1. (10pts) Calculate the following partial derivatives.
   (a) (5pts) Given $w = 4e^x \ln y$, $x = \ln(u \cos v)$, and $y = u \sin v$, calculate $\partial w/\partial u$ and express it in terms of u and v.
   (b) (5pts) Given $xe^{x^2 y} + ye^z + 2\ln x - 2 - 3\ln 2 = 0$, calculate $\partial z/\partial x$.

2. (15pts) Locate the absolute maximum and the absolute minimum of the function

$$f(x, y) = x^2 + 2y^2 - x$$

in the closed and bounded region Ω which is enclosed by the unit circle $x^2 + y^2 = 1$ on the xy plane.

3. (15pts) Calculate the double integral

$$\iint_{\Omega} 2xe^y \mathrm{d}x\mathrm{d}y,$$

where Ω denotes the region enclosed by $y = x^2$, $y = 4$, and $x + y = 2$ on the xy plane.

4. (20pts) Consider two unit circles C_1 and C_2 (of radius 1), with C_1 centered at $(0, 0)$ and C_2 centered at $(1, 0)$. Calculate the area of the portion that is inside C_2 but outside C_1.

5. (20pts) Calculate the volume of the region D bounded by the graphs of

$$z = 4x^2 + y^2 \quad \text{and} \quad z = -4x^2 + 8x - y^2 + 6.$$

6. (20pts) Consider the vector field

$$\vec{F}(x,y,z) = P(x,y,z)\,\vec{i} + Q(x,y,z)\,\vec{j} + R(x,y,z)\,\vec{k},$$

where $P(x,y,z) = e^x \ln y$, $Q(x,y,z) = e^x/y + \sin z$, and $R(x,y,z) = y\cos z$. Verify that $\vec{F}$ is a conservative field and calculate the potential function $f(x,y,z)$ with $\nabla f = \vec{F}$.

1\. a)

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u}$$

$$= 4e^x \ln y \frac{1}{u\cos v}\cos v + 4e^x \frac{1}{y}\sin v$$

$$= 4\cancel{u}\cos v \ln(u\sin v)\frac{1}{\cancel{u\cos v}}\cancel{\cos v} + 4\cancel{u}\cos v \frac{1}{\cancel{u\sin v}}\cancel{\sin v}$$

$$= 4\cos v \ln(u\sin v) + 4\cos v$$

$$= 4\cos v\,[\ln(u\sin v) + 1]$$

b)

$$e^{x^2y} + x e^{x^2y}(2xy) + y e^z \frac{\partial z}{\partial x} + \frac{2}{x} = 0$$

$$(1 + 2x^2y)e^{x^2y} + y e^z \frac{\partial z}{\partial x} + \frac{2}{x} = 0$$

$$\frac{\partial z}{\partial x} = \frac{-(1+2x^2y)e^{x^2y} - \frac{2}{x}}{y e^z}$$

$$= -\frac{(1+2x^2y)x e^{x^2y} + 2}{x y e^z}$$

2. $f_x = 2x-1$, $f_y = 4y$

$f_x = 0$ and $f_y = 0$ $\rightarrow$ critical point $(\frac{1}{2}, 0)$

Note that $(\frac{1}{2}, 0)$ is an interior point

$f(\frac{1}{2}, 0) = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$ $\leftarrow$ abs. min.

At the boundary, $x = \cos t$, $y = \sin t$.

$f(x, y) = \cos^2 t + 2\sin^2 t - \cos t$

$= 1 + \sin^2 t - \cos t = g(t)$

$g'(t) = 2\sin t \cos t + \sin t = 0$

$\sin t = 0$, $t = 0$ and π

$x = \pm 1$, $y = 0$

and $\cos t = -\frac{1}{2}$, $t = \pm \frac{2}{3}\pi$

$x = -\frac{1}{2}$, $y = \pm \frac{\sqrt{3}}{2}$

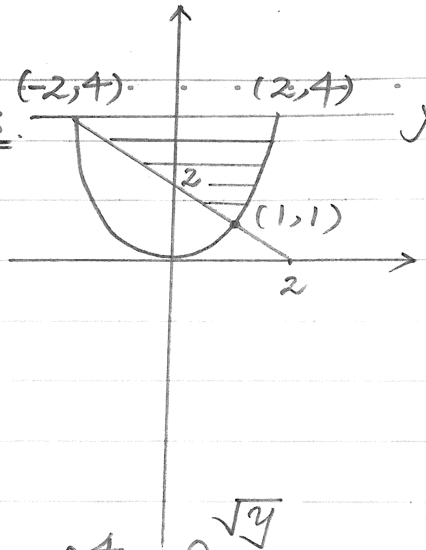
$f(1, 0) = 0$, $f(-1, 0) = 2$

$f(-\frac{1}{2}, \pm\frac{\sqrt{3}}{2}) = \frac{1}{4} + 2(\frac{3}{4}) + \frac{1}{2} = \frac{9}{4}$ $\leftarrow$ abs max.

