

Plato Republic 527.a

Plato, Republic, trans. C. Emlyn-Jones & W. Preddy, Cambridge MA: Harvard University Press, 2013

Οὐ τοίνυν τοῦτό γε, ἣν δ' ἐγώ, ἀμφισβητήσουσιν ἡμῖν ὅσοι καὶ σμικρὰ γεωμετρίας ἔμπειροι, ὅτι αὕτη ἡ ἐπιστήμη πᾶν τούναντίον ἔχει τοῖς ἐν αὐτῇ λόγοις λεγομένοις ὑπὸ τῶν μεταχειριζομένων.

Πῶς; ἔφη.

Λέγουσι μὲν που μάλα γελοίως τε καὶ ἀναγκαίως· ὥς γὰρ πράττοντές τε καὶ πράξεως ἔνεκα πάντας τοὺς λόγους ποιούμενοι λέγουσιν τετραγωνίζειν τε καὶ παρατείνειν καὶ προστιθέναι καὶ πάντα οὕτω φθεγγόμενοι, τὸ δ' ἔστι που πᾶν τὸ μάθημα γνώσεως ἔνεκα ἐπιτηδευόμενον.

“Therefore,” I said, “those who are experienced in the finer points of geometry will not dispute with us this at least: that this knowledge contains everything that’s the opposite to the arguments put forward in it by those who engage in it.”

“How do you mean?” he asked.

“I think the way they argue is quite absurd and is forced on them: I mean, they talk as if they were doing something and making all their terms to fit their activity: **they talk about making the square, applying and adding**, and similarly with everything else; but in my view the subject as a whole is studied for the sake of knowledge.”

Euclid Elements 3.prop 35- prop37

Euclid, *Elements*, trans. D. E. Joyce, <http://aleph0.clarku.edu/~djoyce/java/elements/> (Accessed 22nd July 2023)

Proposition 35.

If in a circle two straight lines cut one another, then the rectangle contained by the segments of the one equals the rectangle contained by the segments of the other.

Ἐὰν ἐν κύκλῳ δύο εὐθεῖαι τέμνωσιν ἀλλήλας, τὸ ὑπὸ τῶν τῆς μιᾶς τμημάτων περιεχόμενον ὀρθογώνιον ἴσον ἐστὶ τῷ ὑπὸ τῶν τῆς ἐτέρας τμημάτων περιεχομένῳ ὀρθογώνιῳ.

Proposition 36.

If a point is taken outside a circle and two straight lines fall from it on the circle, and if one of them cuts the circle and the other touches it, then the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference equals the square on the tangent.

Ἐάν κύκλου ληφθῇ τι σημεῖον ἐκτός καὶ ἀπ' αὐτοῦ πρὸς τὸν κύκλον προσπίπτωσι δύο εὐθεῖαι, καὶ ἡ μὲν αὐτῶν τέμνῃ τὸν κύκλον, ἡ δὲ ἐφαπται, ἔσται τὸ ὑπὸ ὅλης τῆς τεμονούσης καὶ τῆς ἐκτὸς ἀπολαμβανομένης μεταξὺ τοῦ τε σημείου καὶ τῆς κυρτῆς περιφερείας ἴσον τῷ ἀπὸ τῆς ἐφαπτομένης τετραγώνῳ.

Proposition 37.

If a point is taken outside a circle and from the point there fall on the circle two straight lines, if one of them cuts the circle, and the other falls on it, and if further the rectangle contained by the whole of the straight line which cuts the circle and the straight line intercepted on it outside between the point and the convex circumference equals the square on the straight line which falls on the circle, then the straight line which falls on it touches the circle.

Ἐάν κύκλου ληφθῇ τι σημεῖον ἐκτός, ἀπὸ δε τοῦ σημείου πρὸς τὸν κύκλον προσμίπτωσι δύο εὐθεῖαι, καὶ ἡ μὲν αὐτῶν τέμνῃ τὸν κύκλον, ἡ δὲ προσπίπτῃ, ἥ δὲ τὸ ὑπὸ [τῆς] ὅλης τῆς τεμονούσης καὶ τῆς ἐκτὸς ἀπολαμβανομένης μεταξὺ τοῦ τε σημείου καὶ τῆς κυρτῆς περιφερείας ἴσον τῷ ἀπὸ τῆς προσπιπτούσης, ἡ προσπίπτουσα ἐφάπεται τοῦ κύκλου.

Euclid *Elements*, 4.def 1- def 7 & prop 1 – prop 16

Euclid, *Elements*, trans. D. E. Joyce, <http://aleph0.clarku.edu/~djoyce/java/elements/> (Accessed 22nd July 2023)

Definition 1

A rectilinear figure is said to be inscribed in a rectilinear figure when the respective angles of the inscribed figure lie on the respective sides of that in which it is inscribed.

Σχήμα εὐθύγραμμον εἰς σχῆμα εὐθύγραμμον ἐγγράφεσθαι λέγεται, ὅταν ἐκάστη τῶν τοῦ ἐγγραφομένου σχήματος γωνιῶν ἐκάστης πλευρᾶς τοῦ, εἰς ὃ ἐγγράφεται, ᾗπνηται.

Definition 2

Similarly a figure is said to be circumscribed about a figure when the respective sides of the circumscribed figure pass through the respective angles of that about which it is circumscribed.

Σχήμα δὲ ὁμοίως περὶ σχῆμα περιγράφεσθαι λέγεται, ὅταν ἐκάστη πλευρὰ τοῦ περιγραφομένου ἐκάστης γωνίας τοῦ, περὶ ὃ περιγράφεται, ᾗπνηται.

Definition 3

A rectilinear figure is said to be inscribed in a circle when each angle of the inscribed figure lies on the circumference of the circle.

Σχήμα εὐθύγραμμον εἰς κύκλον ἐγγράφεσθαι λέγεται, ὅταν ἐκάστη γωνία τοῦ ἐγγραφομένου ἅπτηται τῆς τοῦ κύκλου περιφερείας.

Definition 4

A rectilinear figure is said to be circumscribed about a circle when each side of the circumscribed figure touches the circumference of the circle.

Σχήμα δὲ εὐθύγραμμον περὶ κύκλον περιγράφεσθαι λέγεται, ὅταν ἐκάστη πλευρὰ τοῦ περιγραφομένου ἐφάπτηται τῆς τοῦ κύκλου περιφερείας.

Definition 5

Similarly a circle is said to be inscribed in a figure when the circumference of the circle touches each side of the figure in which it is inscribed.

Κύκλος δὲ εἰς σχῆμα ὁμοίως ἐγγράφεσθαι λέγεται, ὅταν ἡ τοῦ κύκλου περιφέρεια ἐκάστης πλευρᾶς τοῦ, εἰς ὃ ἐγγράφεται, ἅπτηται.

Definition 6

A circle is said to be circumscribed about a figure when the circumference of the circle passes through each angle of the figure about which it is circumscribed.

Κύκλος δὲ περὶ σχῆμα περιγράφεσθαι λέγεται, ὅταν ἡ τοῦ κύκλου περιφέρεια ἐκάστης γωνίας τοῦ, περὶ ὃ περιγράφεται, ἅπτηται.

Definition 7

A straight line is said to be fitted into a circle when its ends are on the circumference of the circle.

Εὐθεῖα εἰς κύκλον ἐναρμόζεσθαι λέγεται, ὅταν τὰ πέρατα αὐτῆς ἐπὶ τῆς περιφερείας ᾗ τοῦ κύκλου.

Proposition 1

To fit into a given circle a straight line equal to a given straight line which is not greater than the diameter of the circle.

Εἰς τὸν δοθέντα κύκλον τῇ δοθείσῃ εὐθείᾳ μὴ μείζονι οὕσῃ τῆς τοῦ κύκλου διαμέτρου ἴσην εὐθεῖαν ἐναρμόσαι.

Proposition 2

To inscribe in a given circle a triangle equiangular with a given triangle.

Εἰς τὸν δοθέντα κύκλον τῷ δοθέντι τριγώνῳ ἰσογώνιον τρίγωνον ἐγγράψαι.

Proposition 3

To circumscribe about a given circle a triangle equiangular with a given triangle.

Περὶ τὸν δοθέντα κύκλον τῷ δοθέντι τριγώνῳ ἰσογώνιον τρίγωνον περιγράψαι.

Proposition 4

To inscribe a circle in a given triangle.

Εἰς τὸ δοθὲν τρίγωνον κύκλον ἐγγράψαι.

Proposition 5

To circumscribe a circle about a given triangle.

Περὶ τὸ δοθὲν τρίγωνον κύκλον περιγράψαι.

Corollary. When the center of the circle falls within the triangle, the triangle is acute-angled; when the center falls on a side, the triangle is right-angled; and when the center of the circle falls outside the triangle, the triangle is obtuse-angled.

Καὶ φανερόν, ὅτι, ὅτε μὲν ἐντὸς τοῦ τριγώνου πίπτει τὸ κέντρον τοῦ κύκλου, ἢ ὑπὸ ΒΑΓ γωνία ἐν μείζονι τμήματι τοῦ ἡμικυκλίου τυγχάνουσα ἐλάττων ἐστὶν ὀρθῆς· ὅτε δὲ ἐπὶ τῆς ΒΓ εὐθείας τὸ κέντρον πίπτει, ἢ ὑπὸ ΒΑΓ γωνία ἐν ἡμικυκλίῳ τυγχάνουσα ὀρθή ἐστίν· ὅτε δὲ τὸ κέντρον τοῦ κύκλου ἐκτὸς τοῦ τριγώνου πίπτει, ἢ ὑπὸ ΒΑΓ ἐν ἐλάττονι τμήματι τοῦ ἡμικυκλίου τυγχάνουσα μείζων ἐστὶν ὀρθῆς.

Proposition 6

To inscribe a square in a given circle.

Εἰς τὸν δοθέντα κύκλον τετράγωνον ἐγγράψαι.

Proposition 7

To circumscribe a square about a given circle.

Περὶ τὸν δοθέντα κύκλον τετράγωνον περιγράψαι.

Proposition 8

To inscribe a circle in a given square.

Εἰς τὸ δοθὲν τετράγωνον κύκλον ἐγγράψαι.

Proposition 9

To circumscribe a circle about a given square.

Περὶ τὸ δοθὲν τετράγωνον κύκλον περιγράψαι.

Proposition 10

To construct an isosceles triangle having each of the angles at the base double the remaining one.

Ἴσοσκελὲς τρίγωνον συστήσασθαι ἔχον ἑκατέραν τῶν πρὸς τῇ βάσει γωνιῶν διπλασίονα τῆς λοιπῆς.

Proposition 11

To inscribe an equilateral and equiangular pentagon in a given circle.

Εἰς τὸν δοθέντα κύκλον πεντάγωνον ἰσόπλευρόν τε καὶ ἰσογώνιον ἐγγράψαι.

Proposition 12

To circumscribe an equilateral and equiangular pentagon about a given circle.

Περὶ τὸν δοθέντα κύκλον πεντάγωνον ἰσόπλευρόν τε καὶ ἰσογώνιον περιγράψαι.

Proposition 13

To inscribe a circle in a given equilateral and equiangular pentagon.

Εἰς τὸ δοθὲν πεντάγωνον, ὃ ἐστὶν ἰσόπλευρόν τε καὶ ἰσογώνιον, κύκλον ἐγγράψαι. @1 Proposition 14

Proposition 14

To circumscribe a circle about a given equilateral and equiangular pentagon.

Περὶ τὸ δοθὲν πεντάγωνον, ὃ ἐστὶν ἰσόπλευρόν τε καὶ ἰσογώνιον, κύκλον περιγράψαι.

Proposition 15

To inscribe an equilateral and equiangular hexagon in a given circle.

Εἰς τὸν δοθέντα κύκλον ἑξάγωνον ἰσόπλευρόν τε καὶ ἰσογώνιον ἐγγράψαι.

Corollary. The side of the hexagon equals the radius of the circle. And, in like manner as in the case of the pentagon, if through the points of division on the circle we draw tangents to the circle, there will be circumscribed about the circle an equilateral and equiangular hexagon in conformity with what was explained in the case of the pentagon. And further by means similar to those explained in the case of the pentagon we can both inscribe a circle in a given hexagon and circumscribe one about it.

Ἐκ δὴ τούτου φανερόν, ὅτι ἡ τοῦ ἐξαγώνου πλευρὰ ἴση ἐστὶ τῇ ἐκ τοῦ κέντρου τοῦ κύκλου. Ὁμοίως δὲ τοῖς ἐπὶ τοῦ πενταγώνου ἐὰν διὰ τῶν κατὰ τὸν κύκλον διαιρέσεων ἐφαπτομένης τοῦ κύκλου ἀγάγωμεν, περιγραφήσεται περὶ τὸν κύκλον ἐξαγώνον ἰσόπλευρόν τε καὶ ἰσογώνιον ἀκολουθῶς τοῖς ἐπὶ τοῦ πενταγώνου εἰρημένοις. καὶ ἔτι διὰ τῶν ὁμοίων τοῖς ἐπὶ τοῦ πενταγώνου εἰρημένοις εἰς τὸ δοθὲν ἐξαγώνον κύκλον ἐγγράψομεν τε καὶ περιγράψομεν· ὅπερ ἔδει ποιῆσαι.

Proposition 16

To inscribe an equilateral and equiangular fifteen-angled figure in a given circle.

Εἰς τὸν δοθέντα κύκλον πεντεκαίδεκάγωνον ἰσόπλευρόν τε καὶ ἰσογώνιον ἐγγράψαι.

Corollary. And, in like manner as in the case of the pentagon, if through the points of division on the circle we draw tangents to the circle, there will be circumscribed about the circle a fifteen-angled figure which is equilateral and equiangular. And further, by proofs similar to those in the case of the pentagon, we can both inscribe a circle in the given fifteen-angled figure and circumscribe one about it.

Ὁμοίως δὲ τοῖς ἐπὶ τοῦ πενταγώνου ἐὰν διὰ τῶν κατὰ τὸν κύκλον διαιρέσεων ἐφαπτομένης τοῦ κύκλου ἀγάγωμεν, περιγραφήσεται περὶ τὸν κύκλον πεντεκαίδεκάγωνον ἰσόπλευρόν τε καὶ ἰσογώνιον. ἔτι δὲ διὰ τῶν ὁμοίων τοῖς ἐπὶ τοῦ πενταγώνου δείξεων καὶ εἰς τὸ δοθὲν πεντεκαίδεκάγωνον κύκλον ἐγγράψομεν τε καὶ περιγράψομεν· ὅπερ ἔδει ποιῆσαι.

Euclid Elements, 12.prop 1- prop 2

Euclid, *Elements*, trans. D. E. Joyce, <http://alepho.clarku.edu/~djoyce/java/elements/> (Accessed 22nd July 2023)

Proposition 1

Similar polygons inscribed in circles are to one another as the squares on their diameters.

Τὰ ἐν τοῖς κύκλοις ὅμοια πολύγωνα πρὸς ἀλλήλα ἐστὶν ὡς τὰ ἀπὸ τῶν διαμέτρων τετράγωνα.

Proposition 2

Circles are to one another as the squares on their diameters.

Οἱ κύκλοι πρὸς ἀλλήλους εἰσὶν ὡς τὰ ἀπὸ τῶν διαμέτρων τετράγωνα.

Galen On the Diagnosis and Cure of the Soul's Errors V, 58–103 K.

Galen, *On the Passions and Errors of the Soul*, trans. P. W. Harkins, Ohio: Ohio State University Press, 1963

For example, let us suppose we are instructed to draw a circle around a given square or, in the same way, to draw a square around or within a given circle and, again, to draw a circle around a given pentagon which has equal sides and equal angles. If anyone is able immediately to inscribe or circumscribe each of these figures by the method he has learned, by doing this very thing he will give evidence that he has discovered the object of his search. But the subject matter itself cannot give such evidence in such a question as whether the world did or did not begin to exist; nor can it tell whether the universe is finite or infinite, nor how great is the number of the ocean's waves. No question of this sort is decided by the very subject matter we seem to have here. However, if you are instructed (to draw) a polygon of twelve equal sides and angles (around or within) a circle, you will do it immediately. And in fact, the polygon is clearly seen as inscribed or circumscribed just as the circle is seen as circumscribed or inscribed with respect to the polygon.

οἷον ὅταν περιγράψαι τῷ δοθέντι τετραγώνῳ κύκλον ἐπιταχῶμεν, κατὰ ταῦτα δὲ καὶ τῷ δοθέντι κύκλῳ περιγράψαι τετράγωνον ἢ ἐγγράψαι <καὶ> πάλιν περὶ τὸ δοθὲν πεντάγωνον ἰσόπλευρόν τε καὶ ἰσογώνιον κύκλον περιγράψαι· ἐὰν γὰρ τις ἕκαστον τῶν τοιούτων δι' ἧς ἔμαθεν μεθόδου περιγράψαι παραχρῆμα δυνηθῇ, πρὸς αὐτοῦ τοῦ πράγματος μαρτυρῆσεται τὸ ζητούμενον εὐρηκῶς. οὐ μὴν εἰ γέγονεν ἢ ἀγέννητος ὁ κόσμος τὸ ζητούμενον εὐρηκῶς. οὐ μὴν εἰ γέγονεν ἢ ἀγέννητος ὁ κόσμος ἐστί, δύναται τὸ πρᾶγμα αὐτὸ μαρτυρῆσαι, καθάπερ οὐδὲ εἰ πεπερασμένον ἢ ἄπειρον τὸ πᾶν ἢ πόσος ὁ τῶν κυμάτων ἀριθμός· οὐδὲν γὰρ τῶν τοιούτων ζητημάτων ὑπ' αὐτοῦ τοῦ δοκοῦντος εὐρίσκεισθαι πράγματος ἐπικρίνεται, καθάπερ, ἐὰν ἐπιταχθῇ τῷ κύκλῳ δωδεκάγωνον ἰσόπλευρόν τε καὶ ἰσογώνιον <περιγράψαι | ἢ ἐγγράψαι,> εὐθέως τοῦτο ποιήσει· καὶ γὰρ ἐγγραφόμενόν τε καὶ περιγραφόμενον ἐναργῶς ὁράται, ὥσπερ γε καὶ ὁ κύκλος ἐγγραφόμενός τε καὶ περιγραφόμενος τῷ τοιούτῳ πολυγώνῳ.

https://kb.osu.edu/bitstream/handle/1811/28933/GALEN_ON_THE_PASSIONS_AND_ERRORS_OF_THE_SOUL.pdf?sequence=1

Joannes Philoponis *Commentary on Aristotle's Physics* 1.1-3 1.31.1-32.1

Joannes Philoponis, *Commentary on Aristotle's Physics*, trans. C. Osbourne, London: Bloomsbury Academic, 2006

185a16 “For example it is the business of the geometer to refute the quadrature of the circle that is done by means of segments, but not the one by Antiphon.”

