HKUST

MATH1014 Calculus II

Final Examination (White Version) Name:____________________

25th May 2013 Student ID:____________________

12:30-15:30 Tutorial Section:____________________

Directions:

- This is a closed book examination. No Calculator is allowed in this examination.
- DO NOT open the exam until instructed to do so.
- Turn off all phones and pagers, and remove headphones. All electronic devices should be kept in a bag away from your body.
- Write your name, ID number, and Tutorial Section in the space provided above, and also in the **Multiple Choice Item Answer Sheet** provided.
- **Write the color version of your exam paper at the top left corner of the Multiple Choice Item Answer Sheet**. (Green/Orange/Yellow/White)
- DO NOT use any of your own scratch paper. Use only the scratch papers provided by the examination. Write also your name on every scratch paper you use, and do not take any scratch paper away from the examination venue.
- When instructed to open the exam, please check that you have **11** pages of questions in addition to the cover page.
- Answer all questions. Show an appropriate amount of work for each long problem. If you do not show enough work, you will get only partial credit.
- You may write on the backside of the pages, but if you use the backside, clearly indicate that you have done so.
- **Cheating is a serious violation of the HKUST Academic Code. Students caught cheating will receive a zero score for the examination, and will also be subjected to further penalties imposed by the University.**

Please read the following statement and sign your signature.

I have neither given nor received any unauthorized aid during this examination. The answers submitted are my own work.

I understand that sanctions will be imposed, if I am found to have violated the University's regulations governing academic integrity.

Student's Signature :

____

Question No.	Points	Out of
Q. 1-20		60
Q. 21		10
Q. 22		10
Q. 23		10
Q. 24		10
Total Points		100

Part I: Answer all of the following multiple choice questions.

- **Do not forget to put your name, student ID number, and the color version of your exam paper on the Multiple Choice Item Answer Sheet.**
- **Use an HB pencil to mark your answers to the MC questions on the Multiple Choice Item Answer Sheet provided.**
- **Enter also your MC answers to the following boxes for back-up use only. The marking will be completely based on the answers on the Multiple Choice Item Answer Sheet.**

Question	1	2	3	4	5	6	7	8	9	10
Answer										
Question	11	12	13	14	15	16	17	18	19	20
Answer										

Each of the following MC questions is worth 3 points. No partial credit.

1. Evaluate the improper integral $\displaystyle\int_0^4 \frac{\ln x}{\sqrt{x}}dx$.

   (a) $32\ln 2 - 16$ (b) $\ln 2 + 4$ (c) $8\ln 2 - 8$ (d) $4e^2 - 4$ (e) divergent

2. Find the definite integral $\displaystyle\int_0^2 \frac{x^3}{\sqrt{x^2+4}}dx$.

   (a) $\dfrac{-8\sqrt{2}}{3}$ (b) $\dfrac{16-10\sqrt{2}}{3}$ (c) $\dfrac{16-8\sqrt{2}}{3}$ (d) $\dfrac{4+2\sqrt{2}}{3}$ (e) $\dfrac{8-3\sqrt{2}}{3}$

3. Find the definite integral $\int_0^{\frac{\pi}{2}} \sin(2x)\cos(3x)\sin(4x)dx$.

(a) $\frac{16}{45}$ (b) $\frac{64}{45}$ (c) $\frac{11}{45}$ (d) $\frac{44}{45}$ (e) 0

4. Which of the following infinite sequences are convergent?

(i) $a_n = \frac{3+5n^2}{2n+n^2}$; (ii) $a_n = n\sin\frac{1}{n}$; (iii) $a_n = \frac{\sin 2n}{1+\sqrt{n}}$; (iv) $a_n = \frac{n!}{3^n}$

(a) Only (i) and (ii) are convergent.

(b) Only (i) and (iii) are convergent.

(c) Only (i), (ii) and (iii) are convergent.

(d) Only (i), (iii), and (iv) are convergent.

(e) All are convergent.

5. Find the limit of the bounded decreasing sequence defined by

$$a_1 = 3, \qquad a_{n+1} = \frac{1}{4-a_n} + 1.$$

(a) $\frac{1-\sqrt{5}}{2}$ (b) $\frac{1+\sqrt{5}}{2}$ (c) $\frac{5-\sqrt{5}}{2}$ (d) $\frac{5+\sqrt{5}}{2}$ (e) Does not exist.

