Course Code : MCS-212
Course Title : Discrete Mathematics
Assignment Number : MCA_NEW(1)/212/Assign/2023

Last Dates for Submission : 30thApril 2023 (for January Session)

31st October 2023 (for July Session)

Q1: Attempt the following, all questions are compulsory, and each question carries 2 marks for each

a)Prove by mathematical induction that ∑ 1 $i(i+1)$ = $n/(n+1)$

Ans:-

Prove $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$

Prove $\sum_{i=1}^{n} \frac{1}{i(i+1)} = n/(n+1)$

Proof

Base Case

$L.H.S = \sum_{i=1}^{1} \frac{1}{i(i+1)} = \frac{1}{1(1+1)} = \frac{1}{2}$

$R.H.S = \frac{1}{1+1} = \frac{1}{2}$, So it is true

Now, let's assume that the equation holds true for some positive integer K, that is:

$$\sum_{i=1}^{K} \frac{1}{i(i+1)} = \frac{K}{K+1}$$

We need to prove that it also holds for K+1, that is:

$$\sum_{i=1}^{K+1} \frac{1}{i(i+1)} = \frac{K+1}{K+2}$$

Page No.
Date

$$\sum_{i=1}^{k+1} \frac{1}{i(i+1)} = \sum_{i=1}^{k} \frac{1}{i(i+1)} + \frac{1}{(k+1)(k+2)}$$

$$= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)} = \frac{k(k+2)+1}{(k+1)(k+2)}$$

$$= \frac{k^2+2k+1}{(k+1)(k+2)} = \frac{(k+1)^2}{\cancel{(k+1)}(k+2)}$$

$$= \frac{k+1}{k+2}$$

So k+1 is true ||

By Principle of Mathematical induction its true.

b) Verify whether √11 is rational or irrational.

Ans:-

To determine whether √11 is rational or irrational, we need to check if it can be expressed as the quotient of two integers (i.e., a rational number) or not.

Suppose √11 is a rational number, then we can express it as √11 = p/q, where p and q are integers with no common factors (except 1). We can also assume that q ≠ 0 since we cannot divide by zero.

Squaring both sides of the equation, we get:

11 = p^2/q^2

Multiplying both sides by q^2, we get:

11q^2 = p^2

This implies that p^2 is a multiple of 11. Therefore, p must also be a multiple of 11 since the square of any integer (including p) is either a multiple of 11 or leaves a remainder of 1 when divided by 11.

Let p = 11k, where k is an integer. Substituting this into the equation above, we get:

11q^2 = (11k)^2

11q^2 = 121k^2

q^2 = 11k^2

This implies that q^2 is also a multiple of 11, and hence, q must be a multiple of 11.

However, we assumed that p and q have no common factors (except 1), so this is a contradiction. Therefore, our initial assumption that √11 is rational is false.

Hence, we can conclude that √11 is irrational, i.e., it cannot be expressed as the quotient of two integers.

c) What are Demorgan's Law? Explain the use of Demorgen's law with example.

Ans:-

De Morgan's laws are a set of two logical rules that relate the negation of a conjunction (AND) or disjunction (OR) of two statements. These laws are as follows:

1.The negation of a conjunction is equivalent to the disjunction of the negations of the statements. In other words:

$\sim(p \land q) \Leftrightarrow \sim p \lor \sim q$

2.The negation of a disjunction is equivalent to the conjunction of the negations of the statements. In other words:

$\sim(p \lor q) \Leftrightarrow \sim p \land \sim q$

Demorgan's laws are useful in simplifying complex logical expressions by breaking them down into simpler parts. For example, consider the expression $\sim(p \land q)$. Applying the first Demorgan's law, we can rewrite this as $\sim p \lor \sim q$. Similarly, if we have $\sim(p \lor q)$, we can rewrite this as $\sim p \land \sim q$ using the second Demorgan's law.

d) Make truth table for followings:

i) p→ (~ q ∨ ~ r) ∧ (p ∨ r)

ii) p→(~ r ∧ q) ∧ (p ∧ ~ q)

