Stability of three ball juggling patterns as an indicator of a juggler's level

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Abstract

This paper examines the correlation between the stability of a three ball juggling pattern and a juggler's prowess measured by the amount of balls he can juggle. In order to find a correlation, several metrics for the stability of a juggling pattern have been investigated. From all these metrics there is not one that suggests a correlation between the stability of a three ball pattern and the juggler's level.

Introduction

In this paper we research if there is a correlation between the stability of a three ball juggling pattern and the total amount of balls a juggler can juggle. A metric for the stability of a juggling pattern must first be defined in order to find a correlation. If a metric exists such that there is a correlation, than this metric can be used by both amateur jugglers and circus schools to track progress of juggling technique.

Methods

The sample data required were the x and y coordinates of each individual ball in the three ball juggling pattern. We let 42 jugglers (see table 1) juggle two three ball patterns - the basic three ball cascade (*1*) and a high version of the three ball cascade with the same dwell time[1] as the normal cascade. By using Shannon's Juggling Theorem (*2*), this means the hands will spend more time being empty. The siteswaps[2] (*3)*of these patterns are 3 and 900 and from this point on in the paper they will be referred using this notation (*4*). The juggling was performed with a green, a blue and a yellow ball in front of a wall. The patterns were recorded using a 60fps camera in full HD.

To analyze the patterns we used an open source colour tracking program[3] to track each ball individually. The program saved the x- and y-coordinates of the balls in every frame. The library matplotlib in Python was used to plot graphs. A lissajous figure as seen in Fig. 1 was used to check if the balls were successfully tracked. In Fig. 2 the height of the balls against the time can be seen. In order to fourier transform the y positions, we used a python library called numpy. A fourier transform of a single ball's height can be seen in Fig. 3.

[1] Dwell time is the time a ball spends in the hand.
[2] Siteswap is the name of the mathematical notation of a juggling pattern.
[3] https://github.com/smeschke/juggling/blob/master/tracking/colorspaces_tracking.py

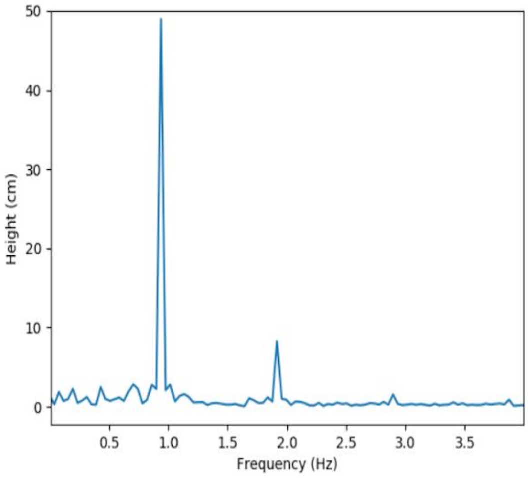
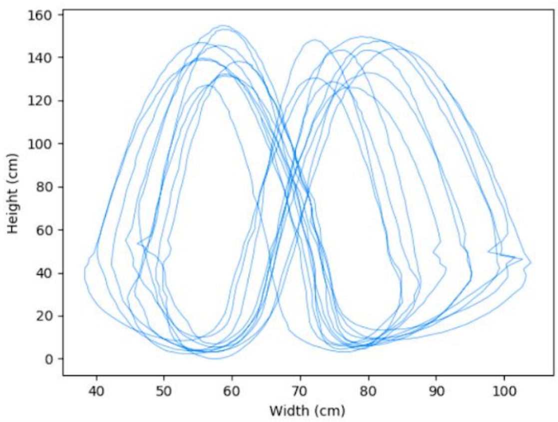


Figure 3. Fourier transform of a ball's y position



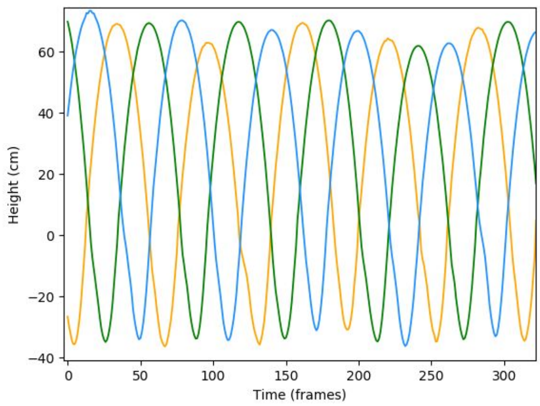


Fig. 1. X and y position of a single ball in a 3 ball cascade over time.

