HKUST

MATH2011 *Introduction to Multivariable Calculus*

Midterm Examination (One Hour)

March 28, 2015

Name:__________________

Student I.D.:__________________

Lecture Section:__________________

Please note the following:

- Do not open the exam until instructed to do so.
- Have your student ID ready for checking.
- Do not use a calculator during the exam.
- You may write on both sides of the examination papers.
- You must show the steps in order to receive full credits.

Question No.	Points	Out of
1		20
2		25
3		20
4		20
5		15
Total		100

1. (20pts) Given a curve $\mathcal{C}$ determined by the parametric equations

$$\begin{cases} x = x(t) = t + 0.25t^2 \\ y = y(t) = 2^t \end{cases},$$

and a point A $(-0.75, 0.5)$ on the curve,
(a) (10pts) Calculate the slope of the tangent line at point A;
(b) (10pts) Find the equation of this tangent line in the form of $y = f(x)$.

2. (25pts) Given the equation of a closed curve $\mathcal{C}$ in polar coordinates as

$$r = r(\theta) = 1.5 + \cos\theta, \quad -\pi \le \theta \le \pi,$$

(a) (10pts) Make a sketch of this curve;
(b) (15pts) Calculate the area of the portion which is inside the closed curve $\mathcal{C}$ and outside the unit circle (of radius 1) centered at the origin.

3. (20pts) Given the vector-valued functions $\vec{u}(t) = 2t^3\vec{i} + (t^2 - 1)\vec{j} - 8\vec{k}$ and $\vec{v}(t) = e^t\vec{i} + 2e^{-t}\vec{j} - e^{t^2}\vec{k}$, find the derivatives of the following functions with respect to t:

(a) (10pts) $\vec{v}\left(\sqrt{t}\right)$;

(b) (10pts) $\vec{u}(t) \cdot \vec{v}(t)$.

4. (20pts) Given a curve determined by the vector-valued function

$$\vec{r}(t) = \left[\frac{1}{3}\cos(t^3) - 3\right]\vec{i} + \frac{1}{3}\sin(t^3)\vec{j} - t^2\vec{k},$$

calculate its arc length for $0 \leq t \leq \sqrt{5}$.

5. (15pts) The graph of $y = f(x)$ in the xy-plane has the parametrization $x = x$, $y = f(x)$, and the vector-valued function $\vec{r}(x) = x\vec{i} + f(x)\vec{j}$. Use $\vec{r}(x)$ to derive a formula for the curvature of the graph. Here it is assumed that $f'(x)$ and $f''(x)$ both exist.