Entanglement witness by quantum circuits

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Machine learning algorithms are applied to the entanglement witness problem.

CONTENTS

I. INTRODUCTION

Entanglement [1] is the key ingredient of quantum computation, quantum communication, and quantum cryptography [ref].

II. PRELIMINARY

A. Notations

B. Entanglement witness

Definition 1 (Entangled state)**.**

Definition 2 (Bipartite state)**.**

Definition 3 (Multi-partite state)**.**

Definition 4 (Fully separable state)**.**

Definition 5 (Genuine entangled state)**.**

Definition 6 (Graph state)**.**

[2]. MBQC [3]

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FIG. 1: (a) entanglement witness, PPT criteria, SVM (kernel). (b) with training data

Definition 7 (Entanglement witness)**.** entanglement witness $\hat{W}$

$$\mathrm{Tr}\left(\hat{W}\hat{\rho}\right) \geq 0, \forall \text{ separable} \ ; \quad \mathrm{Tr}\left(\hat{W}\hat{\rho}\right) < 0, \text{ for some entangled} \tag{1}$$

Problem 1 (Entanglement witness with prior)**.** [4]

C. Shadow tomography

Inspired by [5], Huang et. al [6]

Definition 8 (classical shadow)**.**

Theorem 1.

III. QUANTUM ALGORITHMS

A. Machine learning assisted

separability classifier by neural network [7]. rigorous quantum advantage of quantum kernel method in SVM [8]. classical machine learning with classical shadow [9].

Definition 9 (SVM)**.**

Definition 10 (Kernel)**.**

B. Variational quantum circuits

ansatz

C. Upper bounds and lower bounds

quantum advantage

- input encoding problem [10]
-

IV. NUMERICAL SIMULATION

V. CONCLUSION AND DISCUSSION

todo

- experiment

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