Math 1014
Midterm Exam Spring 2022

Part I: Multiple Choice Questions

1. Find the area under the graph of the function $f(x) = 3x\sqrt{4-x^2}$ over the interval $[0, 2]$.

2. Evaluate the integral $\displaystyle\int_0^2 4\cos(\pi x)\cos(2\pi x)\cos(3\pi x)dx$.

3. Evaluate the integral $\displaystyle\int_0^\pi (\pi - x)\sin x\cos^2 x dx$.

4. Evaluate the integral $\displaystyle\int_2^\infty \frac{5}{x^2\sqrt{x^2+5}}dx$.

5. For which constant k can the improper integral $\int_0^{\infty} \left(\frac{3x^2+x-1}{2x^3+1} - \frac{kx+2}{2x^2+5} \right) dx$ be convergent.

6. Which of the following improper integral is convergent?

(i) $\int_1^{\infty} \frac{\sqrt{x}}{1+x} dx$

(ii) $\int_1^{\infty} \frac{\ln x^2}{4+x^2} dx$

(iii) $\int_1^{\infty} \frac{2^x}{x+2^x} dx$

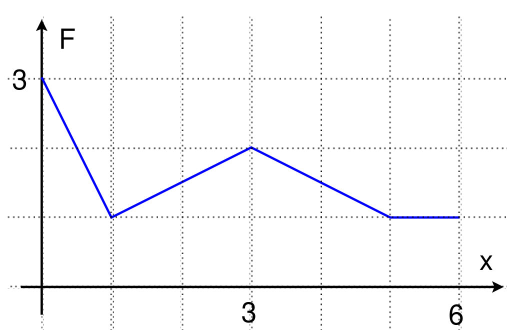
(iv) $\int_1^{\infty} \frac{1}{1+x\ln x} dx$

(v) $\int_1^{\infty} \frac{1}{x\ln\sqrt{x}} dx$

7. The region under the graph of $y = 2xe^{-x^3/6}$ over the interval $[0, \infty)$ is rotated about the x-axis to generate a solid of revolution. Find the volume of the solid..

8. The base of a solid sitting on the xy-plane is the region bounded enclosed by the graphs of $y = 9\sin x$ and $y = \sin x$, where $0 \le x \le \pi$. Suppose that the cross sections of the solid perpendicular to the x-axis are semi-discs. Find the volume of the solid.

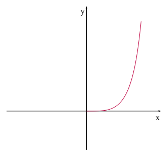
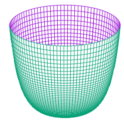
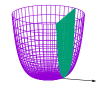
9. The graph of a force function (in newtons) is given as below. How much work (in Joules) is done by the force in moving an object from $x = 0$ to $x = 5$ (in meters)?



10. The length of the graph of a positive continuous function $y = f(x)$ over the interval $[a, b]$ is 2 units. Suppose the area of the surface of revolution obtained by rotating the graph of f about the x-axis is 2π square units. Find the area of the surface of revolution obtained by rotating the graph of $y = f(x) + 1$ about the x-axis.

Part II

1. ([25 points]) A bowl is in the shape of a surface of revolution obtained by rotating the graph of the function $y = 6\tan^2 x^2$ about the y-axis, where $0 \le x \le \frac{\sqrt{\pi}}{2}$. ($x$, y are in meters.)

(a) Find the volume of the bowl. [14 pts]

(b) Consider the cross sections of the solid region contained by the bowl which are perpendicular to the x-axis. Find the average value of their areas. [4 pts]

(c) Suppose the bowl is full of water. Express the work required to pumped all water in the bowl to an outlet at the top of the bowl by a definite integral. **Do not need to evaluate the integral**. (You may denote the density of water by ρ, and the gravity acceleration by g, both in SI units.) [7 pts]

Part III

1. ([25 points]) Let $I_n = \displaystyle\int_0^2 \frac{1}{(x^2+4)^n}\,dx$, where $n = 1, 2, 3, \ldots$ is a positive integer.

   (a) Using integration by parts, or otherwise, find $A(n)$, $B(n)$, which are expressions depending on n, such that

   $$I_{n+1} = A(n) I_n + B(n) \ .$$

   (Hint: Start with I_n.) [12 pts]

   (b) Using (a), or otherwise, evaluate the integral [7 pts]

   $$\int_0^2 \left[\frac{8}{(x^2+4)^5} - \frac{7}{4(x^2+4)^4}\right] dx \ .$$

(c) If Simpson's rule on four subintervals is used to approximate

$$\pi = \int_0^2 \frac{8}{x^2+4} dx \ ,$$

a rational approximate value of π can be found as

$$\pi \approx \frac{1}{3}\left[1 + \frac{64}{a} + \frac{8}{b} + \frac{64}{c} + \frac{1}{d}\right]$$

where a, b, c, d are positive integers. Find a, b, c, d.

[6 pts]