Q1. $\begin{cases} r = 3+2\sin\theta \\ r = 2 \end{cases}$

$3+2\sin\theta = 2$, $\sin\theta = -\frac{1}{2}$

$\theta = \frac{7}{6}\pi$, $\frac{11}{6}\pi$ at intersections

$$\text{Area} = \frac{1}{2}\int_{\frac{7}{6}\pi}^{\frac{11}{6}\pi}\left[2^2 - (3+2\sin\theta)^2\right] d\theta$$

$$= \frac{1}{2}\int_{\frac{7}{6}\pi}^{\frac{11}{6}\pi}(4-9-12\sin\theta-4\sin^2\theta)\,d\theta$$

$$= \frac{1}{2}\int_{\frac{7}{6}\pi}^{\frac{11}{6}\pi}(-5-12\sin\theta-4\sin^2\theta)\,d\theta$$

$$= \frac{1}{2}\int_{\frac{7}{6}\pi}^{\frac{11}{6}\pi}(-5-12\sin\theta-2+2\cos 2\theta)\,d\theta$$

$$= \frac{1}{2}\int_{\frac{7}{6}\pi}^{\frac{11}{6}\pi}(-7-12\sin\theta+2\cos 2\theta)\,d\theta$$

$$= -\frac{7}{\cancel{2}_1}\left(\frac{\cancel{4}^{\cancel{2}\,1}\pi}{\cancel{6}_3}\right) + 6\cos\theta\Big|_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} + \frac{1}{2}\sin 2\theta\Big|_{\frac{7\pi}{6}}^{\frac{11\pi}{6}}$$

$$= -\frac{7}{3}\pi + 6\left(\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right) + \frac{1}{2}\left(-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}\right)$$

$$= -\frac{7}{3}\pi + \frac{11}{2}\sqrt{3}$$

Q2. $$\text{Proj}_{\vec{a}}\vec{b} = \frac{(\vec{b}\cdot\vec{a})\vec{a}}{\vec{a}\cdot\vec{a}}$$

$$(\vec{b} - \text{Proj}_{\vec{a}}\vec{b})\cdot\vec{a}$$

$$= \vec{b}\cdot\vec{a} - \frac{(\vec{b}\cdot\vec{a})(\vec{a}\cdot\vec{a})}{(\vec{a}\cdot\vec{a})} = 0$$

Q3

(1) $c = 1$

(2) $c = -1$

(3) $c = 0$

Q4

$$\vec{r}(t) = \langle \sin t, \sqrt{3}\sin t, 2\cos t \rangle$$

$$\vec{r}'(t) = \langle \cos t, \sqrt{3}\cos t, -2\sin t \rangle$$

$$\vec{r}\left(\tfrac{\pi}{4}\right) = \left\langle \frac{\sqrt{\cancel{4}2}}{2}, \frac{\sqrt{6}}{2}, \sqrt{2} \right\rangle$$

$$= \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{6}}{2}, \sqrt{2} \right\rangle$$

$$\vec{r}'\left(\tfrac{\pi}{4}\right) = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{6}}{2}, -\sqrt{2} \right\rangle$$

Equation of tangent line

$$\vec{r} = \vec{r}\left(\tfrac{\pi}{4}\right) + \vec{r}'\left(\tfrac{\pi}{4}\right)t$$

$$= \left\langle \frac{\sqrt{2}}{2}(1+t), \frac{\sqrt{6}}{2}(1+t), \sqrt{2}(1-t) \right\rangle$$

Q5

$$|\vec{r}'(t)| = \sqrt{4} = 2$$

$$L = \int_0^{\pi/2} 2\, dt = \pi$$

Q6

A $(2,1,0)$

B $(1,1,1)$

C $(3,0,1)$

D $(1,0,2)$

(a)

$$\vec{n}_1 = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 0 & 1 \\ 1 & -1 & 1 \end{vmatrix} = \langle 1, 2, 1 \rangle$$

Plane ABC : $\langle 1, 2, 1 \rangle \cdot \langle x-2, y-1, z-0 \rangle = 0$

$$x - 2 + 2y - 2 + z = 0$$

$$x + 2y + z - 4 = 0$$

$$\vec{n}_2 = \vec{BC} \times \vec{BD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 0 \\ 0 & -1 & 1 \end{vmatrix} = \langle -1, -2, -2 \rangle$$

Plane BCD : $-1(x-1) - 2(y-1) - 2(z-1) = 0$

$$x + 2y + 2z - 5 = 0$$

(b) $$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ -1 & -2 & -2 \end{vmatrix} = \langle -2, 1, 0 \rangle$$

$$\vec{V} = \langle -2, 1, 0 \rangle$$

A point on the line $\begin{cases} x + 2y + z = 4 \\ x + 2y + 2z = 5 \end{cases}$

If $x = 3$, then $\begin{cases} 2y + z = 1 \\ 2y + 2z = 2 \end{cases}$ $\begin{cases} y = 0 \\ z = 1 \end{cases}$

$(3, 0, 1)$

$$\vec{r} = \langle 3, 0, 1 \rangle + \langle -2, 1, 0 \rangle t$$