HOMEWORK 01

INTRODUCTION

DC motors are a part of electrical machines that converts direct current electrical power into mechanical power. Controlling a DC motor means applying a suitable voltage V(t) in such a way that the motor [1]:

- pursues, satisfying certain specifications, a desired position reference θ (t) ,
- rejects the unknown disturbances loaded by the torque,
- remains stable.

In particular we will consider two kind of problems:

1. speed control, i.e. control of the mechanical dynamics with feedback of the angular speed of the motor;
2. position control, i.e. control of integral dynamics with feedback of the motor's angular position of the angular position of the motor.

A simple model of a DC motor is shown in figure.

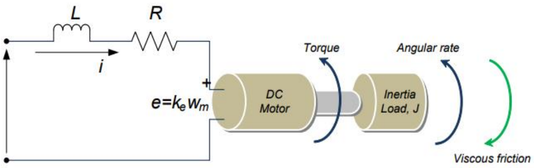

Fig. 1 A typical DC motor model [1]

The two major outputs of a motor are speed and torque.
The relationship between speed and torque is an important part of selecting and operating a DC motor: $T = \frac{V \cdot \omega \cdot k}{R}\ k$ (where T = motor torque, V = supply voltage, ω = rotational speed, k = motor constant and R = resistance).
When the load on the motor is constant, speed is proportional to supply voltage. And when supply voltage is constant, speed is inversely proportional to the load on the motor.
Considering the disturbance torque on the motor shaft while designing the control makes the system more robust to load changes [2].

PROPOSED MODEL

The DC circuit model in Fig.1 can be expressed by a DC motor block which represents the electrical and torque characteristics. We should minimize the speed variations induced by such disturbances.

In this case will be considered an armature-controlled DC motor linear model as in Fig.2

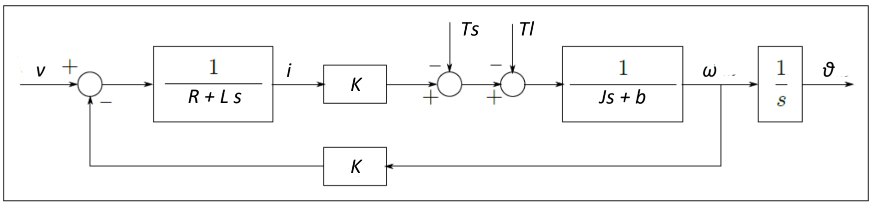

Fig. 2 DC motor linear model

As parameters have been used [1]:

J = 0.01;
b = 0.01;
K = 0.5;
R = 1;
L = 2.5;

And in Simulink, the block schemes have been represented as follow:

