HKUST

MATH2011 *Introduction to Multivariable Calculus*

Midterm Examination (One Hour)

March 24, 2018

Full Name:____________________

Student I.D.:____________________

Lecture Section:____________________

Please note the following:

- Do NOT open the exam until instructed to do so.
- Have your student ID ready for checking.
- Do NOT use a calculator during the exam.
- You may write on both sides of the examination papers.
- You must show the steps in order to receive full mark.

Question No.	**Points**	**Out of**
1		20
2		15
3		15
4		15
5		15
6		20
Total		100

1. (20 pts)

   Compute the area of the region outside $r = 3 + 2\sin\theta$ and inside $r = 2$ in polar coordinates. [Hint: $\sin(\pi/6) = 1/2$. Have a sketch of the two curves before your computation.]

   Solution:

2. (15 pts)

(1) Write down the formula for the orthogonal projection of vector $\boldsymbol{b}$ onto vector $\boldsymbol{a}$ using dot products.

(2) Denote the orthogonal projection of $\boldsymbol{b}$ onto $\boldsymbol{a}$ by $Proj_{\boldsymbol{a}}\boldsymbol{b}$. Prove that $\boldsymbol{b} - Proj_{\boldsymbol{a}}\boldsymbol{b}$ is perpendicular to $\boldsymbol{a}$.

Solution:

3. (15 pts)

Given $\boldsymbol{u} = \langle 1, 2, 3 \rangle$ and $\boldsymbol{v} = \langle 4, 5, 6 \rangle$, find the constant c defined as follows:

(1) $c = \dfrac{\boldsymbol{u} \cdot \boldsymbol{v}}{\boldsymbol{v} \cdot \boldsymbol{u}}$.

(2) $\boldsymbol{u} \times \boldsymbol{v} = c\boldsymbol{v} \times \boldsymbol{u}$.

(3) $c = (a\boldsymbol{u} + b\boldsymbol{v}) \cdot (\boldsymbol{u} \times \boldsymbol{v})$, where a and b are two arbitrary constants.

Solution:

4. (15 pts)

Find the vector-valued function for the line tangent to the curve

$$\boldsymbol{r}(t) = \sin t\ \boldsymbol{i} + \sqrt{3}\sin t\ \boldsymbol{j} + 2\cos t$$

at $t = \pi/4$.

Solution:

5. (15 pts)

Compute the length of the curve $\boldsymbol{r}(t) = \sin t\ \boldsymbol{i} + \sqrt{3}\sin t\ \boldsymbol{j} + 2\cos t\ \boldsymbol{k}$ for the segment of $0 \le t \le \pi/2$.

Solution:

6. (20 pts)

Given four points $A(2,1,0),\ B(1,1,1),\ C(3,0,1),\ D(1,0,2)$.

(1) Find the equations of the plane ABC and plane BCD respectively.

(2) Find the vector-valued function for the intersection line of the above two planes.

Solution: