Documentation for Buy Vs. Rent Calculator

1 Mortgage Calculations

Notation:

- *Monthly* interest rate: r
- Mortgage principal: P
- Mortgage duration in *months*: N

Assuming this is a fixed interest mortgage, the constant monthly payment to be made is

$$c = \frac{r}{1-(1+r)^{-N}} P \tag{1}$$

After k months with outstanding principal P_k the monthly payment can be decomposed into an interest part and a payment towards the principal for the next month:

- Interest: rP_k
- Payment towards principal: $c - rP_k$

In particular, after month $k+1$ the remaining principal is $P_k - (c - rP_k) = (1+r)P_k - c$. Summing up these monthly contributions towards the principal P we get that the remaining principal that still has to be paid off is:

$$P_k = (1+r)^k P - \left(1 + (1+r) + (1+r)^2 + \cdots + (1+r)^{k-1}\right) c \tag{2}$$

$$= \frac{1-(1+r)^{k-N}}{1-(1+r)^{-N}} P. \tag{3}$$

And the interest to be paid in month k is then:

$$rP_{k-1} = r\frac{1-(1+r)^{k-N}}{1-(1+r)^{-N}} P. \tag{4}$$

2 Rent vs. Buy Scenario

In each scenario the total financial position after a given number of years is $balance_{rent}$ and $balance_{buy}$ and for small values of monthly rent we would get

$$balance_{rent} < balance_{buy}, \tag{5}$$

i.e. renting would be financially better. For increasing rent the renting sceanrio will get more costly and we are interested in the maximum rent before buying becomes better:

$$balance_{rent}(rent_{max}) = balance_{buy}. \tag{6}$$

2.1 Renting

To calculate the function $balance_rent$ we start with one-off costs associated with buying a home that represent available money when renting:

$$balance_{rent,0} = HousePrice * (DownPayment + CostSelling), \tag{7}$$

where $DownPayment$ and $CostSelling$ are percentages of the house price. This is capital that is available from year 0 in the rent scenario. To get the available capital in the next year we use the recursion:

$$balance_{rent,n+1} = balance_{rent,n}(1 + InvRate) + BuyOutRunning_n - 12 * rent * (1 + RentGrowth)^n. \quad (8)$$

Each year the existing balance is assumed to grow with the constant investment rate $InvRate$. Running costs that are due in the buying scenario, $BuyOutRunning_n$, can be saved in the rental scenario. Finally, outgoing money in the rental scenario each year is the monthly rent which is assumed to grow at a yearly rate of $RentGrowth$.

We can use this formula to calculate the total balance when renting for each year once we determine the running costs in the buy scenario. For year n these are:

$$BuyRunningOut_n = 12c + HousePrice * (r_{main} + r_{ins}) * (1 + i)^{n-1}. \quad (9)$$

Here, c is the monthly mortgage payment from (1) and we assume that maintenance and insuarance are required at a rate of r_{main} and r_{ins}, rspectively. The latter grow with an inflation rate i.

2.2 Buying

In the buy scenario the initally available money (7) is spent in the process of buying and is thus no longer available. If the property is then sold after n years the balance is:

$$balance_{buy,n} = HousePrice * (1 + HomePriceGrowthRate)^n * (1 - CostSelling) - P_n, \quad (10)$$

where we assume a growth rate of property value of $HomePriceGrowthRate$, take into account the cost of selling as a percentage, $CostSelling$, and P_n is the remaining principal (2) that still has to be paid off.

3 Overall assumptions

The above modelling assumes:

1. Any additional savings that remain from any income are treated equally for buying and renting.