Hippocrates was a merchant of Chios, who was a victim of piracy and lost everything. He travelled to Athens to bring a case against the pirates and while he was staying in Athens, for a long time on account of the court case, he attended philosophy classes, and reached such a high standard in geometry that he attempted to discover the quadrature of the circle. In fact he did not discover that; having squared the lune, he

thought, wrongly, that he could go on from there to square the circle. For he thought that one could also deduce the quadrature of the circle from the quadrature of the lune.

The other man, Antiphon, also attempted to square the circle, but without preserving the first principles of geometry. This is how he tried to do it: 'Suppose I construct a circle', he says, 'and inscribe a square in it; then I divide in half the segments of the circle resulting from the square; then I draw straight lines in either direction from this division to the extremities of the segments, and make an octagonal figure. Now suppose that again we divide the segments that encompass the angles each in half, and again draw straight lines from these divisions to the extremities of the segments in either direction, we shall make a polygonal figure. Suppose, therefore, that we do this over and over, the result will be a figure of a great many angles, having angles that are extremely slight, whose encompassing straight sides coincide with the circle on account of being so small. Granted that any given rectilinear figure can be squared, if I square this polygon, since it coincides with the circle I shall have squared the circle as well'. Hence Antiphon denies the principles of geometry; for it is a geometrical principle that a straight line never coincides with an arc of a circle, but Antiphon allows that, due to smallness, a certain straight line coincides with a certain arc. So Hippocrates, setting out from geometrical principles, and having squared some lunate segment of the circle, drew his next conclusion wrongly, in that he wanted to deduce the quadrature of the circle from this as well. Whereas Antiphon drew his next conclusion by denying the principles of geometry, that a straight line never coincides with an arc. So Aristotle says that, as regards proving that the quadrature of the circle is invalid, it is the business of the geometrician to refute Hippocrates' quadrature of the circle, since Hippocrates preserves the principles of geometry; but the geometer will not go on to refute Antiphon's, because he derives his conclusion by denying the principles of geometry.

Οἷον τὸν τετραγωνισμὸν τοῦ κύκλου τὸν μὲν διὰ τῶν τμημάτων γεωμετρικοῦ διαλῦσαι, τὸν δὲ Ἀντιφώντος οὐ.

Ἰπποκράτης Χιός τις ὢν ἔμπορος, ληστρικῇ νηὶ περιπεσὼν καὶ πάντα ἀπολέσας, ἦλθεν Ἀθήναζε γραψόμενος τοὺς ληστές, καὶ πολλὴν παραμένων ἀπολέσας, ἦλθεν Ἀθήναζε γραψόμενος τοὺς ληστές, καὶ πολλὴν παραμένων ἐν Ἀθήναις διὰ τὴν γραφὴν χρόνον, ἐφοίτησεν εἰς φιλοσόφους, καὶ εἰς τοσοῦτον ἔξεως γεωμετρικῆς ἦλθεν, ὥς ἐπιχειρῆσαι εὐρεῖν τὸν κύκλου τετραγωνισμόν. καὶ αὐτὸν μὲν οὐχ εὔρε, τετραγωνίσας δὲ τὸν μηνίσκον ᾧ ἦθη ψευδῶς ἐκ τούτου καὶ τὸν κύκλον τετραγωνίζειν. ἐκ γὰρ τοῦ τετραγωνισμοῦ τοῦ μηνίσκου καὶ τὸν τοῦ κύκλου τετραγωνισμὸν ᾧ ἦθη συλλογίζεσθαι. ὁ δὲ Ἀντιφών καὶ αὐτὸς ἐπεχείρησε τετραγωνίσαι τὸν κύκλον, ἀλλ' οὐ σφίζων τὰς γεωμετρικὰς ἀρχάς. ἐπεχείρησε δὲ οὕτως. ἐάν, φησί, ποιήσω κύκλον καὶ γράψω ἐντὸς τετράγωνον, τέμω δὲ τὰ τμήματα τοῦ κύκλου τὰ γενόμενα ἐκ τοῦ τετραγώνου δίχα, εἴτα ἀγάγω εὐθείας ἀπὸ τῆς τομῆς ἐκατέρωθεν ἐπὶ τὰ πέρατα τοῦ τμήματος, ποιῶ ὀκτάγωνον σχῆμα. ἐάν δὲ πάλιν τὰ περιέχοντα τὰς γωνίας τμήματα τέμωμεν δίχα, καὶ πάλιν ἀγάγωμεν ἀπὸ τῶν τομῶν εὐθείας ἐκατέρωθεν ἐπὶ τὰ πέρατα τῶν τμημάτων, ποιούμεν πολύγωνον σχῆμα. ἐάν οὖν ἐπὶ πολὺ τοῦτο ποιῶμεν, γίνεται πολυγωνότατον σχῆμα

μικρὰς πάννυ ἔχον τὰς γωνίας, ἃς αἱ περιέχουσιν εὐθεῖαν διὰ τὸ μικρὰς πάννυ εἶναι ἐφαρμόσουσι τῷ κύκλῳ. ἐπεὶ οὖν δέδοται πᾶν τὸ δοθὲν εὐθύγραμμον σχῆμα τετραγωνίσαι, ἐὰν τετραγωνίσω τὸ πολύγωνον τοῦτο, ἐπειδὴ ἐφαρμόζει τῷ κύκλῳ, τετραγωνίσας ἔσομαι καὶ τὸν κύκλον. οὗτος οὖν ἀναιρεῖ τὰς γεωμετρικὰς ἀρχάς· ἀρχὴ γὰρ ἐστὶ γεωμετρικὴ μηδέποτε ἐφαρμόζειν εὐθεῖαν περιφέρειαν, οὗτος δὲ δίδωσι, διὰ σμικρότητα, τινὰ εὐθεῖαν ἐφαρμόζειν τινὶ περιφέρειαν. ὁ μὲν οὖν Ἱπποκράτης, ἐκ γεωμετρικῆς εὐθεῖαν ἐφαρμόζειν τινὶ περιφέρειαν. ὁ μὲν οὖν Ἱπποκράτης, ἐκ γεωμετρικῶν ἀρχῶν ὀρμηθεὶς καὶ τετραγωνίσας μηνοειδὲς τι τοῦ κύκλου τμήμα, κακῶς τὸ ἐξῆς συνεπέρανεν, ἐκ τούτου καὶ τὸν τοῦ κύκλου τετραγωνισμόν συλλογίσασθαι βουλευθείς· ὁ μὲντοι Ἀντιφῶν ἀνελών τὰς γεωμετρικὰς ἀρχάς, τὸ μηδέποτε περιφέρειαν εὐθεῖαν ἐφαρμόζειν, οὕτω τὸ ἐξῆς συνεπέρανεν. φησὶν οὖν ὅτι τὸν τετραγωνισμόν τοῦ κύκλου ἐλέγξει ψευδῆ ὄντα τὸν μὲν Ἱπποκράτους γεωμετρικοῦ ἐστὶ διαλύσαι, ὡς φυλάττοντος τοῦ Ἱπποκράτους τὰς γεωμετρικὰς ἀρχάς, τὸν δὲ Ἀντιφῶντος οὐκέτι διαλύσει ὁ γεωμέτρης, ἐπεὶ ἀνηρημένων τῶν γεωμετρικῶν ἀρχῶν οὕτω συνήκται.

<https://www.bloomsburycollections.com/book/philoponus-on-aristotle-physics-1-1-3/philoponus-on-aristotle-physics-1-1-3>

Joannes Philoponis *Commentary on Aristotle's Posterior Analytics* 1.9-18 111.3-116.1

Joannes Philoponis *Commentary on Aristotle's Posterior Analytics*, trans. C. Osbourne, London: Bloomsbury Academic, 2006

75b37-41 Since it is obvious that it is not possible to demonstrate each thing except from its own principles, if what is being proved belongs to it qua that very thing, it is not possible to know(e) 'this' [if it is proved from premises that are true, indemonstrable and immediate. For if that is so, it is possible to construct proofs the way Bryson proved the squaring [of the circle].

In addition to what he has already proved about scientific knowledge(e), he also adds that taking premises that are true and immediate is not enough to make a demonstration; they also need to be appropriate to the subject of demonstration. For if I say that every stone is coloured, and every coloured thing is a body, therefore every stone is a body, I have taken premises that are true and also immediate (for I need no middle term to demonstrate either that a stone is coloured or that what is coloured is a body), but the middle term is not appropriate to the subject, since being coloured belongs to many other things too. But as has been said many times, the demonstration must be based on principles that are appropriate to each thing. This is in order that the middle term be appropriate to the extremes and common to nothing else. And so, he says, the premises must be taken from things that are not only true and immediate, but also appropriate to the conclusion. Since that way, he says, it is even possible to prove Bryson's quadrature on the basis of things that are common and are not principles appropriate to the claim in question.

Aristotle says only so much about Bryson's quadrature, but Alexander says that Bryson attempted to square the circle as follows. A circle is larger than any rectilinear figure inscribed in it and smaller than [any rectilinear figure] circumscribed [about it]. (A rectilinear figure drawn inside a circle is said to be inscribed in it, and one [drawn] outside [is said] to be circumscribed.⁴) Also a rectilinear figure drawn between the inscribed and the circumscribed rectilinear figures is smaller than the circumscribed one and larger than the inscribed one. But things that are larger and smaller than the same things are equal to one another. Therefore the circle is equal to the rectilinear figure drawn between the inscribed and the circumscribed 30 [figures]. But we can construct a square equal to any given rectilinear figure. Therefore it is possible to make a square equal to the circle. Thus Alexander.