Absolute minimum at $(\frac{1}{2}, 0)$

Absolute maxima at $(-\frac{1}{2}, \pm\frac{\sqrt{3}}{2})$

3.



$$\iint_{\Omega} f(x,y)\,dA = \int_1^4 \int_{2-y}^{\sqrt{y}} f(x,y)\,dx\,dy$$

$$\int_1^4 \int_{2-y}^{\sqrt{y}} 2xe^y\,dx\,dy = \int_1^4 e^y\, x^2 \Big|_{x=2-y}^{x=\sqrt{y}} dy$$

$$= \int_1^4 e^y(-4+5y-y^2)\,dy$$

① $\int_1^4 e^y\,dy = e^4 - e$

② $\int_1^4 ye^y\,dy = \int_1^4 y\,de^y = ye^y\Big|_1^4 - \int_1^4 e^y\,dy$

$$= ye^y\Big|_1^4 - e^y\Big|_1^4 = (4e^4-e)-(e^4-e)$$
$$= 3e^4$$

③ $\int y^2e^y\,dy = \int y^2\,de^y = y^2e^y - \int e^y\,dy^2$

$$= y^2e^y - 2\int ye^y\,dy = y^2e^y - 2ye^y + 2e^y + C$$

$$\int_1^4 y^2e^y\,dy = (y^2-2y+2)e^y\Big|_1^4 = 10e^4 - e$$

$$\int_1^4 e^y(-4+5y-y^2)\,dy$$

$$= -4(e^4-e) + 5(3e^4) - (10e^4-e)$$
$$= e^4 + 5e$$

4. $x^2+y^2=1$, $r=1$, for C_1

$(x-1)^2+y^2=1$, $(r\cos\theta-1)^2+(r\sin\theta)^2=1$

$r^2\cos^2\theta - 2r\cos\theta \cancel{+1} + r^2\sin^2\theta = \cancel{1}$

$r = 2\cos\theta$ for C_2

Intersection : $2\cos\theta = 1$, $\theta = \pm\frac{\pi}{3}$

$$\text{Area} = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \int_{1}^{2\cos\theta} r \, dr \, d\theta$$

$$= \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{2} r^2 \Big|_{1}^{2\cos\theta} d\theta$$

$$= \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \left(2\cos^2\theta - \frac{1}{2}\right) d\theta$$

$$= \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \left(\cos 2\theta + \frac{1}{2}\right) d\theta$$

$$= \frac{1}{2}\sin 2\theta \Big|_{-\frac{\pi}{3}}^{\frac{\pi}{3}} + \frac{\pi}{3} = \frac{\sqrt{3}}{2} + \frac{\pi}{3}$$

5. Intersection $4x^2+y^2 = -4x^2+8x-y^2+6$

$(x-\frac{1}{2})^2 + \frac{y^2}{4} = 1$

region R enclosed by the above ellipse on the xy plane

$y = \pm\sqrt{3-4x^2+4x}$ for $x \in [\frac{1}{2}-1, \frac{1}{2}+1]$

$$\text{Volume} = \int_{-\frac{1}{2}}^{\frac{3}{2}} \int_{-\sqrt{3-4x^2+4x}}^{\sqrt{3-4x^2+4x}} [-4x^2+8x-y^2+6-(4x^2+y^2)]\,dy\,dx$$

$$= \int_{-\frac{1}{2}}^{\frac{3}{2}} \int_{-\sqrt{\cdots}}^{\sqrt{\cdots}} (-8x^2+8x+6-2y^2)\,dy\,dx$$

$$= \int_{-\frac{1}{2}}^{\frac{3}{2}} \left[(-8x^2+8x+6)y - \frac{2}{3}y^3\right]\Bigg|_{-\sqrt{-4x^2+4x+3}}^{\sqrt{-4x^2+4x+3}} dx$$

$$= \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{8}{3}(-4x^2+4x+3)^{\frac{3}{2}}\,dx$$

Note $-4x^2+4x+3 = 4[1-(x-\frac{1}{2})^2]$

$$= \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{64}{3}[1-(x-\tfrac{1}{2})^2]^{\frac{3}{2}}\,dx \qquad u = x-\frac{1}{2}$$

$$= \frac{64}{3}\int_{-1}^{1}(1-u^2)^{\frac{3}{2}}\,du \qquad u = \sin t$$

$$= \frac{64}{3}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^4 t\,dt$$

$$\cos^4 t = (\cos^2 t)^2 = \left(\frac{1+\cos 2t}{2}\right)^2 = \frac{1+2\cos 2t+\cos^2 2t}{4}$$

$$= \frac{1}{4}\left(1+2\cos 2t + \frac{1+\cos 4t}{2}\right)$$

$$\text{Volume} = \frac{64}{3}\times\frac{3}{8}\pi = \underline{8\pi}$$ #

6 $\quad \dfrac{\partial Q}{\partial z} = \cos z$, $\dfrac{\partial R}{\partial y} = \cos z$ equal

$\dfrac{\partial R}{\partial x} = 0$, $\dfrac{\partial P}{\partial z} = 0$ equal

$\dfrac{\partial P}{\partial y} = \dfrac{e^x}{y}$, $\dfrac{\partial Q}{\partial x} = \dfrac{e^x}{y}$ equal.

Conservative

$$f_x = P = e^x \ln y$$

$$f = \int f_x \, dx = e^x \ln y + g(y,z)$$

$$f_y = \frac{e^x}{y} + g_y = Q = \frac{e^x}{y} + \sin z$$

$$g_y = \sin z$$

$$g = \int g_y \, dy = y \sin z + h(z)$$

$$f = e^x \ln y + y \sin z + h(z)$$

$$f_z = y \cos z + h'(z) = R = y \cos z$$

$$h'(z) = 0 , \qquad h(z) = \text{Const.}$$

$$f = e^x \ln y + y \sin z + \text{Const.}$$