6. Find the sum of the series $\sum_{n=1}^{\infty} \left(\frac{2^{n+1} - 3^{n-1}}{5^n} \right)$.

(a) $\frac{5}{2}$ (b) $-\frac{5}{2}$ (c) $-\frac{5}{6}$ (d) $\frac{5}{6}$ (e) divergent

7. Find the sum of the series: $1 - e + \frac{e^2}{2!} - \frac{e^3}{3!} + \cdots + \frac{(-1)^n e^n}{n!} + \cdots$.

(a) $\frac{1}{e^e}$ (b) e^e (c) $\frac{1}{1+e}$ (d) $\frac{1}{1-e}$ (e) divergent

8. Find the largest value of p for which the series $\sum_{n=1}^{\infty} \frac{\ln(n+1)}{(n+1)^{p+1}}$ diverges.

(a) -1 (b) 0 (c) 1 (d) 2 (e) 3

9. To find an approximate value of the sum of the infinite series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}$ so that the error does not exceed $\frac{1}{600}$, the smallest number of terms of the series you need to add is:

   (a) the first three terms

   (b) the first four terms

   (c) the first five terms

   (d) the first six terms

   (e) the first seven terms

10. Using the part of the Maclaurin series of $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$ up to only the second term, i.e., the x^2 term, an approximate value of the integral $\int_0^1 \sqrt{x}\cos(x^2)dx$ can be found as:

    (a) $\frac{4}{15}$ (b) $\frac{7}{15}$ (c) $\frac{16}{33}$ (d) $\frac{17}{33}$ (e) $\frac{19}{33}$

11. Find the coefficient of the x^7 term in the Maclaurin series (Taylor series centered at 0) of the function $f(x) = \int_0^x \frac{1}{1+t^3}\,dt$, where $-1 < x < 1$.

    (a) 0 (b) $\frac{1}{6}$ (c) $\frac{1}{7}$ (d) $-\frac{1}{6}$ (e) $-\frac{1}{7}$

12. Find the angle between the vectors $\langle 1, 2, -2\rangle$ and $\langle 5, 5, 0\rangle$.

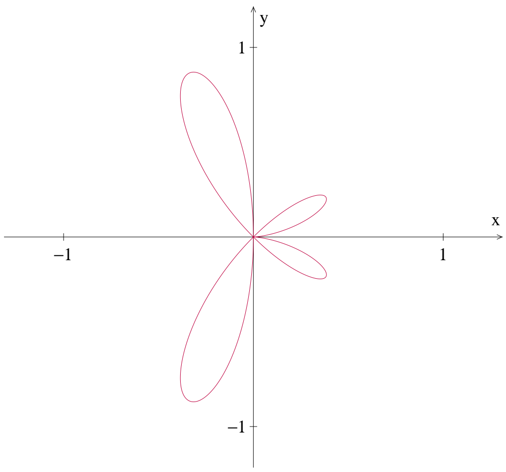
(a) $\dfrac{\pi}{3}$ (b) $\dfrac{2\pi}{3}$ (c) $\dfrac{\pi}{6}$ (d) $\dfrac{5\pi}{6}$ (e) $\dfrac{\pi}{4}$

13. Find the orthogonal projection of the vector $\langle 5, 4, 5\rangle$ onto the vector $\langle 1, 2, 1\rangle$.

(a) $\langle -1, -2, -1\rangle$ (b) $\langle 2, -2, 2\rangle$ (c) $\langle 3, 6, 3\rangle$ (d) $\langle 2, 4, 2\rangle$ (e) $\langle 0, 0, 0\rangle$

14. Find the area enclosed by the curve defined by the polar equation $r = \sin\theta \sin(4\theta)$.

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{3\pi}{4}$ (c) $\dfrac{\pi}{8}$ (d) $\dfrac{3\pi}{8}$ (e) $\dfrac{5\pi}{8}$



15. A planar region bounded by the curves $y = \dfrac{1}{x^2+3x+2}$, x-axis and the lines $x = 0$ and $x = 3$ is rotated about the y-axis to generate a solid region. Find the volume of the solid region.