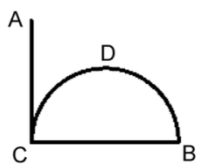
But the philosopher Proclus said that his teacher objected to Alexander's explanation because if that is how Bryson squared the circle, he concurred with Antiphon's quadrature. For Antiphon too made the figure drawn between the inscribed and the circumscribed rectilinear figure fit with the circumference of the circle to the point where he made (as he claimed) a straight line coincide with a circumference, which is impossible. This was discussed in the *Physics*. Therefore Aristotle would not contrast Bryson's quadrature as different from Antiphon's if this is how Bryson squared [the circle]. But, says Proclus, I declare the axiom to be false. For it is not true that things larger and smaller than the same things are equal to one another. Ten is larger than eight and smaller than twelve, but likewise nine is smaller than twelve and larger than eight, but of course it is not the case that ten and nine are equal just because they are both larger and smaller than the same things, namely twelve and eight. Therefore it is not the case that even if the circle and the rectilinear figure drawn between an inscribed [figure] and a circumscribed [figure] are larger and smaller than the same things, namely, the inscribed and the circumscribed [figures], they are for that reason automatically equal to one another too – unless as was already stated, someone claims that the rectilinear figure drawn between the inscribed and the circumscribed [figures] à la Antiphon, coincides with the circle, which is impossible. For a straight line never coincides with a circumference.

Against Proclus' [account] it can be said that if this is how Bryson constructed the quadrature of the circle, he did not construct it at all, but begged the question. For those who attempted to square the circle did not investigate whether it is possible for there to be a square equal to a circle, but supposing that it is possible for there to be one, they attempted to generate the square equal to the circle. But what was just said by Proclus, as our teacher⁹ said, proves that it is possible for there to be a square equal to the circle, even if this is in fact granted. But he still did not draw a square equal to the circle nor did he teach how this might come to be – which is what those who attempted to square the circle wanted to do. And Aristotle spoke as if the circle had been squared by Bryson, even though it was not done geometrically. And so Proclus' explanation does not appear to be naturally fitting.

Therefore Proclus said that Bryson squared the circle as follows. A circle, he declares, is larger than any inscribed rectilinear figure and smaller than any circumscribed one. When there are things that are larger and smaller than something, there is also something equal to it. But there are rectilinear figures larger and smaller than the circle. Therefore there is also one equal to it.

But even if someone were to grant that this is how Bryson constructed [his quadrature], it is possible to claim against it that the account is true for things of the same kind – that when there are things that are larger and smaller than something, there is also something equal to it – but this is not true for things not of

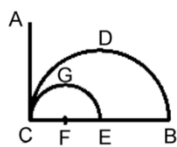
Diagram 1



the same kind. To be sure it is proved by the Geometer that straight line AC drawn at right angles at the end of CB, a diameter of semicircle CDB [Diagram 1] lies entirely outside the circle, and that of the two angles formed by the

circumference and the diameter and by the [line] drawn at right angles and the circumference – I mean the exterior angle ACD and the interior angle DCB¹² – the exterior angle is smaller than any acute rectilinear angle and the interior angle is larger than any acute rectilinear angle. And notice that here al- though we have proved that it is larger and smaller than the same acute rectilinear angle, we still cannot find [an angle] equal [to it] because the magnitudes are not of the same kind. For the angles in question are composed of a straight line and a circumference, and we call them horn angles. And the surprising thing is that even though the exterior angle can be increased and the interior angle decreased ad infinitum, and – vice versa –

Diagram 2

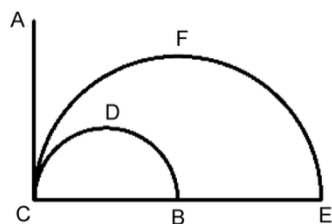


the interior angle can be increased and the exterior angle decreased ad infinitum,¹⁵ the exterior angle as it is being increased ad infinitum will never become equal to any rectilinear acute angle, but will always be smaller than

any [given rectilinear acute angle] nor will the interior angle as it is being increased ad infinitum ever become equal to a right angle. We will increase the exterior angle by drawing smaller circles [Diagram 2].

For example if I cut diameter CB at point E and [if I cut] straight line CE in two at point F, and with centre F and radius FC draw a circle whose semicircle is CGE, the exterior angle ACG has been increased, none the less it is still smaller than any acute angle for the reason that has been stated. For the theorem has been proved by the Geometer for every circle. In the same way, if I again cut the diameter of the inner circle and inscribe a smaller circle, and do this ad infinitum,¹⁸ I will always increase the exterior angle and decrease the interior angle, but the exterior angle will never become equal to an acute rectilinear angle nor will the

Diagram 3



interior angle, but the exterior angle will always be smaller and the interior angle always larger. This is how I will increase the exterior angle and decrease the interior angle.

And, vice versa, I will increase the interior angle and decrease the exterior angle by circumscribing larger circles as follows [Diagram 3]. I produce diameter CB in a straight line to E and with centre B and radius BC draw a circle whose semicircle is CFE. And it is clear that semicircle CFE will fall inside straight line AC because it has been proved that a line drawn at right angles at the end of a diameter lies entirely outside the circle.²¹ Further, it is clear from the following that no part of the outer semicircle CFE coincides with any part of the inner semicircle CDB. For if it coincides, let straight lines GB and GH be drawn from G, the point at which they coincide, to B and H, the centres of the circles. Now since point H is the centre of the inner semicircle, HG is equal to HC, and since B is the centre of the outer semicircle CFE, BG is equal to BC. But BH and CH are equal to HG. Therefore GB is equal to BH and HG. Therefore the two sides GH and BH of triangle GHB are equal to one side, GB, which is impossible. Therefore it is not the case that any part of the outer circle coincides with any part of the inner [circle]. Therefore the outer circle cuts angle ACG. And in the same way by drawing outer circles ad infinitum I will decrease the exterior angle and increase the interior angle ad infinitum, and the interior angle being increased will never become equal to a right angle but always becomes larger than any acute rectilinear angle. Now if it has been proved that something can be larger and smaller than the same thing without being equal because of the dissimilarity of the magnitudes, Bryson was wrong to assume that if the circumscribed rectilinear figure is larger than the circle and the inscribed [rectilinear figure] is smaller, it follows that what is between the inscribed and the circumscribed [figures] is equal. For here too the magnitudes are dissimilar – I mean the rectilinear figure [is dissimilar to] the circle – and so they will not be equal either.

Ἐπεὶ δὲ φανερόν ὅτι ἕκαστον ἀποδείξαι οὐκ ἔστιν ἀλλ' ἢ ἐκ τῶν ἐκάστου ἀρχῶν, ἂν τὸ δεικνύμενον ὑπάρχῃ ἢ ἐκεῖνο, οὐκ ἔστι τὸ ἐπίστασθαι τοῦτο.

Ἔτι καὶ τοῦτο προστίθῃσι τοῖς περὶ ἐπιστήμης δεδειγμένοις, ὅτι οὐκ ἀρκεῖ <εἰς> τὸ ποιῆσαι ἀπόδειξιν τὸ ἀληθεῖς τε καὶ ἀμέσους λαβεῖν προτάσεις, ἀλλὰ καὶ δεῖ οἰκείας τοῦ ὑποκειμένου ἀποδεικτοῦ εἶναι ταύτας. ἐὰν μὲν γὰρ εἴπω οὕτως, ὅτι πᾶς λίθος κέχρωσται, πᾶν τὸ κεχρωσμένον σῶμά ἐστι, πᾶς ἄρα λίθος σῶμά ἐστιν, ἔλαβον μὲν προτάσεις ἀληθεῖς, ἀλλὰ καὶ ἀμέσους (οὐδενὸς γὰρ ὅρου δέομαι μέσου πρὸς τὸ ἀποδείξαι ἢ ὅτι ὁ λίθος κέχρωσται ἢ ὅτι τὸ κεχρωσμένον σῶμά ἐστιν), οὐ μὴν οἰκείος ὁ μέσος ὅρος τῷ ὑποκειμένῳ· τὸ γὰρ κεχρῶσθαι πολλοῖς καὶ ἄλλοις ὑπάρχει. δεῖ δέ, ὡς ἀλλὰ καὶ ἀμέσους (οὐδενὸς γὰρ ὅρου δέομαι μέσου πρὸς τὸ ἀποδείξαι ἢ ὅτι ὁ λίθος κέχρωσται ἢ ὅτι τὸ κεχρωσμένον σῶμά ἐστιν), οὐ μὴν οἰκείος ὁ μέσος ὅρος τῷ ὑποκειμένῳ· τὸ γὰρ κεχρῶσθαι πολλοῖς καὶ ἄλλοις ὑπάρχει. δεῖ δέ, ὡς πολλάκις εἴρηται, τὴν ἀπόδειξιν ἐκ τῶν ἐκάστου οἰκείων ἀρχῶν γίνεσθαι, τουτέστιν ἵνα ὁ μέσος ὅρος οἰκείος ᾖ τοῖς ἄκροις καὶ μηδενὶ ἄλλῳ κοινός. ὥστε, φησί, δεῖ οὐ μόνον ἐξ ἀληθῶν καὶ ἀμέσων ἀλλὰ καὶ ἐξ οἰκείων τοῦ συμπεράσματος εἰληθῆαι τὰς προτάσεις· ἐπεὶ οὕτω, φησί, καὶ τὸν Βρύσωνος τετραγωνισμόν δείξαι δυνατόν ἐκ τινων κοινοτέρων καὶ μὴ ἐκ τῶν οἰκείων ἀρχῶν τοῦ προκειμένου. ὁ μὲν οὖν Ἀριστοτέλης περὶ τοῦ Βρύσωνος τετραγωνισμοῦ τοσοῦτόν φησιν. ὁ δὲ Ἀλέξανδρός φησι τὸν Βρύσωνα ἐπιχειρήσαι τετραγωνίσαι τὸν κύκλον τὸν τρόπον τοῦτον. παντός,

