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|---------|---|
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| West on | |

(dumn k m km stage of elimination. Given mx = [-mx,1] Vmx,i emx, mx,i = 0 then

I k mx the stage of elimination. Given mx = [-mx,1] Vmx,i emx, mx,i = 0 then

I k mx, emx, mx,i is multiplied to a minimate the multiplied

Given an Mx as described abor we see

Where Mrsi is multiplier would for you i in step K.

We see this o exactly LK. ... Lm=I-mrent

b) Define
$$m_{K} = -m_{K} = \begin{bmatrix} -m_{K^{1}} \\ -m_{K^{1}} \end{bmatrix}$$
 where $\forall m_{K^{1}} \in m_{K^{1}} = 0$ when $i \leq k$

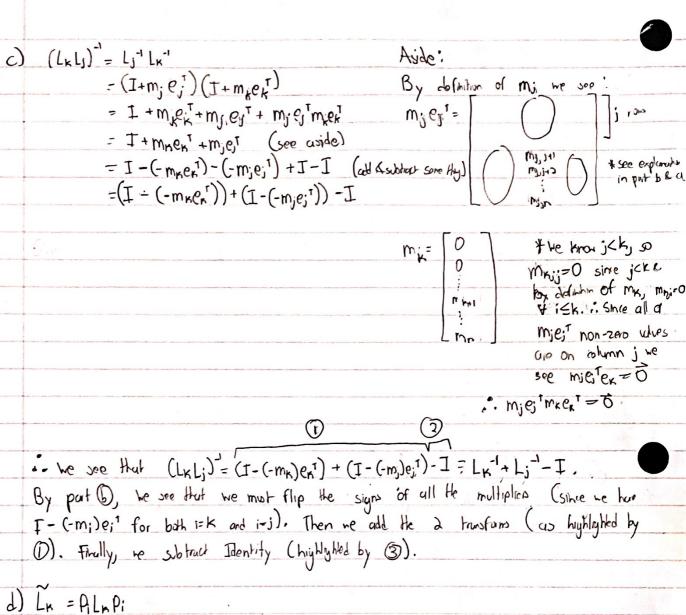
Now we see: $m_{K^{1}} \in m_{K^{1}} = 0$

The state of $m_{K^{1}} = m_{K^{1}} = 0$

The stat

we see we get a gas trunsfam with -1 times its multipliers. So this is exactly LK'

. LK' = I - MK'EK. where MK' = -MK



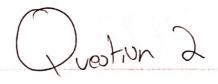
d) Ln = PilnPi = Pi (I-mnen^T) Pi = (Pi - Pimnen^T) Pi - PiA - Pimnen^T Pi = I - Pimnen^T Pi

7F 13 K:

Then by post-multiplying by A, we supply columns in the shapping rows that contain multipliers, so she are shapping multipliers man, and man, and man of maken, he are simply shapping columns in the all Os (since only robin k has) all the subtraction from I he get the which o the nith multipliers it is shapped.

If i < k:

The shapping R he shapped is \$1 & she i < k, he are shapping a O row with another row, since by the or mark, all rows < K are O row. Hence we are not shapping multipliers.



$$A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \\ 0 & 2 \end{bmatrix}$$

$$L_{1} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -21 & 0 & 1 & -8 \\ 0 & 0 & 1 & -8 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

Define
$$L_{k} = \begin{cases} a_{ij} = 0 & \text{when } i = j \\ a_{ij} = -2 & \text{when } i = k+1 & 2 = k \\ a_{ij} = 0 & \text{otherwise} \end{cases}$$

$$\begin{array}{lll}
V_{i} V_{j} &= \begin{cases}
a_{ij} = 1 & \text{when } i = j \\
a_{ij} = (-2) & \text{into when } j = n \text{ and } i \in [l], n-l]
\end{cases}$$

$$\begin{array}{lll}
C_{ij} &= (-2) & \text{into when } j = n \text{ and } i = k
\end{cases}$$

$$\begin{array}{lll}
C_{ij} &= 0 & \text{old}
\end{cases}$$

$$\begin{array}{lll}
C_{ij} &= 0 & \text{old}
\end{cases}$$

$$\begin{array}{lll}
C_{ij} &= 0 & \text{old}
\end{cases}$$

=
$$\begin{cases} a_{ij} = 1 & \text{if } i = j \\ a_{ij} = -2 & \text{if } i = j+1 \end{cases}$$
 where $i \in [l_j, n]$

$$\begin{cases} a_{ij} = 0 & \text{o/w} \end{cases}$$

b) the risk of over flow romos from the compounding or 14 throughout He lost column.