Here are the truth tables for the given logical expressions:

i) p→ (~q ∨ ~r) ∧ (p ∨ r)

p	q	r	¬q	¬r	¬q∨¬r	p∨r	(¬q∨¬r)∧(p∨r)	p⇒(¬q∨¬r)∧(p∨r)
T	T	T	F	F	F	T	F	F
T	T	F	F	T	T	T	T	T
T	F	T	T	F	T	T	T	T
T	F	F	T	T	T	T	T	T
F	T	T	F	F	F	T	F	T
F	T	F	F	T	T	F	F	T
F	F	T	T	F	T	T	T	T
F	F	F	T	T	T	F	F	T

ii) p→(~r ∧ q) ∧ (p ∧ ~q)

p	r	q	¬r	¬r∧q	¬q	p∧¬q	¬r∧q∧p∧¬q	p⇒¬r∧q∧p∧¬q
T	T	T	F	F	F	F	F	F
T	T	F	F	F	T	T	F	F
T	F	T	T	T	F	F	F	F
T	F	F	T	F	T	T	F	F
F	T	T	F	F	F	F	F	T
F	T	F	F	F	T	F	F	T
F	F	T	T	T	F	F	F	T
F	F	F	T	F	T	F	F	T

e) Obtain the truth value of disjunction of " Water is essential for life" and "2+2=4".

Ans.

The truth value of a disjunction (the logical operator "or") is true if at least one of the statements is true.

In this case, "Water is essential for life" is true, as it is a scientifically accepted fact. Similarly, "2+2=4" is also true, as it is a mathematical fact.

Therefore, the disjunction of "Water is essential for life" and "2+2=4" is true.

f) Write the following statements in the symbolic form.

i) Some students can not appear in exam.

ii) Everyone can not sing.

Ans:- i) Let S(x) be "x is a student" and E(x) be "x can appear in the exam". Then the symbolic form of "Some students can not appear in exam" is:

$\exists x\ (S(x) \wedge \neg E(x))$

This can be read as "there exists at least one x such that x is a student and x cannot appear in the exam."

ii) Let P(x) be "x is a person" and S(x) be "x can sing". Then the symbolic form of "Everyone can not sing" is:

$\forall x\ (P(x) \rightarrow \neg S(x))$

This can be read as "for all x, if x is a person, then x cannot sing."

g) Draw logic circuit for the following Boolean Expression: (x y z) + (x+y+z)'+(x'zy')

Ans:-

$(x\ y\ z) + (x + y + z)' + (x'\ z\ y')$

$= (x\ y\ z) + (x'\ y'\ z') + (x'\ z\ y')$ (De Morgan's law)

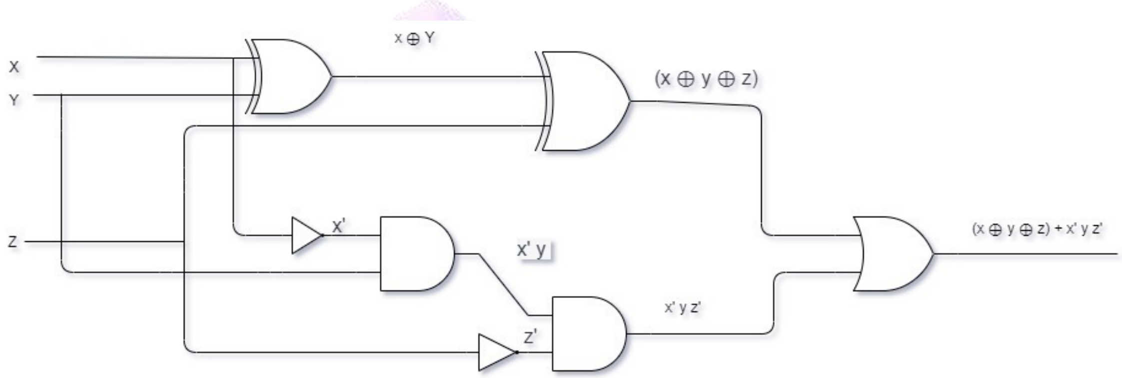
$= (x'\ y'\ z') + (x\ y\ z) + (x'\ z\ y')$ (commutative property of addition)

$= (x'\ y'\ z') + (x\ y\ z) + (x'\ y\ z')$ (commutative property of addition)

$= (x'\ y'\ z' + x\ y\ z) + x'\ y\ z'$ (distributive property of addition)

$= (x \oplus y \oplus z) + x'\ y\ z'$ (using XOR operation)

Therefore, the simplified Boolean expression is $(x \oplus y \oplus z) + x'\ y\ z'$, where $\oplus$ represents the XOR operation.



h) What is dual of a boolean expression? Explain with the help of an example.

The dual of a Boolean expression is obtained by interchanging the logical operations AND and OR, and also interchanging 0's and 1's. In other words, the dual of an expression is obtained by replacing each operator with its complement and each operand with its complement.