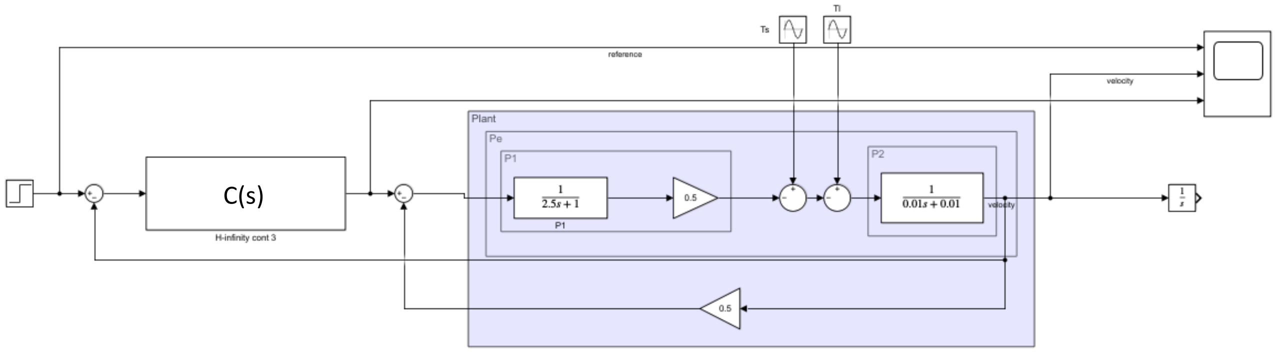

Fig. 3 Velocity control problem: proposed block scheme model

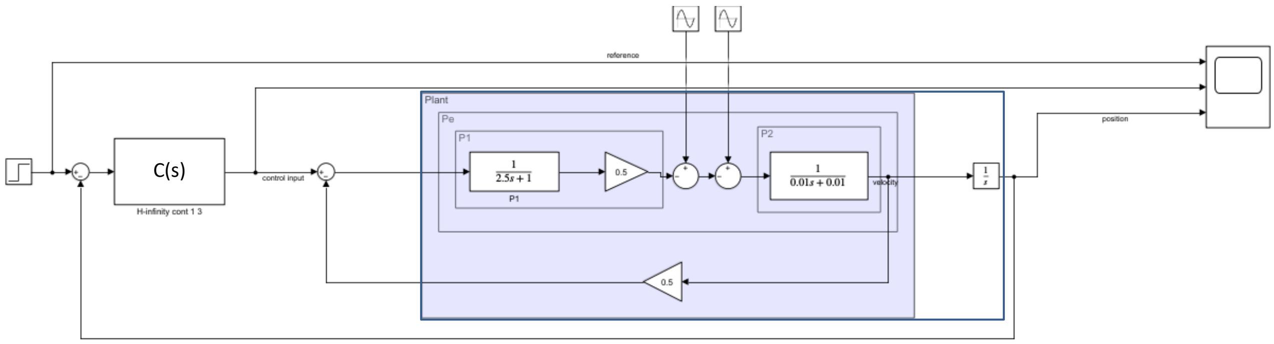

Fig. 4 Position control problem: proposed block scheme model

VELOCITY PROBLEM

To solve the problem, we need to find a controller that can reduce the error between the reference and the output as much as possible, while minimising the control effort.

To do this we will find an H-infinity norm controller, found using the analysis of the sensitivity function and appropriately weighing the weights wS, wT, wU.

We denote by:

- S = 1/1+L the sensitivity function
- T = L/1+L the complementary sensitivity function

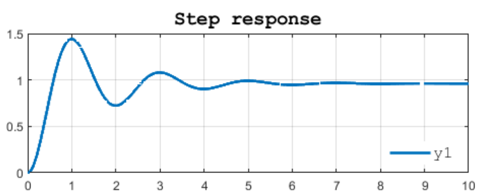

Fig. 5 step response without Controller

As can be seen in the figure, the step response without any controller has several oscillations, although it tends to stabilise for values t>7 in any case.

The optimal controller which minimizes the $min\ ||\cdot||_\infty$ is such that $|\,wBS1(j\omega)\ S(j\omega)| = constant$ where $wBS1$ is the bandwidth B3S.

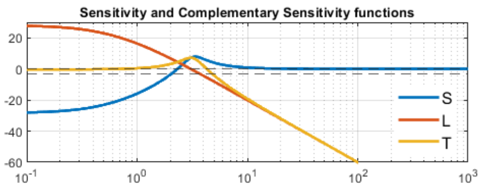

Fig. 6 sensitivity and complementary sensitivity functions

The following values are found from the plot:

```
A1 = 0.039;    % low frequency behaviour of specs of |S|
wBS1 = 2.06;   % is the lower bound on B3S
M1 = 2.5;   % is the upper bound for |S(jω)| (peak of S)

wS1 = (s/M1 +wBS1)/(s+wBS1*A1)  % sensitivity weight
```

Thus, the gamma constant found is equal to GAM1 = 0.4030, which is much lower than 1.
The H-inf controller found in this way is:

$$C1 = \frac{2.576e05\ s\text{^}2 + 3.607e05\ s + 2.679e06}{s\text{^}3 + 1521\ s\text{^}2 + 1.024e06\ s + 8.228e04}$$

