MATH2011 Intro to Multivariable Calculus
Tutorial 2: Vectors

Definitions and Results

Formula 1. The distance between the points $P = (p_1, p_2, p_3)$ and $Q = (q_1, q_2, q_3)$ is the length of the vector between the two points.

Definition 2. A vector-valued function is a function of the form $\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle$.

Definition 3. The positive direction of a curve $\mathbf{r}(t)$ is the direction this curve forms when t increases.

Formula 4. Let $\mathbf{r}(t) = \langle f_1(t), f_2(t), f_3(t)\rangle$ and $\mathbf{L}= \langle L_1, L_2, L_3\rangle$. We say $\mathbf{r}(t)$ approaches $\mathbf{L}$ as t approaches a if $\| \mathbf{r}(t) - \mathbf{L} \| \longrightarrow 0$, $t \longrightarrow a$ or equivalently $f_i(t) \longrightarrow L_i$ for every i.

Definition 5. If $\mathbf{r}(t) = \langle f_1(t), f_2(t), f_3(t)\rangle$, where $f_1(t), f_2(t), f_3(t)$ are differentiable functions, then the we call the vector $\mathbf{r}'(t) = \langle f_1'(t), f_2'(t), f_3'(t)\rangle$ the derivative of $\mathbf{r}(t)$.

Formula 6. The unit tangent vector of $\mathbf{r}(t)$ is $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}(t)\|}$

Definition 7. Let $\mathbf{r}(t) = \langle f_1(t), f_2(t), f_3(t)\rangle$ and $F_1(t), F_2(t), F_3(t)$ are the respective antiderivatives. Then the indefinite integral of $\mathbf{r}$ is $\int \mathbf{r}(t)dt = \mathbf{R}(t) + \mathbf{C} = \langle F_1(t), F_2(t), F_3(t)\rangle$, $\mathbf{C}$ is any constant vector.

Examples

Example 2

For the following $\mathbf{r}(t)$ and $\mathbf{R}(t)$, find a line perpendicular to them both which passes through their intersection. Show that this line is unique.

(A) $\mathbf{r}(t) = \langle -2 + 3t, 2t, 3t\rangle$, $\mathbf{R}(s) = \langle -6 + s, -8 + 2s, -12 + 3s\rangle$
(B) $\mathbf{r}(t) = \langle 4t, 1 + 2t, 3t\rangle$, $\mathbf{R}(s) = \langle -1 + s, -7 + 2s, -12 + 3s\rangle$

Solution

Example 3

Let $\mathbf{u}(0) = \langle 0, 1, 1\rangle$, $\mathbf{u}'(0) = \langle 0, 7, 1\rangle$, $\mathbf{v}(0) = \langle 0, 1, 1\rangle$, and $\mathbf{v}'(0) = \langle 1, 1, 2\rangle$.
Evaluate the following at $t = 0$:
(A) $\frac{d}{dt}(\mathbf{u} \bullet \mathbf{v})$
(B) $\frac{d}{dt}(\cos t\ \mathbf{u}(t))$

Solution

Example 4

Find the points t at which $\mathbf{r}(t)$ is orthogonal to $\mathbf{r}'(t)$ for the following:
(A) $\mathbf{r}(t) = \langle a\cos t, a\sin t\rangle$
(B) $\mathbf{r}(t) = \langle at^2 + 1, t\rangle$
(C) $\mathbf{r}(t) = \langle \cos t, \sin t, t\rangle$

Solution

Example 1

Let Σ be a sphere with the North Pole N and the South Pole S, and Π the Euclidean plane tangent to the sphere at the south pole S. Consider the stereographic projection $Proj$ (will be explained in tutorial, or google it) that is a function on the sphere to the plane. Define γ as the equator of Σ.

Consider the following scenario. Let $\overline{AB}$ be a chord of γ and P be the vertical plane containing $\overline{AB}$. P intersects Σ and forms a circular arc $\widehat{AB}$. Prove that $Proj(\widehat{AB})$ is a Poincare line of the circle $Proj(\gamma)$.

Do not worry about the technical terms; you will not need to know them. We will focus on how vectors help us solve this problem.

Solution