For example, consider the Boolean expression:

A AND (B OR C)

The dual of this expression can be obtained by replacing AND with OR, OR with AND, A with its complement A', B with its complement B', and C with its complement C'. Thus, the dual expression is:

A' OR (B' AND C')

Notice that the dual expression has the same variables as the original expression, but the logical operations and the complementation of the variables are different.

The dual of a Boolean expression can be useful in simplifying or analyzing circuits or logic systems. By applying the duality principle, one can obtain a different perspective on the same problem and potentially find a simpler or more efficient solution.

i)Show using truth table whether (P ∧ Q ∨ R) and (P ∨ R) ∧ (Q ∨ R) are equivalent or not.

Ans To determine whether the two expressions (P ∧ Q ∨ R) and (P ∨ R) ∧ (Q ∨ R) are equivalent, we can construct a truth table for both expressions and compare the outputs for all possible combinations of truth values for P, Q, and R.

Here is the truth table for the expression (P ∧ Q ∨ R):

P	Q	R	P∧Q	P∧QVR
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	F
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

Here is the truth table for the expression (P ∨ R) ∧ (Q ∨ R):

P	R	Q	PVR	QVR	(PVR)∧(QVR)
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	F	T	F
F	F	F	F	F	F

By comparing the outputs for both expressions in the truth tables, we can see that the two expressions are equivalent since they have same outputs for same input combinations. Therefore, the two expressions are equivalent.

j) Explain whether (P ∧ Q) → (Q → R) is a tautology or not.

To determine whether the expression (P ∧ Q) → (Q → R) is a tautology or not, we can construct a truth table for the expression and check if it evaluates to true for all possible combinations of truth values for P, Q, and R.

Here is the truth table for the expression (P ∧ Q) → (Q → R):

P	Q	R	P∧Q	Q⇒R	P∧Q⇒Q⇒R
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	F	T	T
F	T	F	F	F	T
F	F	T	F	T	T
F	F	F	F	T	T

By examining the truth table, we can see that the expression (P ∧ Q) → (Q → R) is not a tautology because it evaluates to false for the input combination where P is T, Q is T, and R is F. Therefore, the expression is not true for all possible input combinations, and hence, it is not a tautology.

Q2: Attempt the following, all questions are compulsory, and each question carries 2 marks for each

a)Set A,B and C are: A = {1, 2, 3,5, 7, 9 11,13}, B = { 1,2, 3 ,4, 5,6, 7,8,9 } and C { 1,2 ,4,5,6,7,8,10, 13}. Find A ∩ B ∩ C , A ∪ B ∪ C, A ∪ (B ∩ C)

Ans:-

To find A ∩ B ∩ C, we need to identify the elements that are common to all three sets A, B, and C:

A ∩ B ∩ C = {1, 2, 5, 7}

To find A ∪ B ∪ C, we need to combine all the elements from sets A, B, and C without duplicates:

A ∪ B ∪ C = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13}

To find A ∪ B ∩ C, we first find the intersection of sets B and C, and then combine it with set A without duplicates:

B ∩ C = {1, 2, 4, 5, 6, 7, 8}

A ∪ (B ∩ C) = {1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13}

Therefore, A ∩ B ∩ C = {1, 2, 5, 7}, A ∪ B ∪ C = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13}, and A ∪ B ∩ C = {1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13}.

b)What is power set? Write power set of set A={1,2,3,4,5,6,7,9}.

The power set of a set is the set of all subsets of that set, including the empty set and the set itself. In other words, if a set A has n elements, then its power set P(A) contains 2nsubsets.

To find the power set of the set A = {1, 2, 3, 4, 5, 6, 7, 9}, we need to list all possible subsets of A, including the empty set and the set itself:

