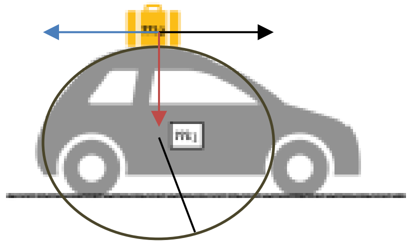
HOMEWORK 01



Problem 1

Consider the system described by

$$m_1\ddot{x}_1 = b(\dot{x}_2 - \dot{x}_1) + f$$
$$m_2\ddot{x}_2 = -b(\dot{x}_2 - \dot{x}_1) + k(x_2 - x_1)$$

where x_1 and x_2 represent the positions of the masses m_1 and m_2 respectively, b is a viscous friction coefficient, k is the elastic coefficient of a repulsive spring and f is the control force applied to the mass m_1.

Suppose

- m1 = 1000Kg
- m2 = 10 kg (hand luggage weight)
- $b = 17{,}1 \cdot 10^{-6}\ [1] + 0.001 \cong 0.001$ (Where 0,001 = friction coef. between plastic and steel)
- $K = \frac{\Delta x}{R} \to 0$

Consider three possible outputs:

1. the absolute position of the suitcase (that is x_1)
2. the absolute position of the car (that is x_2)
3. the relative position of the suitcase with respect to the car (that is $x_1 - x_2$).

The system can be rewritten as:

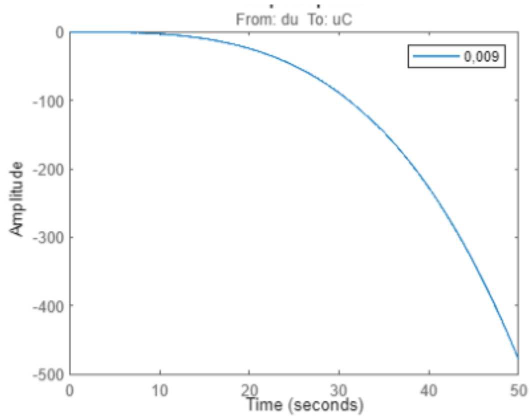
$$x1 = (m2 * s\string^2 + B * s - k)/((s\string^2) * (m1 * m2 * s\string^2 + B * (m1 + m2) * s - k * m1))$$
$$x2 = (B * s - k)/((s\string^2) * (m1 * m2 * s\string^2 + B * (m1 + m2) * s - k * m1));$$
$$x1 - x2 = m2/(m1 * m2 * s\string^2 + B * (m1 + m2) * s - k * m1);$$

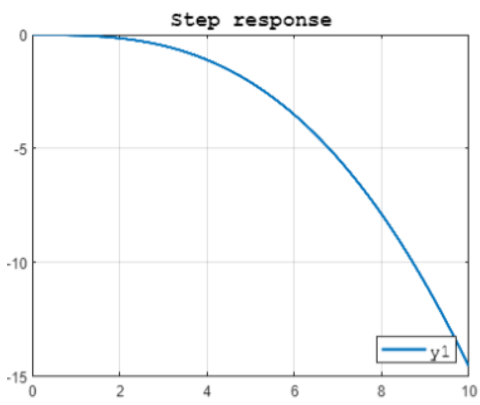
CASE 1:

```
      0.001 (s+0.05193) (s-0.001926)            0.001
x1 =  ------------------------------     ≅      -----
       s^2 (s+0.05241) (s-0.001908)              s^2
```

in this case is not possible to find a controller. Both in the case of applying an H-inf controller and in the case of the controller chosen properly as: $\frac{a\,s^3 + b\,s^2 + c\,s + d}{s^4 + e\,s^3 + f\,s^2 + g\,s + h}$

The step response diverges, more or less rapidly, to infinity.





