CS 240: Reasoning Under Uncertainty (Fall 21)	Nov 8
Lecture 17	
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1 Introduction

In this lecture, we introduced the Game Theory.

2 Game Theory

Game theory studies what happens when self-interested players interact. A player's benefits can be represented using a payoff matrix that maps the actions to real numbers.

2.1 Elements In A Game

There are several elements in a game

- We limit the number of players to 2, so we have player A and player B.
- Each player has a set of strategies. We denote the strategy chosen by player A as a.
- A **strategy profile** $s = (a, b)$ specifies the strategy that is chosen by each player.
- Each player has a payoff function (denoted as u_A and u_B.) Given the strategy profile, $u_A(s)$ returns the payoff for A, and $u_B(s)$ returns the payoff for B.

We assume each player knows the payoff function throughout the game including the payoff functions for the other players.

2.2 Terminologies

Payoff Function

Dominant Strategies

1. Strategy a **strictly dominates** a':
   Choosing a always gives a better outcome than choosing a', no matter what the other players do.
2. Strategy a is **strictly dominant**:
   Strategy a strictly dominates any other strategies of player A.
3. Strategy a' is **strictly dominated** by strategy a:
   Strategy a strictly dominates a'.

Nash equilibrium is a stable state of the system that involves several interacting participants in which no player can gain by a change of strategy as long as all the other participants remain unchanged.

Theorem 1. *Every game where each player has a finite number of strategies has at least one Nash equilibrium.*

A strategy profile is a Nash equilibrium if A's strategy a is a 'best response' to what B does and vice versa.

2.3 Prisoner Dilemma

In this setting, it's easy to see what will happen. Whatever A does, B is better off confessing. Whatever B does, A is better off confessing. So they will both confess. This is because both players in the game have a dominant strategy (confess).

2.4 Hawks and Doves

In this setting, Neither A nor B has a dominant strategy. Even though there is no equilibrium in dominant strategies, it turns out that there exist at least two Nash equilibria: (H, D) and (D, H). This shows there might exist multiple Nash equilibria in the game.

2.5 Mixed Strategy

Intuitively, a Nash equilibrium is established when all the players stick to a strategy profile because each player cannot get a better payoff by changing his/her strategy if other players stay the same.

The intuition of the mixed strategy is each player randomizes his/her strategy to nullify the other player's efforts to maximize their expected payoff. For example, player A randomizes his/her strategy such that player B's expected payoff is irrelevant to how B chooses his/her strategy. Player B can randomize his/her strategy such that player A's expected payoff is irrelevant to how A chooses his/her strategy. Under this scenario, both A and B do not have incentive to change their strategies and this is a Nash equilibrium by definition.

3 Problem

1. Find the Nash equilibrium of a game with the following payoff matrix:

A, B	i	j	k
i	6, 6	10,15	3, 7
j	12,3	5, 5	2, 8
k	8, 0	20, 3	4, 4

2. Suppose A and B play tennis. If A serves to a side where B stands he loses the point, if A serves to a side where B does not stand, he wins the point. The payoff matrix for this game is as follows:

A, B	Left	Right
Left	-1,1	1, -1
Right	1,-1	-1, 1

Find a Nash equilibrium for this setting?
3. For the Hawk and Dove example discussed in class, the following payoff matrix is given. If A and B play Hawk with probability p = q = 0.2 is a mixed-strategy Nash equilibrium, what is the value of a?

A, B	Left	Right
Left	-25,-25	50, 0
Right	0, 50	a, a

4 Answer

1. Nash equilibrium at ¡k, k¿

    1. For player A: k dominates i

    2. For player B: k dominates i and j

    3. For player A: k dominates j

2. Let's assume A randomize his strategy so he plays left with probability t and right with probability 1 - t and player B's payoff is irrelevant to his/her strategy. Then

$$t + (1 - t)(-1) = (-1)t + (1 - t)$$

$$t = 0.5$$

We can obtain B's mixed strategy in the same way. So both players play each strategy with a probability of 0.5.
3.

$$-25q + 50(1 - q) = 0 * q + a(1 - q)$$

$$a = \frac{175}{4}$$