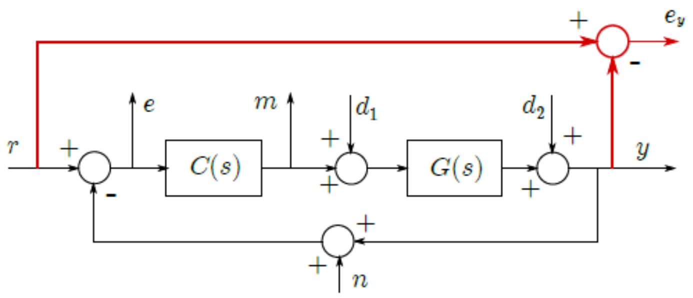
HOMEWORK III

Problem 1

Consider the following system:

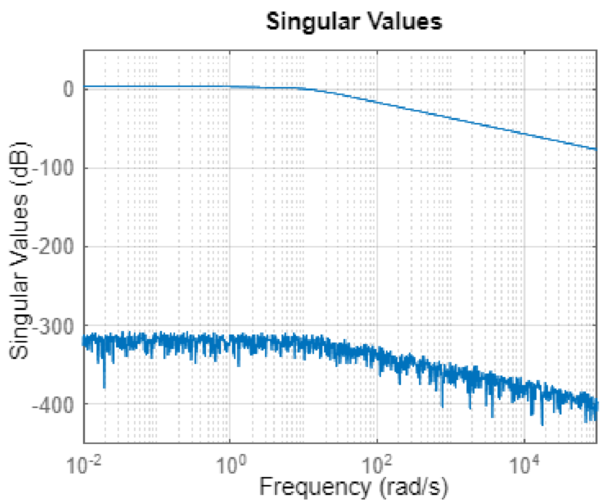


$$G(s) = \frac{s}{(s+4)^2}, \qquad C(s) = \frac{10}{s+10}$$

For MIMO feedback systems, two sets of transfer functions are needed to describe the behavior of the closed loop system (because G(s) and C(s) are matrices). To do this I used connect command to build the augmented plant.

In our case we have:

1) considering r and n as inputs and y and m as outputs:
   in this case Pe1 is stable and plotting the singular values of the frequency response of the system I have:



the singular value plot shows min(Nu,Ny) lines on the plot corresponding to each singular value of the frequency response matrix.

Moreover, by performing a singular value decomposition of matrix Pe1_0 = U*S*V' I obtain:

$$U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = (u1 \quad u2)$$
$$\Sigma = \begin{pmatrix} 1.4142 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \bar{\sigma} & 0 \\ 0 & \underline{\sigma} \end{pmatrix}$$
$$V = \begin{pmatrix} 0.7071 & 0.7071 \\ -0.7071 & 0.7071 \end{pmatrix} = (v1 \quad v2)$$

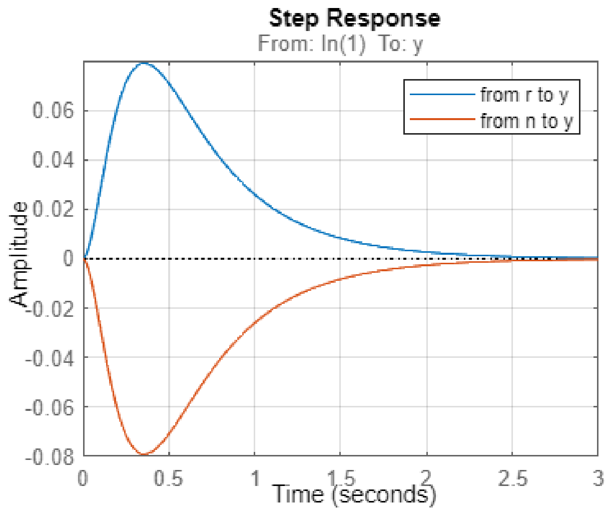
We know the Maximum value of σ is obtained when the input is along the v1 direction. Instead, the smallest value σ is obtained if the input is in the v2 direction.

We have to guarantee that:

$$\bar{\sigma}(w_s(j\omega)S_o(j\omega)) < 1 \quad \Rightarrow \quad \underline{\sigma}(L_o(j\omega)) > |w_s(j\omega)| \text{ if } \underline{\sigma}(L_o(j\omega)) \gg 1 \text{ (nominal performance)}$$

$$\bar{\sigma}(w_m(j\omega)T_i(j\omega)) < 1 \quad \Rightarrow \quad \bar{\sigma}(L_i(j\omega)) < |w_m(j\omega)| \text{ if } \bar{\sigma}(L_o(j\omega)) \ll 1 \text{ stability robustness}$$

As step response I have:



from r to y, and, from n to y, both output goes to 0.

Same consideration can be don in case 2 and 3.

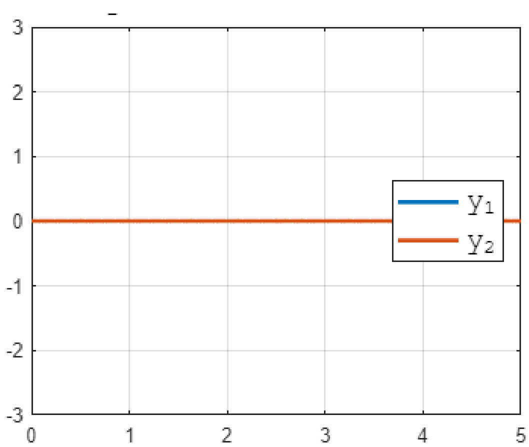
2) Consider now a 2x2 system characterized by the transfer function matrix W(s) previously determined; try to formulate a control problem and solve it (if possible).

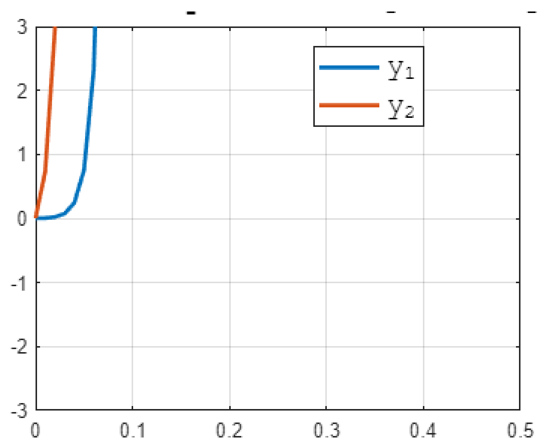
I add some weight performance:

$$w1 = \frac{s + 0.8}{2s + 0.008}$$

$$w2 = \frac{s + 0.4}{2s + 0.004}$$

By adding a H-inf controller I can stabilize the system and the output goes to 0, instead, using a controller as K = `[4*(s+1)/s,0;0,12*(s+1)/s]` , with higher values on the gain, the closed loop system becomes unstable and the output diverges.





Problem 2

In this case consider the problem of exploring all possible control problems involving robustness (stability and performance) of the DC-motor chosen in the first Homework.

Expanding the intervals of the uncertainty with respect to the previous homework I obtain a non robustly stable system for the modeled uncertainty. In particular:

-- It can tolerate up to 86.7% of the modeled uncertainty.

-- There is a destabilizing perturbation amounting to 87.3% of the modeled uncertainty.

-- This perturbation causes an instability at the frequency 3.75 rad/seconds.

-- Sensitivity with respect to each uncertain element is:

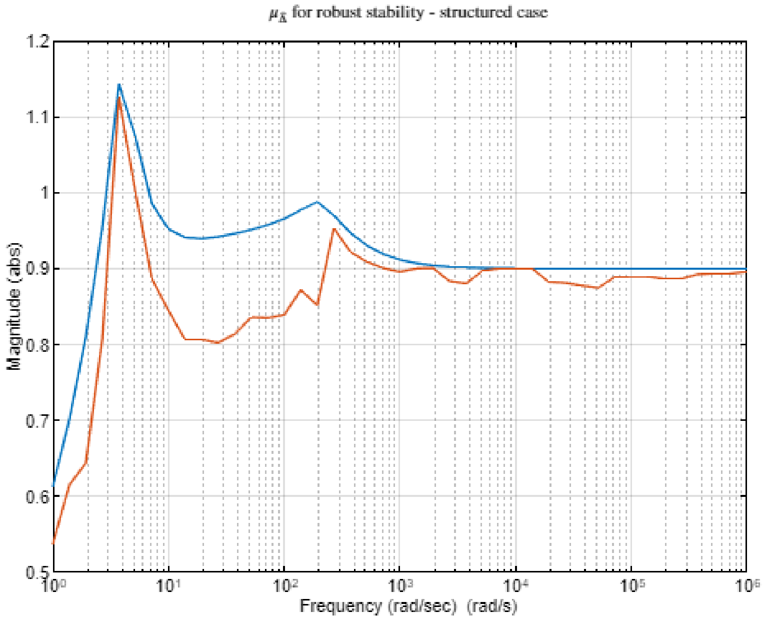
10% for J. Increasing J by 25% decreases the margin by 2.5%.

