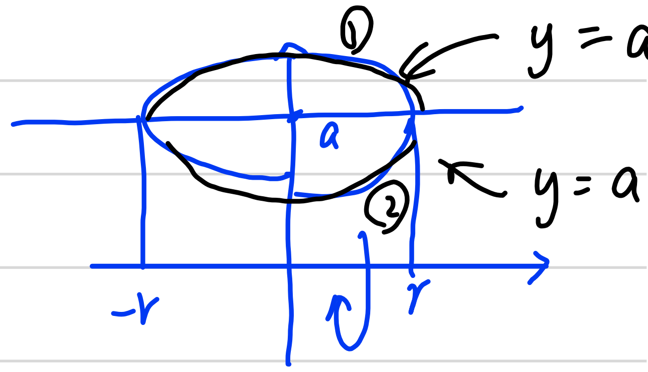
Ex 1 $\quad y=f(x)=\frac{2}{3}(x-1)^{\frac{3}{2}}$ on $[1,2]$

$$L=\int_a^b\sqrt{1+f'(x)^2}\,dx$$

$$f'(x)=\frac{3}{2}\cdot\frac{2}{3}(x-1)^{\frac{1}{2}}=(x-1)^{\frac{1}{2}}$$

$$\text{Arc } L=\int_1^2\sqrt{1+\left((x-1)^{\frac{1}{2}}\right)^2}\,dx$$
$$=\int_1^2\sqrt{x}\,dx$$
$$=\frac{2}{3}x^{\frac{3}{2}}\Big|_1^2$$
$$=\frac{2}{3}\left(2^{\frac{3}{2}}-1\right)\quad\square$$

Ex 2. $\quad x^2+(y-a)^2=r^2 \quad x\in[-r,r]$ revolve about x-axis



$$x^2+(y-a)^2=r^2$$

$$f(x)=y=a\pm\sqrt{r^2-x^2}$$

$$f'(x)=\pm\frac{1}{2\sqrt{r^2-x^2}}(-2x)=\mp\frac{x}{\sqrt{r^2-x^2}}$$

$$S=\int_a^b 2\pi f(x)\sqrt{1+f'(x)^2}\,dx$$

$$=\int_{-r}^{r}2\pi\left(a+\sqrt{r^2-x^2}\right)\sqrt{1+\left(-\frac{x}{\sqrt{r^2-x^2}}\right)^2}\,dx \quad ①$$

$$+\int_{-r+\varepsilon}^{r-\varepsilon}2\pi\left(a-\sqrt{r^2-x^2}\right)\sqrt{1-\left(\frac{x}{\sqrt{r^2-x^2}}\right)^2}\,dx \quad ②$$

$\varepsilon\to 0$ $\quad x=r \quad \frac{x}{0}$

$$=\int_{-r}^{r}2\pi(2a)\sqrt{1+\frac{x^2}{r^2-x^2}}\,dx$$

$$= \int_{-r}^{r} 4\pi a \sqrt{\frac{r^2}{r^2-x^2}}\, dx$$

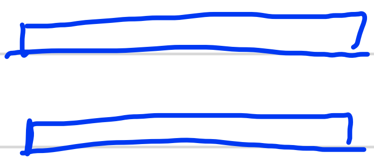
$$= 4\pi a r \int_{-r}^{r} \frac{1}{\sqrt{r^2-x^2}}\, dx$$

$$= 4\pi a r \left(\sin^{-1}\frac{x}{r}\right)\Big|_{-r}^{r}$$

$$= 4\pi a r \left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = 4\pi^2 a r \qquad \square$$

Ex 3.

L



$\rho_A(x) = 4e^{-x}$

$\rho_B(x) = 6e^{-2x}$

$$\text{Mass} = \int_a^b \rho(x)\, dx$$

$$m_A = \int_0^L 4e^{-x} dx = -4e^{-x}\Big|_0^L = 4 - 4e^{-L}$$

$$m_B = \int_0^L 6e^{-2x} dx = -3e^{-2x}\Big|_0^L = 3 - 3e^{-2L}$$

If $m_A > m_B$

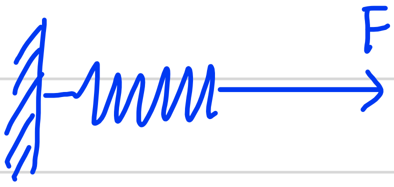
$$4 - 4e^{-L} > 3 - 3e^{-2L}$$

$$3(e^{-L})^2 - 4e^{-L} + 1 = 0$$

$$\Rightarrow (3e^{-L} - 1)(e^{-L} - 1) > 0$$

① $e^{-L} < \frac{1}{3}$ $\qquad$ $L > \ln 3$

② $e^{-2} > 1$ $\quad L < 0$ X $\quad \square$

Ex 4.  $F(x) = 16x - \frac{1}{10}x^3 \quad x \in [0, 7]$

$x = 0 \rightarrow x = 2$

$$W = \int_a^b F(x)\,dx$$

$$= \int_0^2 (16x \frac{1}{10}x^3)\,dx$$

$$= [8x^2 - \frac{1}{40}x^3)\Big|_0^2 dx$$

$$= (32 - \frac{1}{40}\cdot 8) - 0$$

$$= 32 - \frac{1}{5}$$

$$= 31.8 \qquad \square.$$