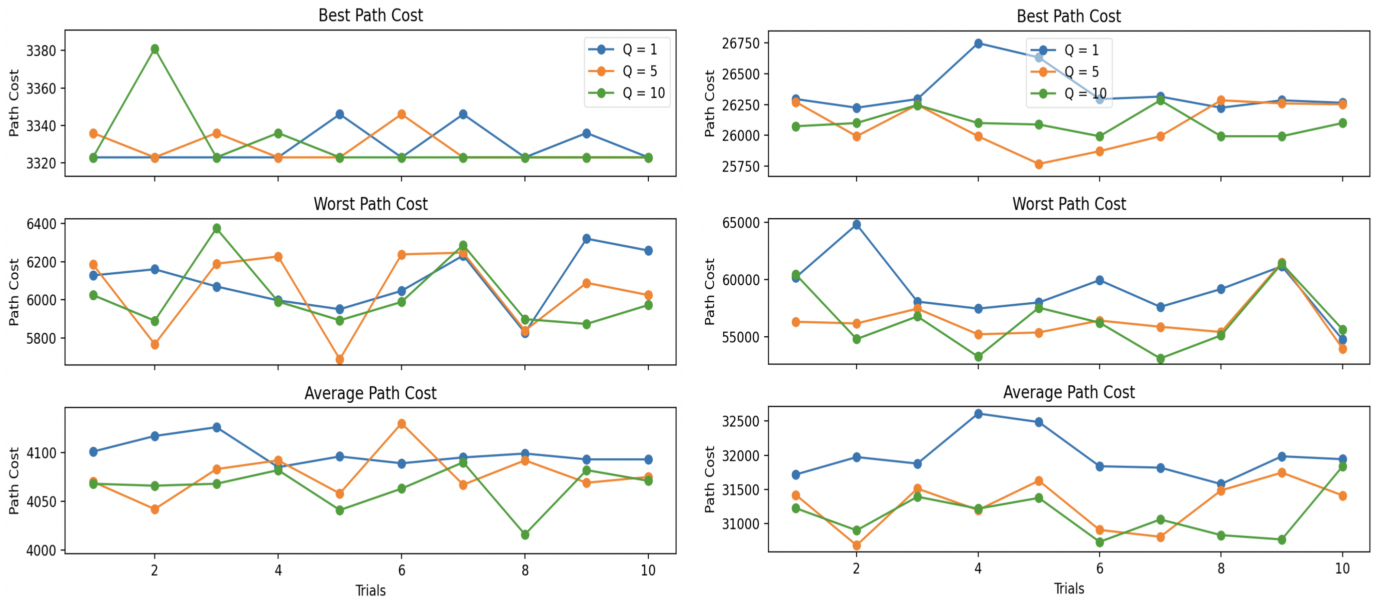



(a) Burma results




(b) Brazil Results

Figure 4: Results with our default values and adjusting the colony size parameter




(a) Burma results

(b) Brazil Results

Figure 5: Results with our default values and adjusting the local heuristic Q value

## Question 1

To find the best parameters we first explored the impact of each parameter on the algorithm, as seen in Figures 1-4. We then furthered this by cross-validating the parameters values to find the best combination. Based on our experimentation of parameter values we can conclude the best parameters were:

$$\rho = 0.5,\ \alpha = 1,\ \beta = 3,\ \text{k} = 50,\ \text{iterations} = 200,$$

## Question 2

The identified combination of parameter yielded the best results in our experiments. This was because a moderate $\rho$ allowed for the preservation of useful pheromone and allowing for exploration, this was particularly seen in Figure 1b. Our parameters also show a greater emphasis on exploration, as with a higher $\beta$ value. This was particular important when the problems size increased, like in Brazil. Having the favour on global exploration allowed for more paths to be explored by the ants in turn leading to a wider range and more consistent optimal set of solutions. Our parameters allowed us to have a good balance between exploitation and exploration.

## Question 3

$\rho$ was a determining factor for how fast the pheromones decay on each edge. A lower $\rho$ meant a slower pheromone decay rate, whereas a higher value increases pheromone decay rate. Having a lower value caused higher worse path costs and average costs as the ants were getting stuck in local optimas and not exploring other potentially better paths, this was clear in our results in Figure 1. The opposed of this was seen with a higher value where the ants were able to find better solutions as had the ability to explore alternative paths.

$\alpha$ influences the balance between pheromone information and heuristic information when picking the next city in the path, $\alpha$ is specifically for pheromone importance. A higher value of $\alpha$ causes historical pheromones to be more favoured over heuristic information. In Figure 2 it is clear that a higher put to more reliance on historical pheromones leading the algorithm to get stuck in local optimas more frequently consequently limiting exploration.

Similar to $\alpha$ $\beta$ is the importance of the heuristic information in the transition rule. A higher $\beta$ prioritises paths with better local heuristic characteristics, and the opposite for lower values. In Figure 3 it was clear having a $\beta$ of 1 caused the algorithm to get stuck in suboptimal solutions as didn't have the favour on exploration of the search space. The moderate value (5) of $\beta$ in both problems allowed for adequate exploration of the search space, whilst still allowing the algorithm to exploit paths already found with pheromones on.

Ant colony size plays a crucial role in the algorithm. Larger colonies allow for potentially more solution to be explored per iteration. In smaller problems larger colonies may not be advantage as paths are more frequently visited making the algorithm converge prematurely, as seen in Figure 4a. Smaller colonies allow for greater convergence as less paths are explored however this comes at the cost of potentially not exploring better paths in the search space.

## Question 4

In our experimentation's we explored the implementation of a simple heuristic Q/d. Varying the value of Q provides more weight to the emphasis on the actual cost of the path, when selecting the next city in the path. The use of the heuristic was more effective with a value of 5, showing a clear improvement in performance compared to a value of 1 seen in Figure 5, which represents the algorithm's performance with the best parameters found in Question 1. A further experiment was conducted using a local search method of hill climbing. However, due to its increased computational expense, especially for larger problems, it did not yield enough significance to warrant the overhead it caused.

## Question 5

The variation we explored was the Elitist ACO variation. This entails designating a specific number of ants as elitist, and due to their elitist status, they deposit a higher amount of pheromones on their paths, making these paths more desirable. After experimenting with this variation, it became evident that a larger colony size was more effective than our initially proposed parameters, and in our experiments, we used a colony size of 100 for this reason. Our results exploring 10%, 15% and 20% of elitist ants demonstrated that having a moderate number of elitist ants, 15%, was best suited as it shown a clear balance between the exploitation of elitist paths and the ability to explore other paths, resulting in a diverse set of solutions. This approach also helped avoid early convergence, a phenomenon observed with a 20% elitist rate. For smaller problems like Burma, this variation didn't yield significant improvements on a variation without it, but for larger problems such as Brazil, it consistently found more optimal solutions at a cost of around 26000 compared to a variation without elitist ants.

## Question 6

Particle Swarm Optimization (PSO) is a promising alternative to ACO for TSP. Primarily due to its capabilities in global exploration, rapid convergence, and efficient parameter tuning. PSO's collective movement of particles allows for a more comprehensive exploration of the solution space, potentially leading to the discovery of diverse and more optimal paths. Moreover, its inherent ability to handle continuous solution spaces is advantageous in TSP instances where solutions can be represented as continuous vectors. The streamlined parameter tuning process in PSO, with fewer parameters to adjust compared to ACO, offers a practical advantage, reducing the complexity of finding optimal configurations. These distinctive attributes make PSO a better choice, particularly when prioritising faster convergence and improved global information exploitation.