P(A) = { {}, {1}, {2}, {3}, {4}, {5}, {6}, {7}, {9}, {1,2}, {1,3}, {1,4}, {1,5}, {1,6}, {1,7}, {1,9}, {2,3}, {2,4}, {2,5}, {2,6}, {2,7}, {2,9}, {3,4}, {3,5}, {3,6}, {3,7}, {3,9}, {4,5}, {4,6}, {4,7}, {4,9}, {5,6}, {5,7}, {5,9}, {6,7}, {6,9}, {7,9}, {1,2,3}, {1,2,4}, {1,2,5}, {1,2,6}, {1,2,7}, {1,2,9}, {1,3,4}, {1,3,5}, {1,3,6}, {1,3,7}, {1,3,9}, {1,4,5}, {1,4,6}, {1,4,7}, {1,4,9}, {1,5,6}, {1,5,7}, {1,5,9}, {1,6,7}, {1,6,9}, {1,7,9}, {2,3,4}, {2,3,5}, {2,3,6}, {2,3,7}, {2,3,9}, {2,4,5}, {2,4,6}, {2,4,7}, {2,4,9}, {2,5,6}, {2,5,7}, {2,5,9}, {2,6,7}, {2,6,9}, {2,7,9}, {3,4,5}, {3,4,6}, {3,4,7}, {3,4,9}, {3,5,6}, {3,5,7}, {3,5,9}, {3,6,7}, {3,6,9}, {3,7,9}, {4,5,6}, {4,5,7}, {4,5,9}, {4,6,7}, {4,6,9}, {4,7,9}, {5,6,7}, {5,6,9}, {5,7,9}, {6,7,9}, {1,2,3,4}, {1,2,3,5}, {1,2,3,6}, {1,2,3,7}, {1,2,3,9}, {1,2,4,5}, {1,2,4,6}, {1,2,4,7}, {1,2,4,9}, {1,2,5,6}, {1,2,5,7}, {1, 2,5,7,9}

c) Give geometric representation for followings:

i) { -3} x R ii) {1, -2) x (2, -3)

Ans:

i) { -3} x R represents a set of all points in the Cartesian plane whose x-coordinate is -3 and whose y-coordinate can take any real value. Geometrically, this corresponds to a vertical line passing through the point (-3, 0) on the x-axis.

ii) {1, -2} x (2, -3) represents a set of all points in the Cartesian plane whose x-coordinate lies between 1 and -2, and whose y-coordinate lies between 2 and -3 (exclusive). Geometrically, this corresponds to a rectangle in the second quadrant with vertices at (1, 2), (1, -3), (-2, -3), and (-2, 2).

d) What is proper subset? Explain with the help of example.

Ans.

A proper subset is a subset of a set that contains only some, but not all, of the elements of the original set. In other words, if set A is a proper subset of set B, then every element of A is also an element of B, but there is at least one element in B that is not in A.

For example, let's say we have two sets:

A = {1, 2, 3}

B = {1, 2, 3, 4}

Here, set A is a subset of set B, because every element of A (1, 2, and 3) is also an element of B. However, set A is not a proper subset of set B, because set B contains all the elements of set A as well as an additional element (4).

Now, let's take another example:

C = {2, 3}

D = {1, 2, 3, 4}

Here, set C is a proper subset of set D, because every element of C (2 and 3) is also an element of D, but D contains additional elements (1 and 4) that are not in C.

To summarize, a proper subset is a subset that contains some, but not all, of the elements of the original set.

e) What is relation? Explain properties of relations with example. $n\ i$ = 1

ans:- In mathematics, a relation is a set of ordered pairs. A relation between two sets A and B is a subset of the Cartesian product A × B. If (a, b) is an ordered pair in the relation R, we write a R b. A relation can exist between any two sets of objects, not necessarily numbers.

There are several properties of relations:

1.Reflexive property: A relation R on a set A is said to be reflexive if every element a ∈ A is related to itself, i.e., (a, a) ∈ R for every a ∈ A. For example, the relation "less than or equal to" (≤) on the set of real numbers is reflexive, since a ≤ a for every real number a.

2.Symmetric property: A relation R on a set A is said to be symmetric if for every pair of elements a, b ∈ A, if (a, b) ∈ R, then (b, a) ∈ R. For example, the relation "is a sibling of" on a set of people is symmetric, since if person A is a sibling of person B, then person B is also a sibling of person A.

3.Transitive property: A relation R on a set A is said to be transitive if for every three elements a, b, and c ∈ A, if (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R. For example, the relation "is taller than" on a set of people is transitive, since if person A is taller than person B and person B is taller than person C, then person A is also taller than person C.

4.Anti-symmetric property: A relation R on a set A is said to be anti-symmetric if for every pair of distinct elements a, b ∈ A, if (a, b) ∈ R and (b, a) ∈ R, then a = b. For example, the relation "divides" on the set of positive integers is anti-symmetric, since if a divides b and b divides a, then a = b.

5.Equivalence relation: A relation R on a set A is said to be an equivalence relation if it is reflexive, symmetric, and transitive. For example, the relation "is congruent to modulo 5" on the set of integers is an equivalence relation.

6.Partial order: A relation R on a set A is said to be a partial order if it is reflexive, anti-symmetric, and transitive. For example, the relation "is a subset of" on the set of all sets is a partial order.