In this way we get the desired closed loop behaviour

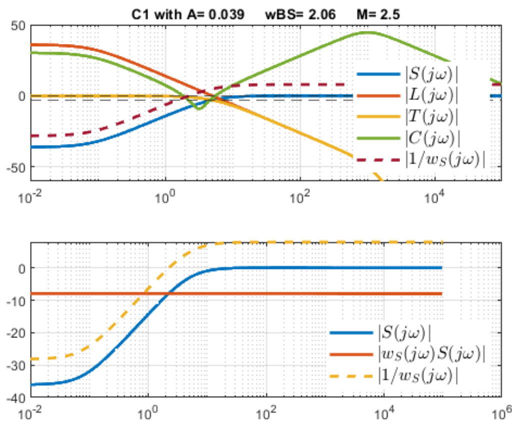

Fig. 7 plot obtained by adding the controller and wS.

- $|S(j\omega)| \le \frac{1}{|w_S(j\omega)|} \le 1$
- $|w_S(j\omega)S(j\omega)|$= constant = γ

In this way we have found a controller that agrees with the required specifications and is able to stabilise the step response in much less time and without oscillations.

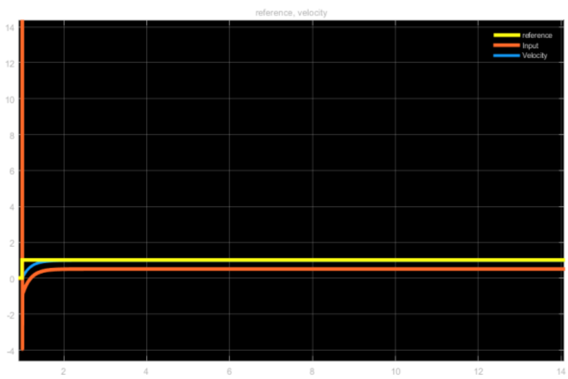

Fig. 8 step response obtained by adding the controller and wS.

Since the GAM1 is much lower than 1, it is possible to increase it by adding a weight wU.

In this way it is possible to increase the GAM1, decrease the control effort but without incurring in instability.
As can be seen, there is a better response and a significantly lower control effort (in red)

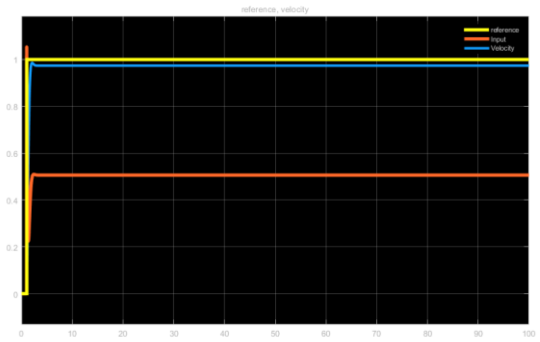

Fig. 9 step response obtained by adding the controller, wS and wU.

The weight wT is not used in this case of study as it is unnecessary due to the absence of noise in the proposed scheme.

It is possible to compare the results with this controller H-inf, using for example a PI controller as
`C3 = pid(0.5, 0.5,0);`

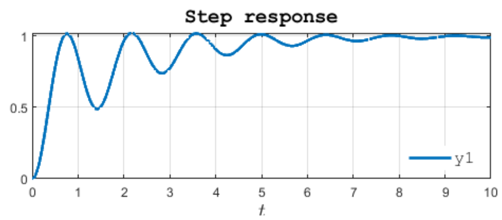

Fig. 10 step response obtained using the PI controller.

In this case the response is worse than the previous one with the controller and has a lot of oscillation.

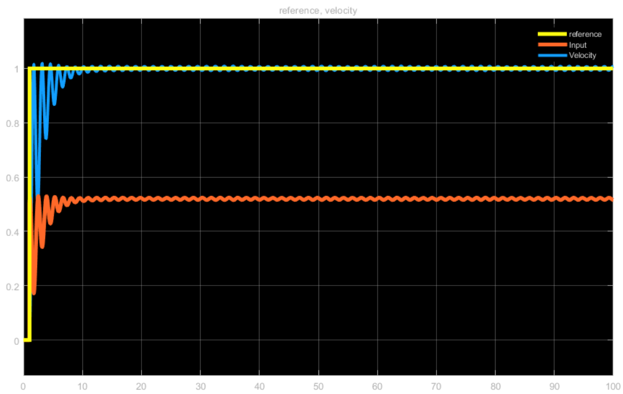

Fig. 11 step response obtained using PI controller.