φησίν, ἐγγραφομένου ἐν τῷ κύκλῳ εὐθυγράμμου σχήματος μείζων ἐστὶν ὁ κύκλος, τοῦ δὲ περιγραφομένου ἐλάττων (ἐγγράφεσθαι δὲ λέγεται ἐν κύκλῳ εὐθύγραμμον τὸ ἐντὸς τοῦ κύκλου γραφόμενον, περιγράφεσθαι δὲ τὸ ἐκτός)· ἀλλὰ καὶ τὸ μεταξὺ τοῦ τε ἐγγραφομένου καὶ περιγραφομένου εὐθυγράμμου γραφόμενον εὐθύγραμμον σχῆμα τοῦ μὲν περιγραφομένου ἐστὶν ἔλαττον τοῦ δὲ ἐγγραφομένου μείζον· τὰ δὲ τοῦ αὐτοῦ μείζονα καὶ ἐλάττονα ἴσα ἀλλήλοις ἐστίν· ὁ κύκλος ἄρα ἴσος ἐστὶ τῷ μεταξὺ γραφομένῳ εὐθυγράμμῳ τοῦ τε ἐγγραφομένου καὶ περιγραφομένου. ἔχομεν δὲ παντὶ δοθέντι εὐθυγράμμῳ ἴσον τετράγωνον συστήσασθαι· τῷ κύκλῳ ἄρα ἴσον τετράγωνον ἔστι ποιῆσαι. ὁ μὲν οὖν Ἀλέξανδρος οὕτως. ἔλεγε δὲ ὁ φιλόσοφος Πρόκλον τὸν

ἦσαι. ὁ μὲν οὖν Ἀλέξανδρος οὕτως. ἔλεγε δὲ ὁ φιλόσοφος Πρόκλον τὸν αὐτοῦ διδάσκαλον ἐπισκῆπτειν τῇ Ἀλεξάνδρου ἐξηγήσει, ὅτι, εἰ οὕτως ἐτετραγώνισεν ὁ Βρύσων τὸν κύκλον, συνέτρεχε τῷ Ἀντιφώντος τετραγωνισμῷ. τὸ γὰρ μεταξὺ τοῦ ἐγγραφομένου καὶ περιγραφομένου εὐθυγράμμου γραφόμενον σχῆμα ἐφαρμόζειν τῇ τοῦ κύκλου περιφερείᾳ, τοῦτο καὶ ὁ Ἀντιφὼν ἐποίει, ἕως οὗ ἐφήρμοσεν, ὡς ἐκεῖνος ἔλεγεν, εὐθεῖαν περιφερείᾳ, ὅπερ ἀδύνατον· εἴρηται δὲ περὶ τούτου ἐν ταῖς Φυσικαῖς. οὐκ ἂν οὖν ὁ Ἀριστοτέλης τὸν Βρύσωνος τετραγωνισμόν ὡς ἕτερον ὄντα παρὰ τὸν Ἀντιφώντος παρετίθει, εἴ γε οὕτως ὁ Βρύσων ἐτετραγώνισεν. ἐγὼ δέ, φησὶν ὁ Πρόκλος, καὶ τὸ ἀξίωμα ψευδὲς εἶναι λέγω· οὐ γὰρ ἀληθὲς τὸ τὰ τοῦ αὐτοῦ μείζονα καὶ ἐλάττονα, ταῦτα ἴσα εἶναι ἀλλήλοις· τὴν γοῦν δεκάδα μείζονα μὲν εἶναι τῶν ὀκτώ, ἐλάττονα δὲ τῶν δώδεκα· ἀλλὰ δὴ καὶ τὰ ἐννέα ὁμοίως τῶν μὲν δώδεκα ἐστὶν ἐλάττονα, μείζονα δὲ τῶν ὀκτώ· καὶ οὐ δήπου τὰ δέκα καὶ τὰ ἐννέα ἴσα ἐστίν, ἐπειδὴ τῶν αὐτῶν, τῶν τε δώδεκα καὶ τῶν ὀκτώ, καὶ μείζονά ἐστι καὶ ἐλάττονα. οὐκ ἄρα, κἂν τοῦ αὐτοῦ, [τὸ μεταξὺ] τοῦ τε ἐγγραφομένου καὶ τοῦ περιγραφομένου, μείζονά ἐστι καὶ ἐλάττονα ὁ κύκλος καὶ τὸ μεταξὺ τοῦ τε ἐγγραφομένου καὶ τοῦ περιγραφομένου γραφόμενον εὐθύγραμμον, ἤδη διὰ τοῦτο καὶ ἀλλήλοις ἐστὶν ἴσα, εἰ μὴ τις, ὅπερ ἤδη εἴρηται, κατὰ τὸν Ἀντιφώντα τὸ μεταξὺ τοῦ ἐγγραφομένου καὶ περιγραφομένου γραφόμενον εὐθύγραμμον ἐφαρμόζειν φησὶ τῷ κύκλῳ, ὅπερ ἐστὶν ἀδύνατον· οὐδέποτε γὰρ εὐθεῖα περιφερείᾳ ἐφαρμόζει. ὁ οὖν Πρόκλος ἐστὶν ἀδύνατον· οὐδέποτε γὰρ εὐθεῖα περιφερείᾳ ἐφαρμόζει. ὁ οὖν Πρόκλος ἔλεγε τετραγωνίζειν τὸν Βρύσωνα τὸν τρόπον τοῦτον· παντός, φησὶ, τοῦ ἐγγραφομένου εὐθυγράμμου μείζων ἐστὶν ὁ κύκλος, τοῦ δὲ περιγραφομένου ἐλάττων· οὐ δὲ ἔστι μείζον καὶ ἔλαττον, τούτου ἔστι καὶ ἴσον· ἔστι δὲ μείζον καὶ ἔλαττον εὐθύγραμμον τοῦ κύκλου· ἔστι ἄρα αὐτοῦ καὶ ἴσον. καὶ πρὸς τὰ Πρόκλου δὲ ἔστιν ἐκεῖνο εἰπεῖν, ὅτι, εἰ οὕτως ὁ Βρύσων κατεσκεύαζε τὸν τοῦ κύκλου τετραγωνισμόν, οὐδὲ κατεσκεύαζεν ὅλως ἀλλὰ τὸ ἐξ ἀρχῆς ἡτέτο. οὐδὲ γὰρ οἱ τὸν κύκλον τετραγωνίζοντες τοῦτο ἐζήτουν, εἰ οἷόν τέ ἐστὶ τῷ κύκλῳ ἴσον τετράγωνον εἶναι, ἀλλ' ὡς οἰόμενοι ὅτι ἐνδέχεται εἶναι οὕτως ἐπειρώντο τετράγωνον ἴσον τῷ κύκλῳ γεννᾶν. τὸ δὲ νῦν παρὰ τοῦ Πρόκλου λεχθέν, ὡς ἔλεγεν ὁ ἡμέτερος διδάσκαλος, ὅτι μὲν ἐνδέχεται ἴσον εἶναι τῷ κύκλῳ τετράγωνον, εἴπερ ἄρα καὶ τοῦτο συγχωρηθῆι, ἔδειξεν· οὐ μὴν δὲ καὶ κατέγραψεν ἴσον τῷ κύκλῳ τετράγωνον οὐδὲ πῶς ἂν τοῦτο γένοιτο ἐδίδαξεν, ὅπερ ποιῆσαι βούλονται οἱ τὸν κύκλον τετραγωνίζοντες. καὶ ὁ Ἀριστοτέλης δὲ ὡς περὶ τετραγωνισθέντος τοῦ κύκλου ὑπὸ τοῦ Βρύσωνος, εἰ καὶ μὴ γεωμετρικῶς, οὕτως εἶπεν. ὥστε οὐδὲ ἡ ἐξηγησις προσφυῆς εἶναι φαίνεται. εἰ δέ τις καὶ συγχωρήσῃ οὕτω τὸν Βρύσωνα κατασκευάζειν, πρὸς αὐτὸν ἔστιν ἀντειπεῖν ὅτι ἐπὶ μὲν τῶν ὁμογενῶν ἀληθὲς ἐστὶν ὁ λόγος, ὅτι οὐ ἔστι μείζον καὶ ἔλαττον, τούτου ἔστι καὶ ἴσον, ἐπὶ μέντοι τῶν ἀνομοιογενῶν οὐκέτι ἀληθὲς τοῦτο. δεικνύται