(a) $\pi(2\ln 5 - 2\ln 2)$ (b) $\pi(4\ln 5 - 8\ln 2)$ (c) $\pi(2\ln 5 - 6\ln 2)$

(d) $\pi(3\ln 5 - 2\ln 2)$ (e) $\dfrac{21}{20}\pi$

16. Find the arc length of the curve defined by $y = \dfrac{x^2}{8} - \ln x$, where $2 \le x \le 4$.

(a) $\dfrac{13}{6} - 6\ln 2$ (b) $\dfrac{3}{2} - \ln 2$ (c) $\dfrac{7}{2} - \ln 2$ (d) $\dfrac{3}{2} + \ln 2$ (e) $\dfrac{13}{3} + 6\ln 2$

17. An observer 2 m above the north pole of a sphere of radius 10 m can see only a part of the sphere. Find the area of that viewable part of the sphere.

(a) $\dfrac{100\pi}{3}$ (b) $\dfrac{200\pi}{3}$ (c) 100π (d) $\dfrac{400\pi}{3}$ (e) $\dfrac{800\pi}{3}$

18. A chain of length 10 m and mass 8 kg hanging from the top of a building has a uniform mass density. Find the work done in lifting the lower end of the chain to the same position as the upper end at the top of the building. ($g = 9.8$ m/s^2 is the acceleration due to gravity.)

(a) $10g$ (in J.) (b) $20g$ (in J.) (c) $30g$ (in J.) (d) $40g$ (in J.) (e) $80g$ (in J.)

19. A curve has slope $\dfrac{y^2}{x^3}$ at any point (x, y) on the curve. Given that $(1, 1)$ is a point on the curve, find the y-coordinate of another point $(4, y)$ on the curve.

(a) $\dfrac{32}{9}$ (b) $\dfrac{32}{5}$ (c) $\dfrac{32}{17}$ (d) 8 (e) 32

20. A spherical tank of radius 2 m is initially **half full** of water. The water is drained through a hole at the bottom of the tank, and the depth of water in the tank at time t (in minutes) is denoted by $h(t)$. If $h = h(t)$ satisfies the differential equation

$$(4h - h^2)\frac{dh}{dt} = -k\sqrt{2gh}$$

where k is a positive constant and g is the acceleration due to gravity, how long (in minutes) will it take for the water to drain completely?

(a) $\dfrac{56}{15k\sqrt{g}}$ (b) $\dfrac{42}{15k\sqrt{g}}$ (c) $\dfrac{36}{15k\sqrt{g}}$ (d) $\dfrac{24}{15k\sqrt{g}}$ (e) $\dfrac{8}{15k\sqrt{g}}$

Part II: Answer each of the following questions.

21. [*10 pts*] Evaluate the following improper integrals.

(a) For any positive integer n, use integration by parts to find a number c_n such that $\displaystyle\int_{-\infty}^{\infty} x^n e^{-x^2}\,dx = c_n \int_{-\infty}^{\infty} x^{n-2}e^{-x^2}\,dx$. Hence or otherwise find the ratio $\dfrac{\int_{-\infty}^{\infty} x^{2n}e^{-x^2}\,dx}{\int_{-\infty}^{\infty} e^{-x^2}\,dx}$. [6 pts]

(b) $\displaystyle\int_2^{\infty} \frac{1}{(x+7)\sqrt{x-2}}\,dx$ [4 pts]

22. [*10 pts*] Determine whether the given series is convergent or divergent. Given brief reason to justify your answer.

(a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n^{\frac{3}{2}}+2}{2n^{\frac{3}{2}}}\right)^n$ [2 pts]

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{1+\frac{1}{2}+\cdots+\frac{1}{n}}$ [2 pts]

(c) $\displaystyle\sum_{n=1}^{\infty}\frac{e^{-n}}{\sqrt{n}}$ [3 pts]

(d) $\displaystyle\sum_{n=2}^{\infty}\frac{\tan^{-1}n}{n(\ln n)^2}$ [3 pts]

23. [*10 pts*] Consider a power series $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n n^{2n+1}}{(2n+1)!}(x-1)^{2n+1}$.

(a) Find the radius of convergence of the given power series. [8 pts]

(b) Find the Taylor Series for the **derivative function** f' centered at 1. [2 pts]

24. [*10 pts*] A cone container of top radius 4 m and height 8 m is fully filled with water. A ball of radius 3 m is inserted slowly as far as possible into the container to expel as much water as possible.

(a) Find the amount of water spilled out of the container. [5 pts]

(b) Take your answer in part (a), or just denote it by V, and remove the ball from the cone container. Express the work required to pump all the remaining water (water density denoted by ρ, gravity acceleration denoted by g) to the top of the container by an integral. **You do not need to evaluate the integral.** [5 pts]