POSITION PROBLEM

It is possible to make the same considerations as in the Velocity problem.
The results will be presented more briefly.

Also in this case, the use of the found controller stabilises the output.

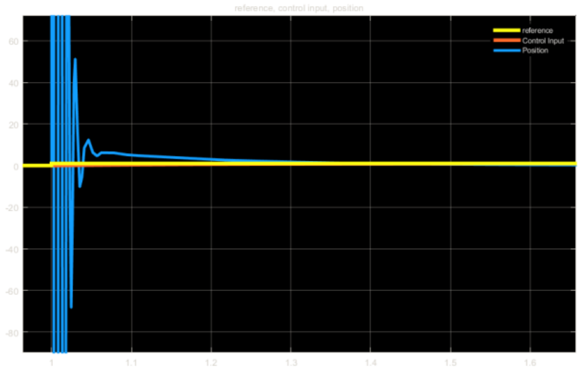

Fig. 12 step response obtained by adding the controller and wS.

As before, by adding an H-inf controller the output stabilises but there are oscillations with very high amplitude in the first-time interval.

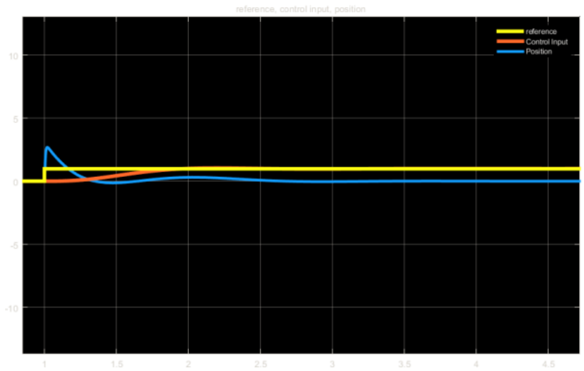

Fig. 13 step response obtained by adding the controller, wS and wU.

On the other hand, by also adding a weight on wU the oscillations are greatly attenuated, even though a very small error is present.

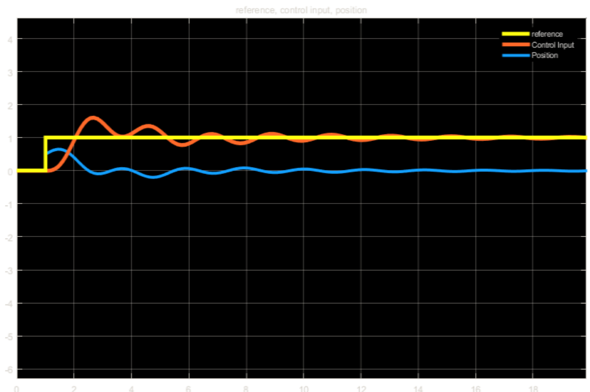

Fig. 14 step response obtained using PI controller.

In this case, comparing with a PI defined as in the previous problem, the output stabilises over a longer interval with the presence of oscillations.

In this case, too, the use of the H-inf controller is the best choice.

References

[1] P. A. G. L. Bonivento Claudio, Sistemi di automazione industriale - Architetture e controllo., McGraw-Hill Education (Italy), 2004.

[2] S. Cholakka, “Load Disturbance Torque Estimation for MotorDrive Systems with Application to Electric PowerSteering System,” University of Windsor, 2009.

[3] W. P. Aung, “Analysis on Modeling and Simulink of DC,” World Academy of Science, Engineering and Technology, 2007.

[4] O. J. Oguntoyinbo, “PID CONTROL OF BRUSHLESS DC MOTOR AND ROBOT TRAJECTORY PLANNING AND SIMULATION WITH MATLAB ® /SIMULINK ® Technology and Communication 2009 ACKNOWLEDGEMENTS”.