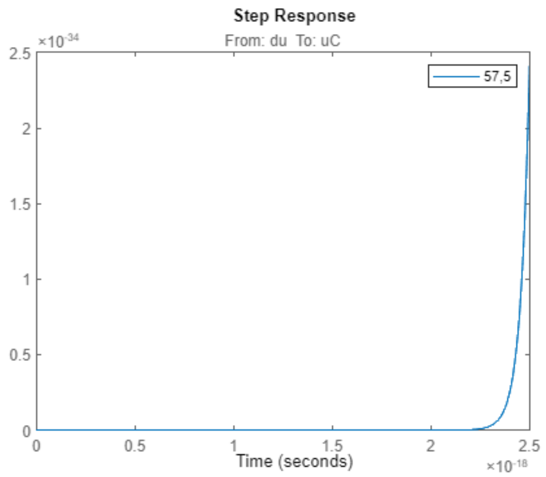
CASE 2:

As in Case 1, no controller is able to stabilise the response.

In this case, following the same procedure as in case 1, the step response diverges towards + infinity

$$X2 = \frac{5e\text{-}05\ (s-0.002)}{s^2\ (s+0.05241)\ (s-0.001908)} \cong \frac{5e\text{-}05}{s^2\ (s+0.05241)}$$

Non-minimum phase but unstable.



CASE 3:

In this case

$$x1\text{-}x2 = \frac{0.001}{(s+0.05241)\ (s-0.001908)}$$

applying $C(s) = \dfrac{as+b}{s^2+cs+d}$ as controller, $p^*(s) = (s+\alpha(1+j))^2(s+\alpha(1-j))^2$
with: $\alpha > 0$

$c = 4.0\,\alpha - 0.0505$

$d = 8.0\,\alpha^2 - 0.202\,\alpha + 0.00265025$

$a = 8000.0\,\alpha^3 - 404.0\,\alpha^2 + 10.601\,\alpha - 0.138887625$

$b = 4000.0\,\alpha^4 + 0.8\,\alpha^2 - 0.0202\,\alpha + 0.000265025$

For α > 0.026 the system is stable.

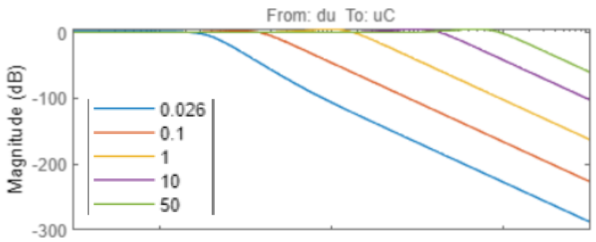


Fig. 2 complementary sensitivity

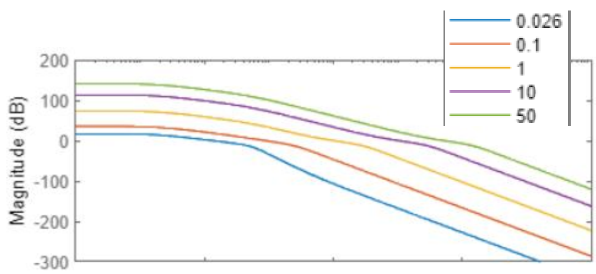
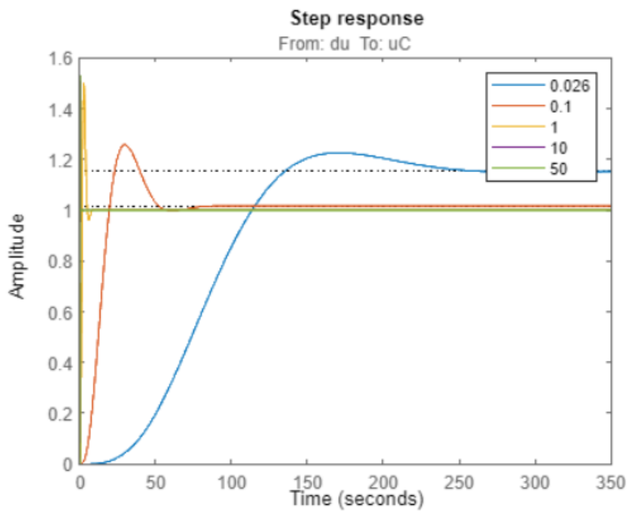


Fig. 1 sensitivity

(plots for different values of α)

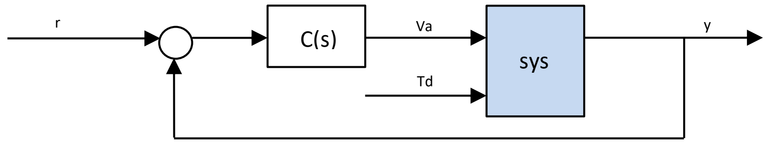
as α increases, plots shift to the right, without peaks.