γοῦν παρὰ τῷ γεωμέτρῃ ὅτι ἐπὶ τοῦ ἡμὶ οὐκέτι ἀληθές τοῦτο. δεικνύται γοῦν παρὰ τῷ γεωμέτρῃ ὅτι ἐπὶ τοῦ ἡμικυκλίου τοῦ ΓΑΒ ἡ ἀπ' ἄκρας τῆς διαμέτρου τῆς ΓΒ πρὸς ὀρθὰς ἀγομένη εὐθεΐα ἢ ΑΓ πάντως ἐκτὸς μὲν πίπτει τοῦ κύκλου, τῶν δὲ δύο γωνιῶν τῶν γινομένων ὑπὸ τῆς περιφερείας καὶ τῆς διαμέτρου καὶ ἔτι ὑπὸ τῆς πρὸς ὀρθὰς ἀχθείσης καὶ τῆς περιφερείας, λέγω δὴ τῆς τε ἐκτὸς τῆς ὑπὸ ΑΓΔ καὶ τῆς ἐντὸς τῆς ὑπὸ ΔΓΒ, ἡ μὲν ἐκτὸς πάσης ὀξείας γωνίας εὐθυγράμμου ἐλάττων ἐστίν, ἡ δὲ ἐντὸς πάσης ὀξείας γωνίας εὐθυγράμμου μείζων ἐστίν. καὶ ἰδοὺ ἐνταῦθα τῆς αὐτῆς ὀξείας εὐθυγράμμου γωνίας μείζονα καὶ ἐλάττονα δεδειχότες ἴσην εὑρεῖν οὐκ ἂν δυνασώμεθα διὰ τὸ ἀνομοιογενῆ εἶναι τὰ μεγέθη· ἐξ εὐθείας γὰρ καὶ περιφερείας ὑπόκεινται αἱ προκείμεναι γωνίαι, ἅς καὶ κερατοειδεῖς καλοῦσι. καὶ τὸ παράδοξον, ὅτι καὶ τῆς ἐκτὸς γωνίας ἐπ' ἄπειρον ἀυξηθῆναι δυναμένης καὶ μειωθῆναι ἐντὸς, καὶ ἔμπαλιν τῆς ἐντὸς ἐπ' ἄπειρον αὐξέσθαι δυναμένης μειοῦσθαι δὲ τῆς ἐκτὸς, οὔτε ἡ ἐκτὸς αὐξομένη ἐπ' ἄπειρον ἴση ποτὲ γενήσεται τῇ ὀξείᾳ γωνίᾳ εὐθυγράμμῳ, ἀλλ' αἰεὶ ἔσται πάσης ἐλάττων, οὔτε ἡ ἐντὸς ἐπ' ἄπειρον αὐξομένη τῇ ὀρθῇ ποτε γενήσεται ἴση. αὐξομεν δὲ τὴν μὲν ἐκτὸς γωνίαν ἐλάττονας κύκλους γράφοντες· οἷον ἐὰν τέμω τὴν ΓΒ διάμετρον κατὰ τὸ Ε σημεῖον καὶ τὴν ΓΕ εὐθεΐαν δίχα κατὰ τὸ Ζ σημεῖον καὶ κέντρῳ μὲν τῷ Ζ διαστήματι δὲ τῷ ΖΓ κύκλον γράψω, οὗ ἡμικύκλιον τὸ ΓΗΕ, ἡ μὲν ἐκτὸς γωνία ἢ ὑπὸ ΑΓΗ ἡυξῆται, καὶ πάλιν οὐδὲν ἤττον πάσης ὀξείας ἐστὶν ἐλάττων διὰ τὸν εἰρημένον λόγον· ἐπὶ παντὸς γὰρ πάσης ὀξείας ἐστὶν ἐλάττων διὰ τὸν εἰρημένον λόγον· ἐπὶ παντὸς γὰρ κύκλου δέδεικται τὸ θεώρημα τῷ γεωμέτρῃ. κατὰ τὸν αὐτὸν δὲ τρόπον πάλιν τὴν τοῦ ἐντὸς κύκλου διάμετρον τεμὼν καὶ ἐλάττονα κύκλον ἐγγράψας καὶ τοῦτο ἐπ' ἄπειρον ποιήσας αἰεὶ μὲν αὐξῶ τὴν ἐκτὸς, μειῶ δὲ τὴν ἐντὸς, καὶ οὔτε ἡ ἐκτὸς γενήσεται ποτε ἴση ὀξείᾳ εὐθυγράμμῳ οὔτε ἡ ἐντὸς, ἀλλ' ἡ μὲν ἐκτὸς αἰεὶ ἔσται ἐλάττων, ἡ δὲ ἐντὸς αἰεὶ μείζων. οὕτω μὲν τὴν μὲν ἐκτὸς αὐξῶ, μειῶ δὲ τὴν ἐντὸς. ἀνάπαλιν δὲ αὐξῶ μὲν τὴν ἐντὸς, μειῶ δὲ τὴν ἐκτὸς μείζονας κύκλους περιγράφων τοῦτον τὸν τρόπον. ἐκβάλλω γὰρ τὴν ΓΒ διάμετρον ἐπ' εὐθείας ἐπὶ τὸ Ε, καὶ κέντρῳ μὲν τῷ Β διαστήματι δὲ τῷ ΒΓ κύκλον γράφω, οὗ ἡμικύκλιον τὸ ΓΖΕ. καὶ δῆλον ὅτι τὸ ΓΖΕ ἡμικύκλιον ἐντὸς πεσεῖται τῆς ΑΓ εὐθείας διὰ τὸ δεδειχθαι ὅτι ἡ ἀπ' ἄκρας τῆς διαμέτρου πρὸς ὀρθὰς ἀγομένη πάντως ἐκτὸς πίπτει τοῦ κύκλου. ὅτι δὲ οὐδὲ μόνον τι τοῦ ἡμικυκλίου τοῦ ἐκτὸς τοῦ ΓΖΕ ἐφάπτεται μορίου τινὸς τοῦ ἐντὸς ἡμικυκλίου τοῦ ΓΑΒ, δῆλον ἐντεῦθεν. εἰ γὰρ ἄπτεται, ἀπὸ τοῦ σημείου, καθ' ὃ ἐφαρμόζουσιν, εἰ τύχοι, τοῦ Η, ἐπεξέυχθωσαν ἐπὶ τὰ κέντρα τῶν κύκλων, τότε Β καὶ τὸ Θ, ἢ ΗΒ, ΗΘ εὐθεΐα. ἐπεὶ οὖν τὸ Θ σημεῖον κέντρον ἐστὶ τοῦ ἐντὸς ἡμικυκλίου, ἴση ἐστὶν ἡ ΘΗ τῇ ΘΓ· πάλιν ἐπεὶ τὸ Β κέντρον ἐστὶ τοῦ ἐκτὸς ἡμικυκλίου τοῦ ΓΖΕ, ἴση ἐστὶν ἡ ΒΗ τῇ ΒΓ. ἀλλὰ ἡ ΒΘ καὶ ἡ ΓΘ ἴσαι εἰσὶ τῇ ΘΗ· ἡ ἄρα ΗΒ ἴση ἐστὶ ταῖς ΒΘ, ΘΗ. τριγώνου ἄρα τοῦ ΗΘΒ αἱ δύο πλευραὶ αἱ ΗΘ, ΒΘ τῇ μιᾷ τῇ ΗΒ ἴσαι εἰσίν, ὅπερ ἀδύνατον· ΗΘΒ αἱ δύο πλευραὶ αἱ ΗΘ, ΒΘ τῇ μιᾷ τῇ ΗΒ ἴσαι εἰσίν, ὅπερ ἀδύνατον· οὐκ ἄρα ἐφαρμόζει μέρος τι τοῦ ἐκτὸς κύκλου μέρει τινὶ τοῦ ἐντὸς. Τέμνει ἄρα ὁ ἐκτὸς κύκλος τὴν ὑπὸ ΑΓΗ γωνίαν. καὶ οὕτως ἐπ' ἄπειρον τῷ αὐτῷ τρόπῳ ἐκτὸς γράφων κύκλους ἐπ' ἄπειρον μὲν τὴν ἐκτὸς γωνίαν μειῶ, αὐξῶ δὲ τὴν ἐντὸς· καὶ οὐδέποτε ἡ ἐντὸς αὐξομένη ἴση γενήσεται τῇ ὀρθῇ, ἀλλ' αἰεὶ πάσης ὀξείας εὐθυγράμμου μείζων γίνεται. εἰ τοίνυν δέδεικται ὅτι ἐνδέχεται τοῦ αὐτοῦ μείζον μὲν τι καὶ ἔλαττον εἶναι, οὐκέτι δὲ καὶ ἴσον διὰ τὴν ἀνομοιότητα τῶν μεγεθῶν, κακῶς ἄρα ὁ Βρύσων ἐλάμβανεν ὅτι, εἰ μείζον τοῦ κύκλου ἐστὶ τὸ περιγραφόμενον εὐθύγραμμον καὶ ἔλαττον τὸ ἐγγραφόμενον, ἔστιν ἄρα καὶ ἴσον τὸ μεταξὺ τοῦ τε ἐγγραφομένου καὶ τοῦ περιγραφομένου· ἀνόμοια γὰρ κἀνταῦθα τὰ μεγέθη, λέγω δὴ τὸ εὐθύγραμμον τῷ κύκλῳ, ὥστε οὐδὲ ἴσα ἔσται.

75b41-76a1 For such arguments prove [their conclusion] in virtue of some common property that belongs to something else as well. [This is why the arguments apply also to other things that do not belong in the same genus.]

That when there are things that are larger and smaller than something, there is also something equal to it – the principle from which Bryson thought he proved the quadrature of the circle – is not proper to geometry, but is common to very many other things as well. It is especially characteristic of dialectic, not geometry, to make use of such [claims], because they do not prove the claim in question on the basis of geometrical principles.

