| CS 240: Reasoning Under Uncertainty (Fall 21) | Nov 3 |
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# Lecture 16

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## 1 Introduction

In this lecture, we introduced the Game Theory.

## 2 Game Theory

Game theory studies what happens when self-interested players interact. A player's benefits can be represented using a payoff matrix that maps the actions to real numbers.

### 2.1 Elements In A Game

There are several elements in a game

- We limit the number of players to 2, so we have player A and player B.
- Each player has a set of strategies. We denote the strategy chosen by player $A$ as $a$.
- A **strategy profile** $s = (a, b)$ specifies the strategy that is chosen by each player.
- Each player has a payoff function (denoted as $u_A$ and $u_B$.) Given the strategy profile, $u_A(s)$ returns the payoff for $A$, and $u_B(s)$ returns the payoff for $B$.

We assume each player knows the payoff function throughout the game including the payoff functions for the other players.

### 2.2 Terminologies

**Payoff Function**

**Dominant Strategies**

1. Strategy $a$ **strictly dominates** $a'$:
   Choosing $a$ always gives a better outcome than choosing $a'$, no matter what the other players do.
2. Strategy $a$ is **strictly dominant**:
   Strategy $a$ strictly dominates any other strategies of player $A$.
3. Strategy $a'$ is **strictly dominated** by strategy $a$:
   Strategy $a$ strictly dominates $a'$.

**Nash equilibrium** is a stable state of the system that involves several interacting participants in which no player can gain by a change of strategy as long as all the other participants remain unchanged.

**Theorem 1.** *Every game where each player has a finite number of strategies has at least one Nash equilibrium.*

A strategy profile is a Nash equilibrium if $A$'s strategy $a$ is a 'best response' to what $B$ does and vice versa.

### 2.3 Prisoner Dilemma

In this setting, it's easy to see what will happen. Whatever A does, B is better off confessing. Whatever B does, A is better off confessing. So they will both confess. This is because both players in the game have a dominant strategy (confess).

### 2.4 Hawks and Doves

In this setting, Neither A nor B has a dominant strategy. Even though there is no equilibrium in dominant strategies, it turns out that there exist at least two Nash equilibria: (H, D) and (D, H). This shows there might exist multiple Nash equilibria in the game.

### 2.5 Mixed Strategy

Intuitively, a Nash equilibrium is established when all the players stick to a strategy profile because each player cannot get a better payoff by changing his/her strategy if other players stay the same.

The intuition of the mixed strategy is each player randomizes his/her strategy to nullify the other player's efforts to maximize their expected payoff. For example, player A randomizes his/her strategy such that player B's expected payoff is irrelevant to how B chooses his/her strategy. Player B can randomize his/her strategy such that player A's expected payoff is irrelevant to how A chooses his/her strategy. Under this scenario, both A and B do not have incentive to change their strategies and this is a Nash equilibrium by definition.

## 3 Problem

1. Find the Nash equilibrium of a game with the following payoff matrix:

| A, B | i | j | k |
|---|---|---|---|
| i | 6, 6 | 10,15 | 3, 7 |
| j | 12,3 | 5, 5 | 2, 8 |
| k | 8, 0 | 20, 3 | 4, 4 |

2. Suppose A and B play tennis. If A serves to a side where B stands he loses the point, if A serves to a side where B does not stand, he wins the point. The payoff matrix for this game is as follows:

| A, B | Left | Right |
|---|---|---|
| Left | -1,1 | 1, -1 |
| Right | 1,-1 | -1, 1 |

Find a Nash equilibrium for this setting?
3. For the Hawk and Dove example discussed in class, the following payoff matrix is given. If A and B play Hawk with probability p = q = 0.2 is a mixed-strategy Nash equilibrium, what is the value of a?

| A, B | Left | Right |
|---|---|---|
| Left | -25,-25 | 50, 0 |
| Right | 0, 50 | a, a |

# 4 Answer

1. Nash equilibrium at ¡k, k¿

   1. For player A: k dominates i

   2. For player B: k dominates i and j

   3. For player A: k dominates j

2. Let's assume A randomize his strategy so he plays left with probability t and right with probability 1 - t and player B's payoff is irrelevant to his/her strategy. Then

$$t + (1 - t)(-1) = (-1)t + (1 - t)$$

$$t = 0.5$$

We can obtain B's mixed strategy in the same way. So both players play each strategy with a probability of 0.5.
3.

$$-25q + 50(1 - q) = 0 * q + a(1 - q)$$
$$a = \frac{175}{4}$$