Fig. 2. The y-positions of all the balls in a 3 ball cascade. Every colour is different ball.

We analyse four aspects of our data:

1) Height of the highest peak in the fourier transform. The more a certain frequency is represented in a graph, the higher the peak will be at that frequency in the fourier transform. Since we expect better jugglers to be more consistent in their frequency of throwing, we expect better jugglers to have higher peaks.
2) The width of the highest peak at half its height in the fourier transform. Following the same reasoning as above, we expect better jugglers to have slimmer peaks.
3) Difference between the three fourier transforms of the balls of one juggler. This is calculated by adding up the absolute differences of all individual frequencies, and then dividing by the length to keep it relative.
4) Difference in height between throws of a ball. This statistic is calculated from the y position against time graph. The absolute difference between all neighbouring peaks are added together, and again divided by length to keep them relative. We expect better jugglers to have a more constant throw height, which results in a lower difference.

Results

A 95% confidence interval is calculated for the mean of every sample group for every metric.[4] The intervals are quite large, which is due to the fact that we have a too small sample size. You can find the confidence intervals in the supplementary materials.

The null hypothesis that we want to reject is that there is no difference between the means of the different groups of jugglers, with respect to any of the given metrics. In other words $\mu_3 = \mu_4 = \mu_5 = \mu_7 = \mu_8$ where μ_k is the mean of the metric for the group of k ball jugglers. Our alternative hypothesis is that there is a difference between the means of the groups. To test this we use a one way anova test with confidence level 0,95. We assume that our samples are all independent, from a normally distributed population and that the standard deviation of all groups are equal. The results can be seen in table 2 and table 3.

With our 95% confidence level we can only reject the null hypothesis with respect to the difference between each ball's Fourier transform of the siteswap 900. For all the other metrics we cannot reject the null hypothesis.

Discussion

The results found are subject to a couple of choices and unexpected circumstances during the process of data collection, which will be discussed here.

The rejected null hypothesis

The null hypothesis that we rejected, that the means of the difference between each ball's Fourier transform are the same, is statistically significant. However, the differences are within 0,0005 Hz of each other and thus they are not clinically important.

Threshold of >50 catches

To be an n-ball juggler your record of n balls had to be higher than 50 catches. This threshold might have been too severe as someone who could do only 20 catches of 5 balls would still be categorised as a 4 ball juggler. This threshold was chosen to be certain someone could really control an n-ball pattern.

Choice of the 3 ball patterns

Since the first pattern most jugglers learn to juggle is a three ball cascade we chose siteswap 3 as an easy pattern. Our hypothesis was that the difference in stability would be too minute to be noticed using one of the metrics. Therefore there was a need for a more difficult pattern. Yet it had to be easy enough that everyone who participated could juggle it for 30 seconds. The siteswap 900 seemed appropriate. Using Shannon's Juggling Theorem (*2*) the time of a hand being empty increases for this pattern, which makes it harder to throw consistent. Additionally it is harder to throw to the same height since the throw height increases.

Confidence Interval

The confidence intervals of almost all listed metrics are very large. This is partially because our total

[4] The sample group of 6 ball jugglers does not have a confidence interval since there was only one 6 ball juggler.

sample group of 42 people is divided further into 6 smaller groups, which results in a large Standard Error, which increases the size of the confidence interval. Secondly, there are too many outliers due to malfunctions in the tracking system. Sometimes it loses track of the ball and gives (0,0) coordinates. Besides that, sometimes it also wrongly identifies a part of the wall as a ball, which results in wrong coordinates as well. A way to solve this would be to manually remove or improve every video.

Conclusion

After analysing four aspects of the path of a juggling ball during two different siteswaps, we cannot conclude that there is a measurable difference between jugglers of different skill levels.