Κατὰ κοινόν τε γὰρ δεικνύουσιν οἱ τοιοῦτοι λόγοι, ὃ καὶ ἐτέρῳ ὑπάρξει.

Τὸ γὰρ οὗ ἔστι μείζον καὶ ἔλαττον, τούτου εἶναι καὶ ἴσον, ἐξ οὗ ἐδόκει δεικνύναι τὸν τοῦ κύκλου τετραγωνισμόν ὁ Βρύσων, οὐκ ἴδιον γεωμετρίας ἀλλὰ κοινὸν καὶ ἄλλων πλείστων, καὶ μᾶλλον διαλεκτικῆς τὸ τοῖς τοιούτοις χρῆσθαι καὶ οὐ γεωμετρίας, διότι μὴ ἐκ τῶν ἀρχῶν τῶν γεωμετρικῶν τὸ προκείμενον δεικνύουσιν.

76a1-3 Therefore, he does not know(e) in virtue of that very subject, but [only] accidentally, for the demonstration would not apply to another genus as well.

If, he says, he proved the quadrature on the basis not of appropriate principles, but of some [principles] that are more general, it follows that he proved it on the basis not of per se attributes, but of accidental ones. What it is to prove per se and not accidentally he goes on [to define] next: when we know(g) something from its appropriate principles and not from some more general ones that can apply to other things as well. And this is why he said above that there is no scientific knowledge(e) or demonstration of perishables except as if accidentally, meaning by ‘accidentally’ constructing the demonstration from some more general [principles].

Οὐκοῦν οὐχ ἢ ἐκεῖνο ἐπίσταται ἀλλὰ κατὰ συμβεβηκός· οὐ γὰρ ἂν ἐφήρμοσεν ἢ ἀπόδειξις καὶ ἐπ’ ἄλλο γένος.

Εἰ μὴ ἐκ τῶν ἀρχῶν, φησί, τῶν οἰκείων ἐδείκνυε τὸν τετραγωνισμόν ἀλλ’ ἐκ τινων κοινοτέρων, οὐκ ἄρα ἐκ τῶν καθ’ αὐτὸ ἀλλ’ ἐκ τῶν κατὰ συμβεβηκός ὑπαρχόντων ἐδείκνυε. τί δέ ἐστι τὸ καθ’ αὐτὸ ἀλλὰ μὴ κατὰ συμβεβηκός δεικνύναι, ἐφεξῆς ἐπήγαγεν, ὅτι ὅταν γινώσκωμέν τι ἐκ τῶν οἰκείων αὐτοῦ ἀρχῶν καὶ μὴ ἐκ τινων κοινοτέρων δυναμένων καὶ ἄλλοις ὑπάρχειν. καὶ διὰ τοῦτο καὶ ἀνωτέρω ἔλεγε μὴ εἶναι τῶν φθαρτῶν ἐπιστήμην μηδὲ ἀπόδειξιν, εἰ μὴ οὕτως ὥσπερ κατὰ συμβεβηκός, κατὰ συμβεβηκός λέγων τὸ ἀπὸ τινων κοινοτέρων ποιεῖσθαι τὴν ἀπόδειξιν.

Marinos of Neapolis *Commentary on Euclid's Data* p240, p242

Marinos of Neapolis *Commentary on Euclid's Data*, trans. C. M. Taisbak, Gylling, Denmark: Museum Tusculanum Press, 2003

The opposite is *aporon* [not available]. For example, squaring a circle, for this is not yet in our power, even if it can be attained and falls under some science, or the scientific knowledge of it has not yet been grasped.

...

For even though they have much in common, e.g. to draw a straight line through two points and a circle through three, and to construct an equilateral <triangle on a line>, still, to square the circle is *tetamenon* [orderly] but is *agnoston* [unknown].

ἄπορον δέ ἐστι τὸ ἀντικειμένως ἔχον, ὡς ὁ τοῦ κύκλου τετραγωνισμός· οὕτω γάρ ἐστιν ἐν πόρῳ, εἰ καὶ οἷόν τε αὐτὸ πορισθῆναι καὶ ἐστὶν ἐπιστητόν· ἐπιστήμη γὰρ αὐτοῦ οὕτω κατεῖληπται.

...

εἰ γὰρ καὶ κοινὰ αὐτοῖς πολλὰ ὑπάρχει, ὡς τὸ διὰ δύο σημείων εὐθεῖαν γράψαι καὶ διὰ τριῶν κύκλον καὶ ἰσόπλευρον συστήσασθαι, ἀλλὰ τὸ τετραγωνίζειν τὸν κύκλον τεταγμένον μὲν, ἄγνωστον δέ·

<https://scaife.perseus.org/reader/urn:cts:greekLit:tlg4075.tlg002.1st1K-grc1:1/>

Heron of Alexandria, *Metrica* I. 26 (p. 66.13-19 [Schönel])

Cohen, M.R. & Drabkin, I.E. (1948), *A Source Book in Greek Science*, Cambridge, MA: Harvard University Press

Archimedes also shows in his book on Plinthis and Cylinders that the ratio of the circumference of any circle to its diameter is greater than 211875:67441, and less than 197888:62351. But since these figures are inconvenient to measurements, they are reduced to the smallest numbers, 22:7.

Ἀρχιμήδης μὲν οὖν ἐν τῇ τοῦ κύκλου μετρήσει δείκνυσιν, ὅτι ἡ τετράγωνος τὰ ἀπὸ τῆς διαμέτρου τοῦ κύκλου ἴσα γίνονται ὡς ἔγγιστα ἰδ κύκλοις· ὥστε ἐὰν δοθῇ ἡ διάμετρος τοῦ κύκλου εἰ τύχοι μονάδων 1, δεήσει τὰ 1 ἐφ' ἑαυτὰ ποιῆσαι· γίνονται 19· ταῦτα ἐπὶ τὰ 11· γίνονται 209· ὡς τὸ 11· γίνονται 22· ὡς τὸ 7·

Ptolemy, *Syntaxis Mathematica* VI. 7 (p. 513.1-5 (Heiberg))

Cohen, M.R. & Drabkin, I.E. (1948), *A Source Book in Greek Science*, Cambridge, MA: Harvard University Press

The ratio [of the circumference to the diameter] remains constant, $3 + \frac{8}{60} + \frac{30}{(60)^2} : 1$. This ratio is almost exactly the mean between the values $3 \frac{10}{71}$ and $3 \frac{1}{7}$ which Archimedes was content to give.

πρὸς τὰς διαμέτρους ὄντος, ὃν ἔχει τὰ γ ἢ λ πρὸς τὸ ἕν· οὗτος γὰρ ὁ λόγος μεταξύ ἐστὶν ἔγγιστα τοῦ τε τριπλασίου πρὸς τῷ ζ' μέρει καὶ τοῦ τριπλασίου πρὸς τοῖς δέκα ἑβδομηκοστομόνοις, οἷς ὁ Ἀρχιμήδης κατὰ τὸ ἀπλούστερον συνεχρήσατο.

Eutokios, *Commentary on Archimedes' Measurement of a Circle* III. 258 (Heiberg)

Cohen, M.R. & Drabkin, I.E. (1948), *A Source Book in Greek Science*, Cambridge, MA: Harvard University Press

Now we must observe that Apollonius of Perga in his *Ocyticion* demonstrated this with other numbers, obtaining a closer approximation [than Archimedes did]. Now while Apollonius' approximation is more accurate, it is not useful for Archimedes' purpose. For Archimedes, as we have said, sought in this book [Measurement of a Circle] to find an approximation for practical purposes.

ἰστέον δέ, ὅτι καὶ Ἀπολλώνιος ὁ Περγαῖος ἐν τῷ Ὀκυτοκίῳ ἀπέδειξεν αὐτὸ δι' ἀριθμῶν ἐτέρων ἐπὶ τὸ σύνεγγυς μᾶλλον ἀγαγών. τοῦτο δὲ ἀκριβέστερον μὲν εἶναι δοκεῖ, οὐ χρησίμων δὲ πρὸς τὸν Ἀρχιμήδους σκοπόν· ἔφαμεν γὰρ αὐτὸν σκοπὸν ἔχειν ἐν τῷδε τῷ βιβλίῳ τὸ σύνεγγυς εὐρεῖν διὰ τὰς ἐν τῷ βίῳ χρείας.

Aristotle *Sophistic Refutations* 171b.35-172a.9

Aristotle *Sophistic Refutations*, trans. E. S. Forster & D. J. Furley, Cambridge, MA: Harvard University Press, (1955)

In a way, the contentious argument relates to the dialectical one as the false-diagrammer is to the geometrician. For it argues fallaciously from the same [premises] as dialectic, and the false-diagrammer from the same [premises] as the geometer. Yet the [false-diagrammer] is not contentious because he makes false diagrams from the principles and conclusions under his art. However, clearly the [argument] falling under dialectic will be contentious applied to other [disciplines]. For example, the squaring of the circle by means of lunules is not contentious, but that of Bryson is. The former cannot carry over to anything other than geometry alone because it is from [geometry's] proper principles but the latter [carries over] against many interlocutors—those who do not know what is possible and impossible in each case; for [the argument] will adapt. Or as Antiphon squared the circle. Or if someone should deny that it is better to walk after supper on account of Zeno's argument, it is not medical. For [the argument] is common.