The step response shows that, as alpha increases, there is a better performance, with a faster response but also a higher but acceptable oscillation peak.

Problem 2

Considering the DC Motor scheme chosen in HW01, it can be represented as:

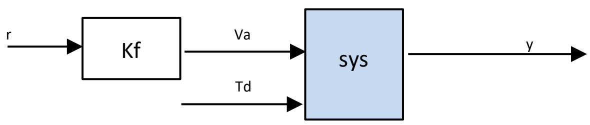


and construct a state-space model of the DC motor with two inputs (Va, Td) and one output (y).

$P1 \ = \ tf(b, [L \ R]);$ $\quad$ *% armature*
$P2 \ = \ tf(1, [J \ Kf]);$ $\quad$ *% eqn of motion*

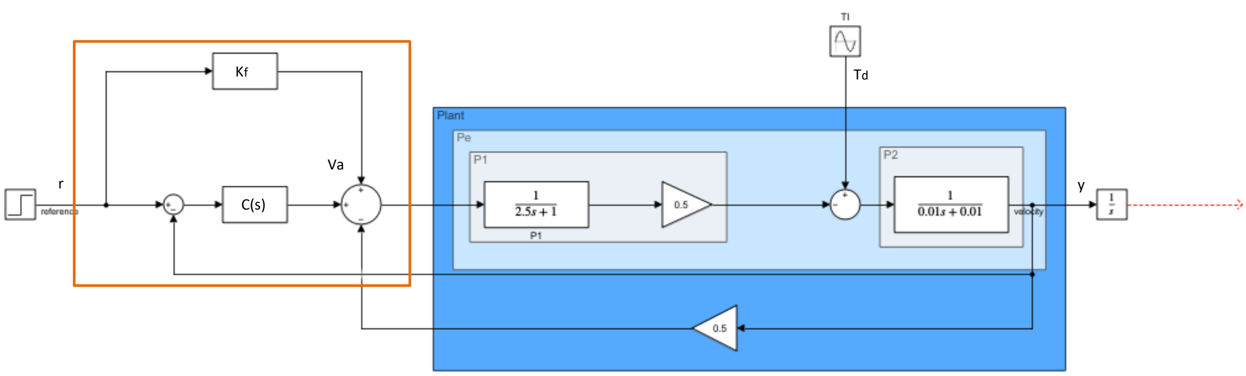
$sys \ = \ feedback(P1 * P2, K);$ $\quad$ *% close back emf loop*

Otherwise, a feedforward reference compensator Kf can be added. Kf in order not to influence the 1DOF scheme must be A.S.
The idea is to add Kf so that the effect of disturbances or noise does not affect the system.

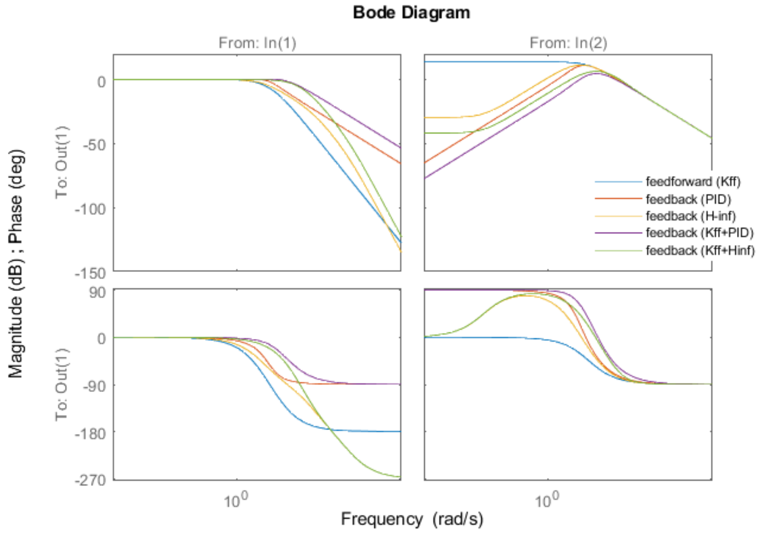


The feedforward gain Kf should be set to the reciprocal of the DC gain from *Va* to *y*.
$Kf \ = \ 1/dcgain(sys(1))$