Future research in this field could include the measuring of brain activity while juggling. As shown in the paper *Towards real-time visualisation of a jugglers brain* (*5*)*,* the brain activity differs between an expert and an amateur juggler while juggling. This concept can be further explored, to see if a computer can predict how good a juggler is based on his brain activity.

Acknowledgements, References

(*1*) P.J. Beeks, "Juggling Dynamics", thesis, Free University Amsterdam (1989), pp. 24-25.

(*2*) C. E. Shannon, Scientific aspects of juggling, Claude Elwood Shannon: Collected Papers, (IEEE Press, 1993), pp. 850-864.

(*3*) B. Polster, *The Mathematics of Juggling*, (Springer, New York, 2000), pp. 7-9.

(*4*) J. Buhler, D. Eisenbud, R. Graham, C. Wright, Juggling drops and descents. *American Mathematical Monthly*. **101**, 507–519, (1994).

(*5*) G. Schiavone, U. Grössekathöfer, S. à Campo, V. Mihajlovic: Towards real-time visualisation of a jugglers brain, *Brain-Computer Interfaces.* **2**, 90-102 (2015).

Nr. of balls	Nr. of jugglers
3	2
4	7
5	18
6	1
7	10
8	4

Table 1. Maximum amount of balls the juggler can juggle for >50 catches.

	one way anova statistic	p-value
Highest peak of Fourier transform	2,163	0,094
Width peak of Fourier transform	0,207	0,933
Difference between each ball's Fourier transform	1,769	0,157
Height difference per ball	0,796 0,742 0,381	0,536 0,570 0,820

Table 2. p-values and anova statistics of siteswap 3

	one way anova statistic	p-value
Highest peak of Fourier transform	2,400	0,069
Width peak of Fourier transform	0,705	0,594
Difference between each ball's Fourier transform	5,103	0,002
Height difference per ball	1,649 1,861 0,927	0,184 0,139 0,459