ὁ δ' ἐριστικός ἐστὶ πως οὕτως ἔχων πρὸς τὸν διαλεκτικὸν ὡς ὁ ψευδογράφος πρὸς τὸν γεωμετρικόν· ἐκ γὰρ τῶν αὐτῶν τῷ διαλεκτικῷ παραλογίζεται, καὶ ὁ ψευδογράφος τῷ γεωμέτρῳ. ἀλλ' ὁ μὲν οὐκ ἐριστικός, ὅτι ἐκ τῶν ἀρχῶν καὶ συμπερασμάτων τῶν ὑπὸ τὴν τέχνην ψευδογραφεῖ· ὁ δ' ὑπὸ τὴν διαλεκτικὴν περὶ τᾶλλα ὅτι ἐριστικός ἐσται δῆλον. οἷον ὁ τετραγωνισμὸς ὁ μὲν διὰ τῶν μηνίσκων οὐκ ἐριστικός, ὁ δὲ Βρύσωνος ἐριστικός· καὶ τὸν μὲν οὐκ ἔστι μετενεγκεῖν ἀλλ' ἢ πρὸς γεωμετρίαν μόνον, διὰ τὸ ἐκ τῶν ἰδίων εἶναι ἀρχῶν, τὸν δὲ πρὸς πολλούς, ὅσοι μὴ ἴσασιν τὸ δυνατόν ἐν ἐκάστῳ καὶ τὸ ἀδύνατον· ἀρμόσει γάρ. ἢ ὡς Ἀντιφῶν ἐτετραγώνιζεν. ἢ εἴ τις μὴ φαίη βέλτιον εἶναι ἀπὸ δειπνοῦ περιπατεῖν διὰ τὸν Ζήνωνος λόγον, οὐκ ἰατρικός· κοινὸς γάρ.

But being simultaneous by nature does not seem to hold good for all things that are relative to something; for the knowable would appear to be prior to the knowledge of it; for the most part we acquire knowledge of what pre-exists; for in few cases, or in no case, would you find knowledge coming into being at the same time as its object. Furthermore when the knowable is removed it removes with it the knowledge, but the knowledge [when removed] does not remove the knowable with it. For if there is no object of knowledge there can be no knowledge of it (for there will no longer be anything for it to be the knowledge of); but even if there is no knowledge that does not mean that there can be no object of knowledge. Take the squaring of the circle: if it is knowable, the knowledge of it does not yet exist, but it is still an object of knowledge. Furthermore when animal is removed, knowledge is removed; but it is possible for there still to be many knowables.

ὡσαύτως δὲ καὶ ἐπὶ τῶν ἄλλων ὅσα τοιαῦτα. οὐκ ἐπὶ πάντων δὲ τῶν πρὸς τι ἀληθὲς δοκεῖ τὸ ἅμα τῇ φύσει εἶναι· τὸ γὰρ ἐπιστητὸν πρότερον ἢ δόξειε τῆς ἐπιστήμης εἶναι. ὥς γὰρ ἐπὶ τὸ πολὺ προϋπαρχόντων 25 τῶν πραγμάτων τὰς ἐπιστήμας λαμβάνομεν· ἐπ' ὀλίγων γὰρ ἢ ἐπ' οὐδενὸς ἴδοι τις ἂν ἅμα τῷ ἐπιστητῷ τὴν ἐπιστήμην γινομένην. Ἔτι τὸ μὲν ἐπιστητὸν ἀναιρεθὲν συναναιρεῖ τὴν ἐπιστήμην, ἢ δὲ ἐπιστήμη τὸ ἐπιστητὸν οὐ συναναιρεῖ· ἐπιστητοῦ μὲν γὰρ μὴ ὄντος οὐκ ἔστιν ἐπιστήμη 30 (οὐδενὸς γὰρ ἔσται ἐπιστήμη), ἐπιστήμης δὲ μὴ οὔσης οὐδὲν κωλύει ἐπιστητὸν εἶναι, οἷον καὶ ὁ τοῦ κύκλου τετραγωνισμὸς εἴγε ἔστιν ἐπιστητὸν, ἐπιστήμη μὲν αὐτοῦ οὐκ ἔστιν οὐδέπω, αὐτὸς δὲ ἐπιστητὸν ἔστιν. ἔτι ζῶου μὲν ἀναιρεθέντος οὐκ ἔσται ἐπιστήμη, τῶν δ' ἐπιστητῶν πολλὰ ἐνδέχεται εἶναι.

66) Following them Hippocrates of Chios, who invented the method of squaring lunules, and Theodorus of Cyrene became eminent in geometry. For Hippocrates wrote a book on elements, the first of whom we have any record who did so.

ἐφ' οἷς Ἱπποκράτης ὁ Χῖος ὁ τὸν τοῦ μηνίσκου τετραγωνισμὸν εὐρών, καὶ Θεόδωρος ὁ Κυρηναῖος ἐγένοντο περὶ γεωμετρίαν ἐπιφανεῖς. πρῶτος γὰρ ὁ Ἱπποκράτης τῶν μνημονευομένων καὶ στοιχεῖα συνέγραψεν.

213) They say that the first to effect reduction of difficult constructions was Hippocrates of Chios, who also squared the lune and made many other discoveries in geometry, being a man of genius when it came to constructions, if there ever was one.

πρῶτον δὲ φασι τῶν ἀπορουμένων διαγραμμάτων τὴν ἀπαγωγὴν ποιήσασθαι Ἱπποκράτην τὸν Χῖον, ὃς καὶ μηνίσκον ἐτετραγώνισε καὶ ἄλλα πολλὰ κατὰ γεωμετρίαν εὗρεν εὐφυῆς περὶ τὰ διαγράμματα εἴπερ τις ἄλλος γενόμενος.

422-423) It is my opinion that this problem is what led the ancients to attempt the squaring of the circle. For if a parallelogram can be found equal to any rectilinear figure, it is worth inquiring whether it is not

possible to prove that a rectilinear figure is equal to a circular area. Indeed, Archimedes proved that a circle is equal to a right-angled triangle when its radius is equal to one of the sides about the right angle and its perimeter is equal to the base. But of this elsewhere; let us proceed to the next propositions.

Ἐκ τούτου δὲ οἶμαι τοῦ προβλήματος ἐπαχθέντες οἱ παλαιοὶ καὶ τὸν τοῦ κύκλου τετραγωνισμόν ἐζήτησαν. εἰ γὰρ παραλληλόγραμμον ἴσον εὐρίσκεται παντὶ εὐθυγράμμῳ, ζητήσεως ἄξιον, μὴ καὶ τὰ εὐθύγραμμα τοῖς περιφερογράμμοις ἴσα δεικνύναι δυνατόν. καὶ ὁ Ἀρχιμήδης ἔδειξεν, ὅτι πᾶς κύκλος ἴσος ἐστὶ τριγώνῳ ὀρθογωνίῳ, οὗ ἡ μὲν ἐκ κέντρου ἴση ἐστὶν μιᾷ τῶν περὶ τὴν ὀρθήν, ἡ δὲ περίμετρος τῇ βάσει. ἀλλὰ ταῦτα ἐν ἄλλοις· ἐπὶ δὲ τὰ ἐξῆς ἴωμεν.

https://www.jstor.org/stable/j.ctv1ovm1rv?saml_data=eyJzYW1sVG9rZW4iOiI2ZTI4NTBhYS1lNTc3LTRL MzktOGYyNyooNTk2ODI4YjJkZGUiLCJpbmNoaXR1dGlvbklkcyI6WyIxN2VlMTcxNC1hNWYyLTRLlOTUtY WY1ZSozMjcyZGNkMDZjMjMiXXo

Proklos, *Commentary on the First Book of Euclid's Elements* 356.6-15

Proklos, *Commentary on the First Book of Euclid's Elements*, trans. G. R. Morrow, Princeton, NJ: Princeton University Press, 1970

This is the way in which other mathematicians also are accustomed to distinguish lines, giving the property of each species. Apollonius, for instance, shows for each of his conic lines what its property is, and Nicomedes likewise for the conchoids, Hippias for the quadratrices, and Perseus for the spiric curves. After a species has been constructed, the apprehension of its inherent and intrinsic property differentiates the thing constructed from all others. In the same way, then, the author of the *Elements* first investigates the properties of parallel lines.

Τοῦτον δὲ τὸν τρόπον εἰώθασιν καὶ οἱ ἄλλοι μαθηματικοὶ διαλέγεσθαι περὶ τῶν γραμμῶν, ἐκάστου εἶδους τὸ σύμπτωμα παραδιδόντες. καὶ γὰρ Ἀπολλώνιος ἐφ' ἐκάστης τῶν κωνικῶν γραμμῶν, τί τὸ σύμπτωμα δείκνυσιν, καὶ ὁ Νικομήδης ἐπὶ τῶν κογχοειδῶν, καὶ ὁ Ἰππίας ἐπὶ τῶν τετραγωνιζουσῶν, καὶ ὁ Περσεὺς ἐπὶ τῶν σπειρικῶν. μετὰ γὰρ τὴν γένεσιν τὸ καθ' αὐτὸ καὶ ἡ αὐτὸ ὑπάρχον ληφθὲν ἀφορίζει τὸ συστάν ἡμῖν εἶδος ἀπὸ τῶν ἄλλων ἀπάντων. κατὰ τὰ αὐτὰ δὲ οὖν καὶ ὁ στοιχειωτὴς τὰ συμπτώματα τῶν παραλλήλων ἀνευρίσκει πρῶτον.

<https://archive.org/details/commentaryonfirsooooproc/page/276/mode/zup?q=hippias>