For example, let us consider the set A = {1, 2, 3}. A relation R on A can be defined as R = {(1, 1), (1, 2), (2, 2), (2, 3)}. This relation is reflexive, since (1, 1) and (2, 2) are in R; it is symmetric, since (1, 2) is in R if and only if (2, 1) is in R; and it is transitive, since (1, 2) and (2, 2) are in R, so (1, 2) is in R. However, it is not anti-symmetric, since both (1, 2) and (2, 1) are in R, but 1 ≠ 2. Hence, the relation R is not a partial order.

f) Explain whether function: f(x) = x2 posses an inverse function or not.

Ans:-

The function f(x) = x^2 is not invertible on its entire domain of real numbers, because it fails the horizontal line test.

To see why, consider the points (1,1) and (-1,1) on the graph of the function. These two points have the same y-value, which means that if we try to draw a horizontal line through the graph that intersects the curve at those points, the line would intersect the curve at two different points. This violates the horizontal line test, which is a necessary condition for a function to have an inverse.

However, we can define a restricted domain of the function that makes it invertible. For example, if we restrict the domain to non-negative real numbers (i.e., x ≥ 0), then the function becomes one-to-one, and we can define its inverse function as f^(-1)(y) = √y.

Similarly, we can restrict the domain to negative real numbers (i.e., x < 0) and define the inverse function as f^(-1)(y) = -√y.

Therefore, the function f(x) = x^2 does not have a global inverse function on its entire domain, but it does have two different inverse functions on its restricted domains.

g) Write the finite automata corresponding to the regular expression (a + b)*ab

Ans.

The regular expression (a + b)*ab represents the language consisting of all strings over the alphabet {a, b} that end with the string "ab".

We can construct a deterministic finite automaton (DFA) that recognizes this language using the following steps:

1. Start with a state q0 that is the initial state of the automaton.
2. Add a transition from q0 to itself on input a or b.
3. Add a new state q1 that is the only accepting state of the automaton.
4. Add a transition from q0 to q1 on input a.
5. Add a transition from q1 to q1 on input a or b.
6. Add a transition from q1 to itself on input b.
7. Add a transition from any state to a dead state (which is not an accepting state) on any input that has not already been defined.

The resulting DFA can be represented by the following state transition table:

State	a	b
q0	q0	q0
q1	q1	q1
D	D	D

where D is a dead state. The initial state is q0 and the only accepting state is q1.

We can also represent this DFA using a diagram:

a,b ┌───▼───┐ a │ q0 ├──►q1 └────────┘ b

In this diagram, the state q0 has transitions to itself on input a or b, and a transition to q1 on input a. The state q1 has transitions to itself on input a or b, and a transition to a dead state on input other than a or b.

h) If L1 and L2 are context free languages then, prove that L1 U L2 is a context free language.

Ans.

To prove that the union of two context-free languages is also a context-free language, we can use the fact that context-free languages are closed under union and concatenation.

Let G1 = (V1, T, S1, P1) be a context-free grammar that generates L1, and let G2 = (V2, T, S2, P2) be a context-free grammar that generates L2. We can construct a new context-free grammar G = (V, T, S, P) that generates the union of L1 and L2 as follows:

- Let V = {S}, where S is a new start symbol.
- Let P be the union of P1 and P2, with the additional rule S -> S1 | S2.

Intuitively, the new grammar G has two "copies" of the original grammars G1 and G2, one for each language L1 and L2. The new start symbol S allows us to choose which grammar to use to generate a string in the union of L1 and L2.

To see that G generates L1 U L2, consider a string w in L1 U L2. Then either w is in L1 or w is in L2. If w is in L1, then G1 generates w by definition of context-free grammar. If w is in L2, then G2 generates w by definition of context-free grammar. Therefore, either G1 or G2 generates w, and the rule S -> S1 | S2 allows us to use either grammar to generate w.

Conversely, to see that every string generated by G is in L1 U L2, consider a string w generated by G. Then the only rule that can be applied to S is S -> S1 or S -> S2, which means that either G1 or G2 generates w. Therefore, w is in L1 or w is in L2, and hence w is in L1 U L2.

Therefore, we have shown that the union of two context-free languages is also a context-free language.

i)Explain Decidable and Undecidable Problems. Give example for each.

Ans In computability theory, a problem is said to be decidable if there exists an algorithm that can solve the problem for any input instance in a finite amount of time. Conversely, a problem is said to be undecidable if there is no algorithm that can solve the problem for all input instances in a finite amount of time.