[(-1)"2" | ≤ 10"00-1 must be true to shore a float.

 $2^{i-1} = 10^{100}$ $i+1 = \log_{2}(10^{100})$ i+1 = 332.19 i = 331.19

in the see Hult He luyest mutrix in the form that can be factored in TR10 (16, 2) before over 10 occurs a order 331

O No He fectorization o not stable, as it will fail when the order of the meetrix is greater than 331 due to the overflow error.

Overtion 3

Let Pij be a parmutation muliix that sups row il; where is

i. we see
$$V = \begin{cases} a_{ij} = 2 & \text{when } i = j & \forall i \in [l_j n - 1] \\ a_{ij} = 1 & \text{whon } i = j - 1 & \forall j \in [l_j n - 1] \\ a_{ij} = 4 + (-1)^{i+1} (2)^{(1-i)} & \text{when } i = n \text{ and } k = n \\ a_{ij} = 0 & o/w \end{cases}$$

$$L_{K} = \begin{cases} a_{i}^{i} = 1 & \text{when } i = j & \forall i \in [1], n \\ a_{i}^{i} = (-1)^{K+1} \cdot 2^{-K} & \text{when } i = K+1 \text{ and } j = k \\ a_{i}^{i} = 0 & \text{o/w} \end{cases}$$

- b) There is no risk of averillar when using philothy, as our definition of U show that he never deal with any large numbers.
- c) Yes the furtherization is stuble since there is no risk for Overflow error

b)
$$A\vec{x} = \vec{b}$$
 $PA\vec{x} = P\vec{b}$
 $L\vec{d} = P\vec{b}$
 $\begin{bmatrix} 1 & 0 & 0 \\ 1/8 & 1 & 0 \\ 1/4 & -3/4 & 1 \end{bmatrix}$
 $\begin{bmatrix} d_1 \\ d_3 \end{bmatrix} = \begin{bmatrix} -16 \\ 6 \\ -7 \end{bmatrix}$

$$\begin{bmatrix}
 0 & 32 & 8 \\
 0 & 4 & 13 \\
 0 & 0 & 17
 \end{bmatrix}
 \begin{bmatrix}
 x_1 \\
 x_2 \\
 x_3
 \end{bmatrix}
 =
 \begin{bmatrix}
 -16 \\
 8 \\
 17
 \end{bmatrix}$$

$$|7x_{3}=|7| |4x_{2}+|3x_{3}=8| 8x_{1}+33x_{2}+8x_{3}=-16| x_{3}=-16| x_{3}=1 |4x_{2}+|3x_{3}=8| x_{4}+3x_{2}+8x_{3}=-16| x_{5}=-16| x_{5}=-16| x_{7}=-16| x_{7}=-16$$

C) Consider the coverminent you have several linear systems that differ only by the right side (ip. some A but different Is in A=Is). It is much charge to first factor out the mutix (which has I the cost of 3 +O(n2)) and then one factors to solve each liver system (each the cody n3000), rather than I doing elimination each tree (lakes 3 +O(n2)) flops out time).

Question 5

Steps:

- ① Since (is invertible, we first get system $C\vec{y} = \vec{x}$ (since $C^{-1}\vec{x} = y$). This will take $\frac{n^3}{3} + O(n^2)$ flop
- 3 Next, me culculate Ax through matrix multiplication. This will take no flops
- 3 Next, we add ARR CMX which we redeclated in OOD, rall this resulting well'this addition tokes o flops

- @ Next, he find 2At winy matrix multiplication, which lokes no Morph
- (B) Next, we cold the 2At culculated in (D& is calculated in (B), call the result of ER" which tops (B== of)
- Total flops $\cdot \frac{n^3}{3} + O(n^2) + n^2 + n^2 + \frac{n^3}{3} + O(n^2)$ $= \frac{2n^3}{3} + O(n^2)$
 - : the complexity of this approach is $\frac{2n^3}{3} + O(n^2)$

Steps: 1 find PA-LV fudricular, which the 3+0(n2) flop

(a) multiply A on both sides.

$$\lambda = \lambda^{-6} \times$$

$$\lambda_{\gamma} = \lambda \lambda^{-1} \lambda^{-5} \times$$

$$\lambda_{\gamma} = \lambda^{-5} \times$$

3 report step @ 5 more tres

AAAAAA i = x

(6) Next, we use the fuctorization in step (1) to solve 6 liver systems involvy A (each hukmy n2 Map)

6 tres he most we LV - Puchorisation to do Solves

1

total flop= n3 + 6n3+06)

i. we see complexity of algorithm 10 = 3+612+0(n)