Table 3. p-values and anova statistics of siteswap 900

Supplementary material

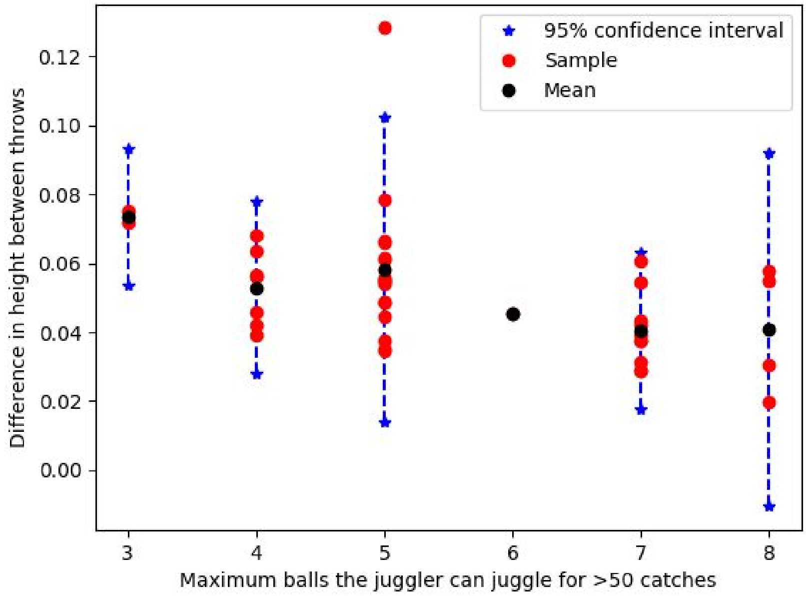


Fig. 8. height difference between each throw of siteswap 3

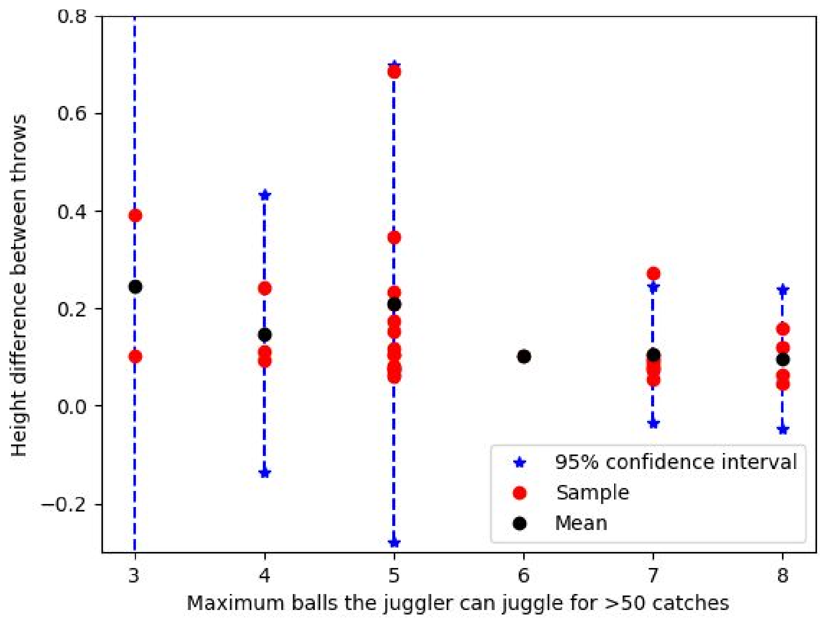


Fig. 9. height difference between each throw of siteswap 900