One classic example of a decidable problem is the problem of determining whether a given number is prime or composite. This problem can be solved using various algorithms, such as the Sieve of Eratosthenes or the Miller-Rabin primality test, which have been proven to run in finite time for any input instance.

On the other hand, the halting problem is an example of an undecidable problem. This problem asks whether a given program, when executed on a given input, will eventually halt or run forever. It can be shown using a diagonalization argument that there is no algorithm that can solve the halting problem for all input instances, even if we restrict ourselves to programs written in a particular programming language or with a particular computational model.

Another famous example of an undecidable problem is the Entscheidungsproblem, which asks whether there is an algorithm that can decide whether a given mathematical statement is provable or not. This problem was first posed by David Hilbert in 1928, and was later shown by Kurt Gödel to be undecidable using his incompleteness theorems.

j) What is equivalence relation? Explain use of equivalence relation with the help of an example.

An equivalence relation is a binary relation that is reflexive, symmetric, and transitive. In other words, it is a relation on a set that satisfies the following three properties:

1.Reflexivity: for any element a in the set, a is related to itself.

2.Symmetry: if a is related to b, then b is related to a.

3.Transitivity: if a is related to b and b is related to c, then a is related to c.

Equivalence relations are used to partition a set into subsets of elements that are related to each other. The equivalence classes are formed by grouping together all elements that are related to each other by the equivalence relation. Equivalence relations are commonly used in various fields of mathematics, including algebra, topology, and number theory.

For example, consider the set of integers Z and the equivalence relation "a is congruent to b modulo 3", denoted by $a \equiv b \pmod 3$. This relation is reflexive, symmetric, and transitive. It partitions the set of integers into three equivalence classes:

•[0] = {..., -6, -3, 0, 3, 6, ...}

•[1] = {..., -5, -2, 1, 4, 7, ...}

•[2] = {..., -4, -1, 2, 5, 8, ...}

Here, [0] represents the set of all integers that leave a remainder of 0 when divided by 3, [1] represents the set of all integers that leave a remainder of 1 when divided by 3, and [2] represents the set of all integers that leave a remainder of 2 when divided by 3.

Equivalence relations are useful because they allow us to study the properties of a set by examining the properties of its equivalence classes. In the example above, we can use the equivalence relation to prove that the sum of two integers that leave a remainder of 1 when divided by 3 will always leave a remainder of 2 when divided by 3, by considering the elements in the equivalence classes [1] and [2].

Q3: Attempt the following, all questions are compulsory, and each question carries 2 marks for each.

a)Suppose we want to choose two persons from a party consisting of 35 members as Manager and Assistant Manager. In how many ways can this be done?

Ans: The number of ways to choose two persons from a group of 35 members as Manager and Assistant Manager is a combination problem. We want to choose two people from 35, so we have 35 choose 2 possible pairs:

35 choose 2 = (35 * 34) / (2 * 1) = 595

Therefore, there are 595 ways to choose two persons from a party consisting of 35 members as Manager and Assistant Manager.

b) There are three Companies, C1, C2 and C3. The party C1 has 4 members, C2 has 5 members and C3 has 6 members in an assembly. Suppose we want to select two persons, both from the same Company, to become president and vice president. In how many ways can this be done?

Ans:-

We need to find the number of ways to select a president and a vice president from the members of each company separately and then add them up.

For company C1, we can choose a president in 4 ways and then choose a vice president in 3 ways (since one person has already been chosen for the president). Therefore, there are 4 * 3 = 12 ways to choose a president and vice president from company C1.

Similarly, for company C2, we can choose a president in 5 ways and then choose a vice president in 4 ways, giving us 5 * 4 = 20 ways to choose a president and vice president from company C2.

And for company C3, we can choose a president in 6 ways and then choose a vice president in 5 ways, giving us 6 * 5 = 30 ways to choose a president and vice president from company C3.

Thus, the total number of ways to choose a president and vice president from any of the three companies is:

12 20 + 30 = 62 ways.

c) Suppose there are five married couples and they (10 people) are made to sit about a round table so that neither two men nor two women sit together. Find the number of such circular arrangements.

Ans;-

Let's first arrange the five couples in a circle. We can do this in 4! ways (since we can rotate the circle to get the same arrangement).

Now we have to seat the 10 people in alternate positions around the circle, such that no two men or two women sit together. We can start by choosing any one of the 5 women to be seated in any one of the 5 seats. Then we can seat her husband in one of the two seats adjacent to her (since no two men can sit together). We can then seat the next woman in one of the remaining 4 seats, and her husband in one of the two seats adjacent to her. Continuing this process, we can seat all 10 people in a valid arrangement.