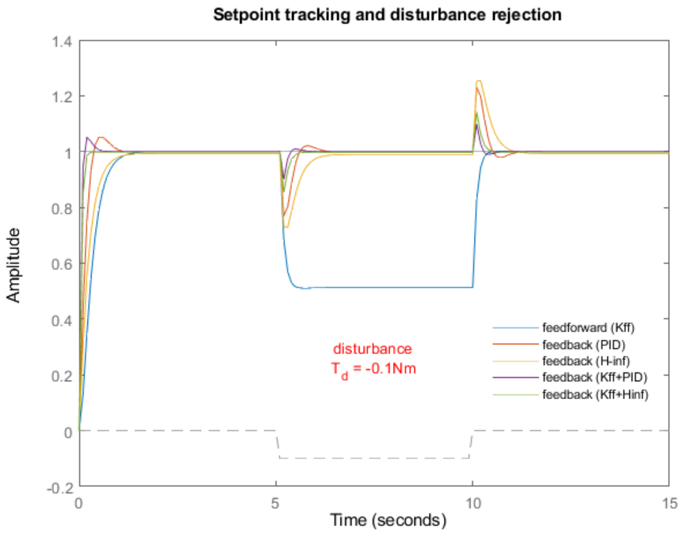
Otherwise, to further improve performance we can consider both, as follow:

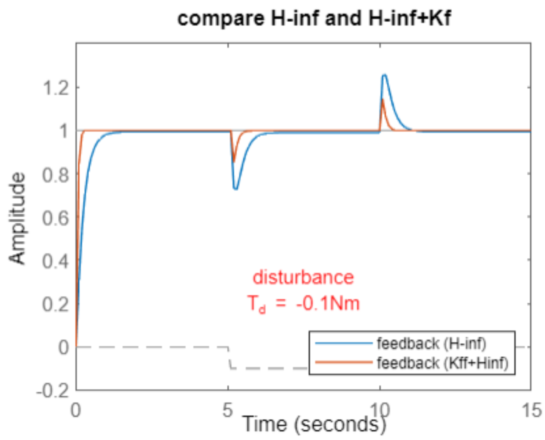


comparing the various schemes results in the end:



as can be seen from the figures, the best performance is achieved by applying an H-inf controller and a feedforward gain Kf. Therefore, adding an additional controller results in a better, faster output response that is able to reject the disturbance.





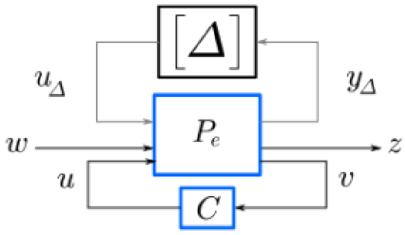
Problem 3

Let's consider the system used in homework 01, with uncertain parameters R, L, K, b, J.

Consider $P_{nominal} + P_{perturbed}$,to find the corresponding weight, we can use the **ucover** command to find the uncertain system $P_m(s) = P_m^{nom}(s)(1 + w_m(s)\Delta_m(s))$

The Robust stability condition is given by: $||w_m(s) \cdot T_n(s)||_\infty < 1$ (nec & suff condition if we consider structured uncertainty).

If I let the parameters varying, I will get several perturbed plants. In this case I have to pull out the uncertainties from the plant:



Where $\Delta(s) = \begin{bmatrix} \Delta_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \Delta_n \end{bmatrix}$ (structured uncertainty)

The corresponding Nyquist plot should go throught the point (-1,0), in this case: