HKUST

MATH1014 Calculus II

Final Examination (Green Version) Name:_____

18th May 2019 Student ID:_____

12:30-15:30 Lecture Section:_____

Directions:

- This is a closed book examination. **No Calculator is allowed in this examination**.
- DO NOT open the exam until instructed to do so.
- Turn off all phones and pagers, and remove headphones. All electronic devices should be kept in a bag away from your body.
- Write your name, ID number, and Lecture Section in the space provided above, and also in the **Multiple Choice Item Answer Sheet** provided.
- **Mark your ID numbers correctly in the I.D. No. box in the Multiple Choice Item Answer Sheet**.
- DO NOT use any of your own scratch paper. Use only the scratch papers provided by the examination. Write also your name on every scratch paper you use, and do not take any scratch paper away from the examination venue.
- When instructed to open the exam, please check that you have **9** pages of questions in addition to the cover page.
- Answer all questions. Show an appropriate amount of work for each long problem. If you do not show enough work, you will get only partial credit.
- You may write on the backside of the pages, but if you use the backside, clearly indicate that you have done so.
- **Cheating is a serious violation of the HKUST Academic Code. Students caught cheating will receive a zero score for the examination, and will also be subjected to further penalties imposed by the University.**

Please read the following statement and sign your signature.

I have neither given nor received any unauthorized aid during this examination. The answers submitted are my own work.

I understand that sanctions will be imposed, if I am found to have violated the University's regulations governing academic integrity.

Student's Signature :

_____

Question No.	Points	Out of
Q. 1		2
Q. 2-16		60
Q. 17		12
Q. 18		12
Q. 19		14
Total Points		100

Part I: Answer all of the following multiple choice questions.

- Mark your answers to the multiple choice questions on the Multiple Choice Item Answer Sheet.
- **For backup purpose and exam paper viewing session, put also your MC answers in the following boxes**. Grading will be based on the answers marked on the MC answer sheet.

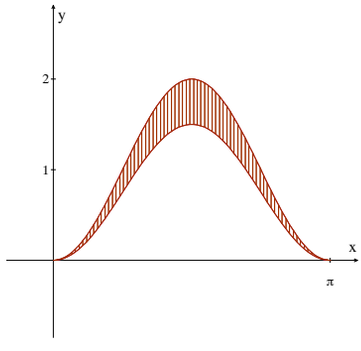
Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Answer																

Each of the following MC questions except Q1 is worth 4 points. No partial credit.

1. What is the colour version of your exam paper? (Read the top left corner of the cover page!) **Make sure that you have also written and marked your ID number correctly in the I.D. No. Box at the top middle of the MC answer sheet**. You will lose the points of this question if you do not do both correctly.

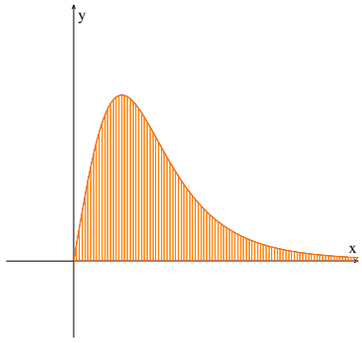
   (a) Green (b) Orange (c) White (d) Yellow (e) None of the previous

2. Find the area of the region enclosed by the graphs of the functions $y = 2\sin^2 x$ and $y = \frac{3}{2}\sin^3 x$ over the interval $0 \le x \le \pi$.



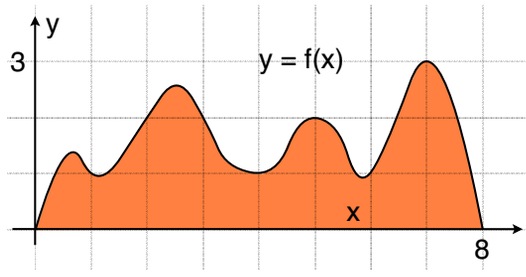
   (a) $2\pi - 2$ (b) $2\pi - 3$ (c) $2\pi - 4$ (d) $\pi - 2$ (e) $\pi - 4$

3. Find the area under the graph of the function $y = \dfrac{8x}{(x^2+2)^3}$ over the interval $0 \le x < \infty$.



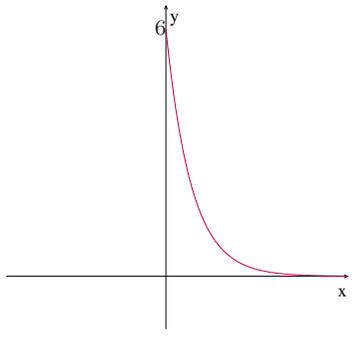
(a) ∞ (b) 1 (c) $\dfrac{3}{2}$ (d) 2 (e) $\dfrac{1}{2}$

4. The area under the graph given below over the interval $[0, 8]$ is rotated about the x-axis to generate a solid of revolution. Express the volume of the solid as a definite integral and then estimate the integral by the Trapezoidal Rule on **four subintervals** of equal length. What is the approximate value T_4 of the volume thus found?



(a) 8π (b) 10π (c) 12π (d) 14π (e) 16π

5. Find the volume of the solid of revolution obtained by rotating the area under the graph of $y = 6e^{-2x}$ over the interval $0 \le x < \infty$ about the y-axis.



(a) π (b) 2π (c) 3π (d) 4π (e) 6π

6. Find the constant C for which the improper integral $\int_0^\infty \left(\frac{3x}{x^2+2} - \frac{C}{x+3}\right) dx$ is convergent.

(a) -1 (b) -3 (c) 1 (d) $\frac{3}{2}$ (e) 3

7. Evaluate the improper integral $\int_3^6 \frac{3}{x\sqrt{x^2-9}}\, dx$.

(a) $\frac{\pi}{2}$ (b) $\frac{2\pi}{3}$ (c) π (d) $\frac{\pi}{3}$ (e) divergent

8. Which of the following improper integrals is divergent?

(a) $\int_1^\infty \frac{2e^{\sin x^3}}{x^2}\, dx$

(b) $\int_2^\infty \frac{1}{(\ln x)^2}\, dx$

(c) $\int_0^\infty \frac{1}{\sqrt{x^6+2}}\, dx$

(d) $\int_1^\infty \frac{\ln x}{e^{3x}}\, dx$

(e) None of the previous

9. Find all infinite sequences from the following which converge to a finite limit as $n \to \infty$?

(i) $\left(\frac{e}{\pi}\right)^n$, (ii) $\sqrt{n^2+3n}-n$, (iii) $\frac{n^n}{n!}$, (iv) $\frac{(\ln n)^2}{n}$, (v) $\frac{(2n)!}{(n!)^2}$

(a) (i) and (iii) only (b) (ii) and (iii) only (c) (i), (ii) and (iv) only

(d) (iii), (iv) and (v) only (e) (i),(iv) and (v) only

10. Find the sum of the series $\sum_{n=1}^{\infty} \frac{4(3^{n+1})-2^{n+1}}{6^n}$.

(a) 8 (b) 11 (c) 13 (d) 15 (e) 16

11. Find the sum of the series: $\sum_{n=1}^{\infty} \left(\frac{1}{3^{\frac{1}{n+1}}} - \frac{1}{3^{\frac{1}{n}}}\right)$.

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{3}{2}$ (e) $\frac{2}{3}$

12. Find **all** convergent infinite series from the following:

(i) $\sum_{n=1}^{\infty} \frac{5^n n!}{(2n)!}$ (ii) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[n]{4}}$ (iii) $\sum_{n=1}^{\infty} \frac{\cos n}{n^2 + e^{-n}}$ (iv) $\sum_{n=1}^{\infty} \frac{3}{(2 + \frac{1}{n})^{2n}}$

(a) Only (i) and (ii) are convergent.

(b) Only (iii) and (iv) are convergent.

(c) Only (i), (ii), and (iii) are convergent.

(d) Only (i), (iii) and (iv) are convergent.

(e) All are convergent.

13. Find the coefficient of the x^5 term in the Maclaurin series (i.e., Taylor series centered at 0) of the function $f(x) = \ln(2 + x^2)^3$.

(a) $\frac{1}{2}$ (b) 0 (c) 1 (d) $\frac{2}{3}$ (e) $\frac{3}{4}$

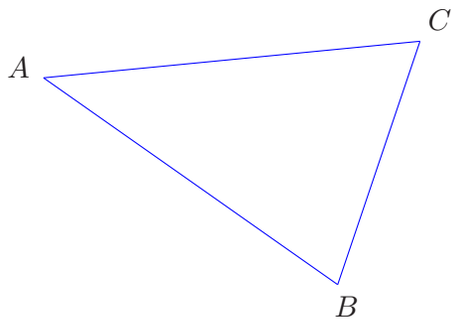
14. Find the sum of the series $\sum_{n=1}^{\infty} \frac{n}{2^{n-1}}$.

(a) 2 (b) 3 (c) 4 (d) 6 (e) 8

15. Find the orthogonal projection of the vector $\langle 7, 7, -3\rangle$ onto the vector $\langle 1, 2, -2\rangle$.

(a) $\langle -1, -2, 2\rangle$ (b) $\langle -2, -4, 4\rangle$ (c) $\langle 2, 4, -4\rangle$ (d) $\langle 3, 6, -6\rangle$ (e) $\langle \frac{3}{2}, 3, -3\rangle$

16. Using the cross product of suitable vectors or otherwise, find the area of the triangle $\triangle ABC$ with vertex coordinates given by $A : (4, 4, 4)$, $B : (3, 4, 2)$ and $C : (2, 2, 2)$.



(a) $\sqrt{6}$ (b) $2\sqrt{6}$ (c) $4\sqrt{6}$ (d) $3\sqrt{6}$ (e) $5\sqrt{6}$

Part II: Answer each of the following questions.

17. [*12 pts*] Let $I = \displaystyle\int_0^1 2x^3 \tan^{-1} x \, dx$, and recall that $\dfrac{d \tan^{-1} x}{dx} = \dfrac{1}{1+x^2}$.

(a) By applying integration by parts, find a constant k such that [8 pts]

$$\int 2x^3 \tan^{-1} x \, dx = k \left(x^4 \tan^{-1} x - \frac{1}{3}x^3 + x \right) - k \int \frac{1}{1+x^2} \, dx$$

Show your steps for full credit.

(b) Using part (a), or otherwise, find the value of $I = \displaystyle\int_0^1 2x^3 \tan^{-1} x \, dx$. [4 pts]

18. [*12 pts*] Determine whether the given series is convergent or divergent. **Give reason to justify your answer for full credit**.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+2}}$ [4 pts]

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n 5^n}{n^n}$ [4 pts]

(c) $\displaystyle\sum_{n=1}^{\infty} \ln \frac{n}{3n+2}$ [4 pts]

19. [*14 pts*] The function f is defined by a power series as follows: $f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(n+2)6^n} (x-3)^n$.

(a) Using Ratio Test or otherwise, find the largest open interval (open interval of convergence) in which the given power series converges absolutely. Show your steps for full credit. [7 pts]

(b) Determine whether the given power series is convergent or not at the boundary points of the open interval of convergence of $f(x)$. [2 pts]

(c) Let $H(x) = (x-3)^2 f(x)$. Find the derivative $H'(5)$. [5 pts]

Math1014 Final Exam Formula Sheet

Trigonometric Identities

$$\cos^2\theta + \sin^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$
$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\sin(A+B) = \sin A\cos B + \sin B\cos A$$
$$\sin(A-B) = \sin A\cos B - \sin B\cos A$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$
$$\sin A\cos B = \tfrac{1}{2}\Big(\sin(A+B) + \sin(A-B)\Big)$$
$$\cos A\cos B = \tfrac{1}{2}\Big(\cos(A+B) + \cos(A-B)\Big)$$
$$\sin A\sin B = \tfrac{1}{2}\Big(\cos(A-B) - \cos(A+B)\Big)$$
$$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B}$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$$

$$\int \cos^n x dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x dx$$
$$\int \sin^n x dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x dx$$
$$\int \sec^n x dx = \frac{1}{n-1}\sec^{n-2}x\tan x + \frac{n-2}{n-1}\int \sec^{n-2}x dx$$