For the first woman, we have 5 choices of seat. For each subsequent woman, we have 4 choices of seat. And for each of the men, we have 2 choices of seat (since they can only sit in the two seats adjacent to their respective wives). Therefore, the total number of valid circular arrangements is:

5 × 2^5 × 4^4 = 40960

Hence, there are 40,960 such circular arrangements

d)How many words can be formed using letter of DEPARTMENT using each letter at most once? i) If each letter must be used, ii) If some or all the letters may be omitted.

Ans:-

i) If each letter must be used, we can arrange the letters of the word DEPARTMENT in a specific order. The first letter can be any of the 10 letters, the second letter can be any of the remaining 9 letters, the third letter can be any of the remaining 8 letters, and so on. The total number of words that can be formed is:

$10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 3{,}628{,}800$

ii) If some or all the letters may be omitted, we can choose any subset of the letters, and then arrange them in any order. We can choose from 0 to 10 letters, giving us a total of 2^10 = 1024 subsets. For each subset, we can arrange the letters in any order. The number of words that can be formed is:

$(2^{10} - 1) \times (9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 1 = 9{,}437{,}425$

The first term in the formula above counts the number of words that can be formed using a non-empty subset of the letters, and the second term counts the word with no letters. The formula subtracts one from the total number of subsets to exclude the empty subset.

e)What is the probability that a 13-card hand has at least one card in each suit?

Ans To calculate the probability that a 13-card hand has at least one card in each suit, we can use the principle of inclusion-exclusion.

The total number of 13-card hands that can be dealt from a standard deck of 52 cards is given by:

$C(52, 13) = 22{,}957{,}480$

To count the number of hands that do not contain cards from a particular suit, we first choose 13 cards from the 39 cards that are not in that suit, which can be done in $C(39, 13)$ ways. We can do this for each of the 4 suits, so there are $4C(39, 13)$ hands that do not contain cards from a particular suit.

However, this counts twice the hands that do not contain cards from two particular suits, and three times the hands that do not contain cards from three particular suits. Therefore, we need to subtract these counts to avoid double counting.

The number of hands that do not contain cards from two particular suits can be counted by choosing 13 cards from the 26 cards that are not in those two suits, which can be done in $C(26, 13)$ ways. There are $C(4, 2) = 6$ ways to choose two suits from the four, so there are $6C(26, 13)$ hands that do not contain cards from two particular suits.

The number of hands that do not contain cards from three particular suits can be counted by choosing 13 cards from the 13 cards in the one suit that is not included, which can be done in C(13, 13) = 1 way. There are C(4, 3) = 4 ways to choose three suits from the four, so there are 4 hands that do not contain cards from three particular suits.

Finally, we need to add back the number of hands that do not contain cards from all four suits, which is simply C(13, 13) = 1.

Therefore, the number of 13-card hands that have at least one card in each suit is:

C(52, 13) - 4C(39, 13) + 6C(26, 13) - 4 + 1 = 6,042,312

The probability of drawing such a hand is then:

P = 6,042,312 / 22,957,480 ≈ 0.2634

So the probability that a 13-card hand has at least one card in each suit is approximately 0.2634 or 26.34%.

f) What is the probability that a number between 1 and 10,000 is divisible by neither 2, 3, 5 nor 7?

Ans

We can approach this problem by first finding the total number of integers between 1 and 10,000, inclusive, and then subtracting the number of integers that are divisible by at least one of 2, 3, 5, or 7.

The total number of integers between 1 and 10,000 is 10,000.

To count the number of integers that are divisible by 2, we divide 10,000 by 2 and round down to get 5,000. Similarly, the number of integers divisible by 3, 5, and 7 are 3,333, 2,000, and 1,428, respectively.

To count the number of integers that are divisible by two of these numbers, we can use the inclusion-exclusion principle. The number of integers divisible by 2 and 3 is 1,666, the number divisible by 2 and 5 is 1,000, the number divisible by 2 and 7 is 714, the number divisible by 3 and 5 is 666, the number divisible by 3 and 7 is 476, the number divisible by 5 and 7 is 285, and the number divisible by 2, 3, and 5 is 333. We do not need to consider numbers that are divisible by 2, 3, and 7, or 2, 5, and 7, or 3, 5, and 7, as these cases are already included in our previous counts.