5% for K. Increasing K by 25% decreases the margin by 1.25%.

8% for L. Increasing L by 25% decreases the margin by 2%.

21% for R. Increasing R by 25% decreases the margin by 5.25%.

53% for b. Increasing b by 25% decreases the margin by 13.2%.



Instead, by changing the controller, adding an integrator C=0.05/s the system is robustly stable for the modeled uncertainty.

-- It can tolerate up to 105% of the modeled uncertainty.

-- There is a destabilizing perturbation amounting to 105% of the modeled uncertainty.

-- This perturbation causes an instability at the frequency 0.532 rad/seconds.

-- Sensitivity with respect to each uncertain element is:

1% for J. Increasing J by 25% decreases the margin by 0.25%.

0% for K. Increasing K by 25% decreases the margin by 0%.

1% for L. Increasing L by 25% decreases the margin by 0.25%.

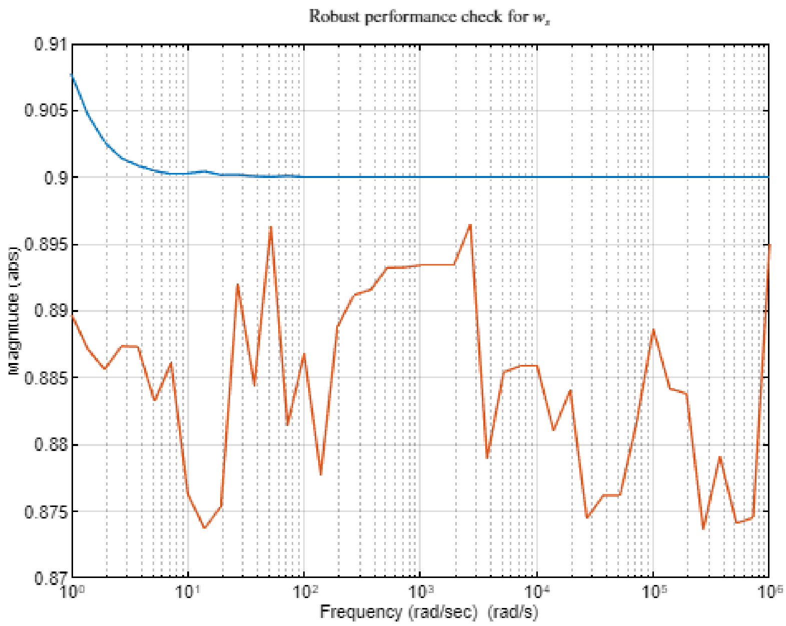
28% for R. Increasing R by 25% decreases the margin by 7%.

82% for b. Increasing b by 25% decreases the margin by 20.5%.

Considering this last controller, and by adding a performance weight.

The structured singular value, or μ, is the mathematical tool used by *robstab* to compute the robust stability margin. I can use the mussv function directly to compute mu as a function of frequency and reproduce the results above. The function mussv is the underlying engine for all robustness analysis commands.
For robust stability analysis, only the channels of M associated with the uncertainty channels are used.

Computing mu(M11) at fixed frequencies, the resulting plot and lower and upper bounds are:



The robust stability margin is the reciprocal of the structured singular value. Therefore upper bounds from mussv become lower bounds on the stability margin. Make these conversions and find the destabilizing frequency where the mu upper bound peaks (that is, where the stability margin is smallest).

For the nominal values of the uncertain elements k and delta, the closed-loop gain exceeds 1 (it is = 9.2325e+07) so in this case the performance margin is 0.