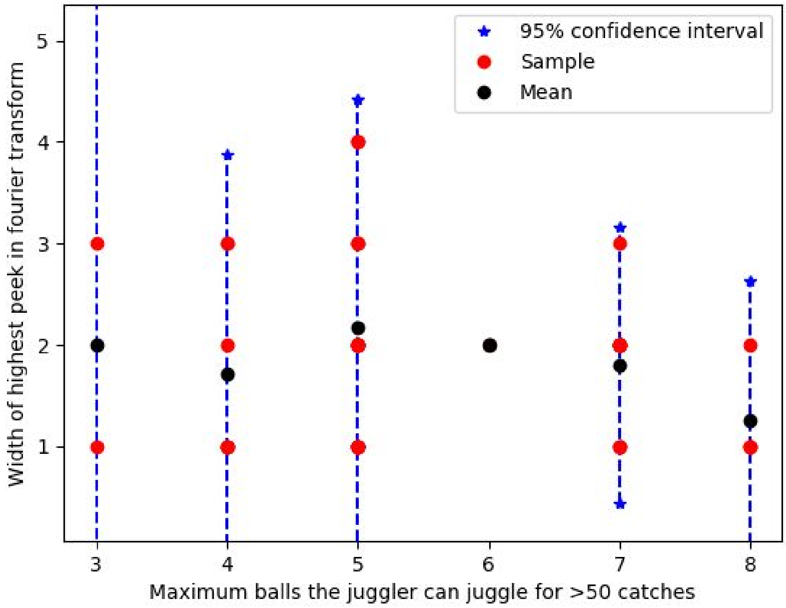


Fig. 10. Peak width of siteswap 3

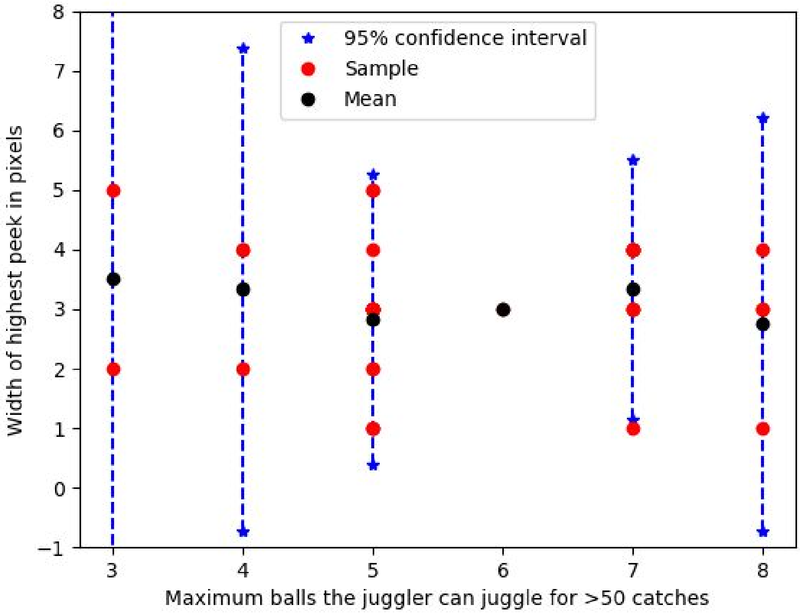


Fig. 11. Peak width of siteswap 900

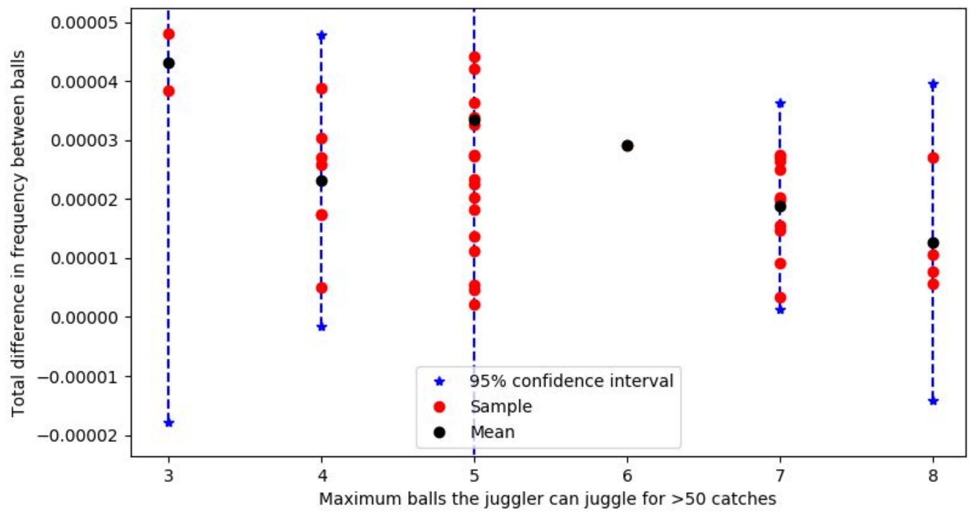


Fig. 12. Difference between each ball's Fourier transform siteswap 3

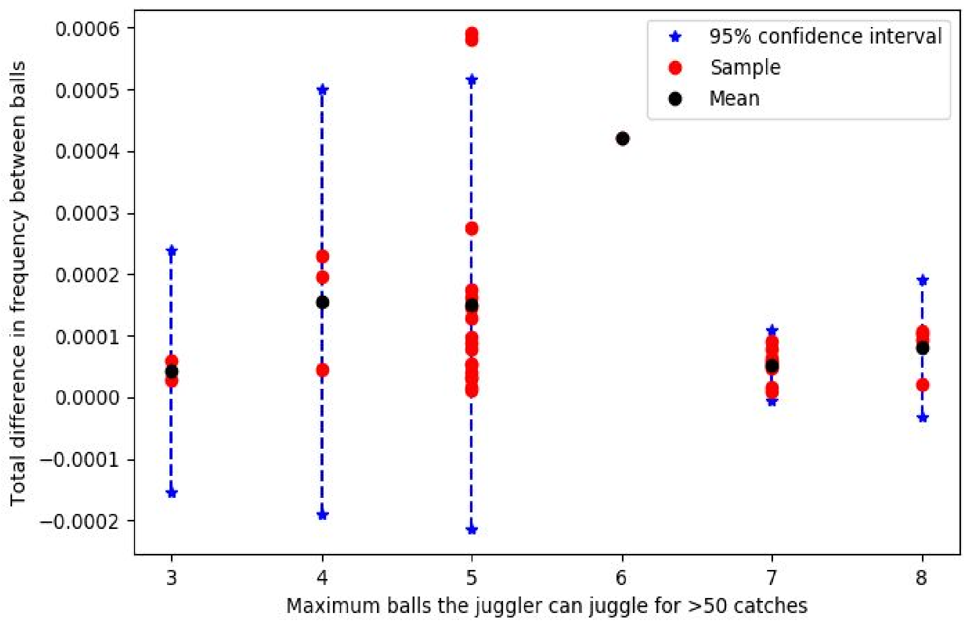


Fig. 13. Difference between each ball's Fourier transform siteswap 900

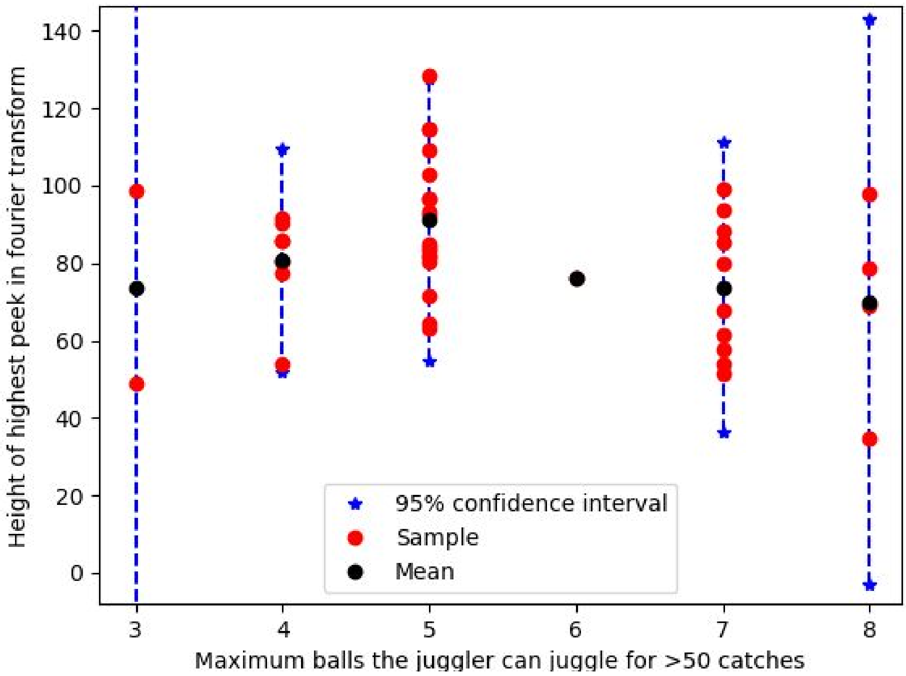


Fig. 14. Highest peak of the Fourier transform of siteswap 3

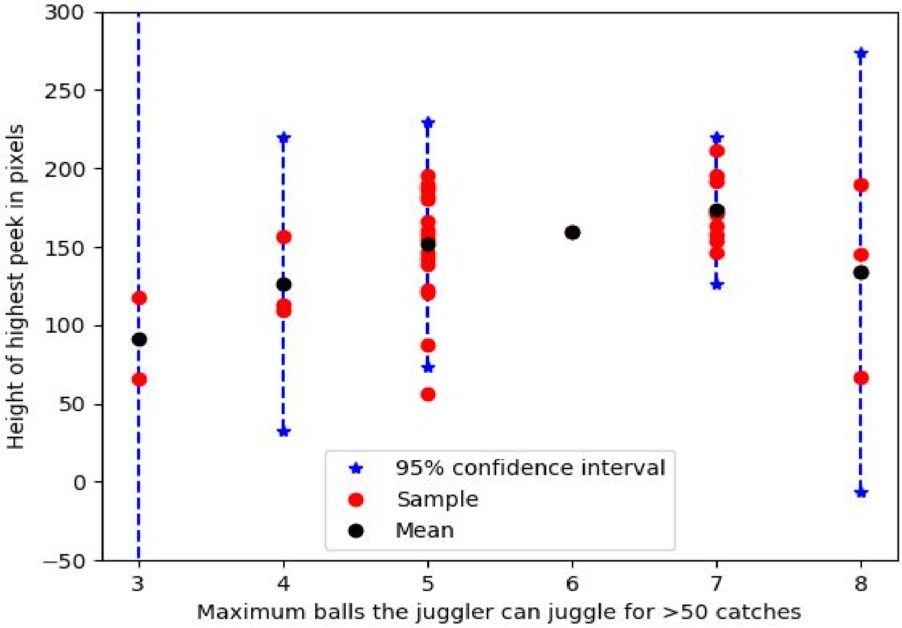


Fig. 15. Highest peak of the Fourier transform of siteswap 900