To count the number of integers that are divisible by three of these numbers, we only need to consider the case where a number is divisible by 2, 3, and 5, which gives us 166.

Finally, we need to count the number of integers that are divisible by all four of these numbers, which is simply 47.

Using the inclusion-exclusion principle, we can compute the total number of integers that are divisible by at least one of 2, 3, 5, or 7:

5000 + 3333 + 2000 + 1428 - 1666 - 1000 - 714 - 666 - 476 - 285 + 333 + 166 - 47 = 6240

Therefore, the number of integers between 1 and 10,000 that are not divisible by any of 2, 3, 5, or 7 is:

10,000 - 6,240 = 3,760

The probability that a number between 1 and 10,000 is not divisible by any of 2, 3, 5, or 7 is then:

P = 3,760 / 10,000 = 0.376

So the probability that a number between 1 and 10,000 is not divisible by any of 2, 3, 5, or 7 is 0.376 or 37.6%.

g)Explain inclusion-exclusion principle and Pigeon Hole Principle with example.

Ans:

Inclusion-Exclusion Principle:

The Inclusion-Exclusion Principle is a counting technique used in combinatorics to determine the size of a set that is a union of multiple other sets. It states that:

$|A1 \cup A2 \cup ... \cup An| = |A1| + |A2| + ... + |An| - |A1 \cap A2| - |A1 \cap A3| - ... - |An-1 \cap An| + ... + (-1)^{n-1}|A1 \cap A2 \cap ... \cap An|$

This formula can be explained as follows: The left-hand side represents the size of the set that is formed by combining all the sets A1, A2, ..., An. To find the size of this set, we first add up the sizes of each individual set A1, A2, ..., An. But, we have counted the elements that are in the intersection of two or more sets multiple times, so we need to subtract the sizes of these intersections from the sum. However, we have now subtracted some elements too many times, so we need to add back in the sizes of the intersections of three or more sets, and so on, until we reach the last term, which alternates between adding and subtracting the size of the intersection of all the sets.

Example:

Suppose we have three sets A, B, and C, where |A| = 5, |B| = 6, |C| = 7, |A ∩ B| = 2, |B ∩ C| = 3, and |A ∩ C| = 4. We want to find |A ∪ B ∪ C|.

Using the Inclusion-Exclusion Principle:

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

$= 5 + 6 + 7 - 2 - 3 - 4 + 0$

$= 9$

Therefore, the size of the set A ∪ B ∪ C is 9.

Pigeonhole Principle:

The Pigeonhole Principle is a counting technique that states that if there are more pigeons than pigeonholes, then at least one pigeonhole must contain more than one pigeon. It can also be stated as follows: If n + 1 objects are placed into n containers, then at least one container must contain two or more objects.

Example:

Suppose we have 7 balls of different colors (red, orange, yellow, green, blue, indigo, and violet), and we want to choose 6 of them. We want to prove that there must be at least two balls of the same color in the group of 6.

Using the Pigeonhole Principle:

There are 7 colors of balls, so there are 7 pigeonholes. We want to choose 6 balls, which are the pigeons. Since there are more pigeons than pigeonholes (6 > 7), there must be at least one pigeonhole that contains more than one pigeon. Therefore, there must be at least two balls of the same color in the group of 6.

h) In a tennis tournament, each entrant plays a match in the first round. Next, all winners from the first round play a second-round match. Winners continue to move on to the next round, until finally only one player is left as the tournament winner. Assuming that tournaments always involve n = 2k players, for some k, find the recurrence relation for the number rounds in a tournaments of n players.

Ans:-

Let R(n) be the number of rounds required for a tournament of n players.

First, consider the base case when n = 2. In this case, only one match is played, and the winner is the tournament winner. Therefore, R(2) = 1.

Now, suppose n > 2. In the first round, there are n/2 matches, and n/2 winners. These n/2 winners will then play in the second round, requiring R(n/2) rounds. Since there are only n/2 players in the second round, the total number of rounds for the tournament of n players is R(n/2) + 1.

Therefore, we can write the recurrence relation:

R(n) = R(n/2) + 1

with the base case R(2) = 1.

We can also use the substitution method to find a closed-form solution for R(n):

R(n) = R(n/2) + 1

= R(n/4) + 2

= R(n/8) + 3

= ...

= R(2^k) + k

Since n = 2^k, we have k = log_2(n), so the closed-form solution is:

R(n) = log_2(n) + 1

Therefore, the number of rounds required for a tournament of n players is log_2(n) + 1.

i)Find an explicit recurrence relation for minimum number of moves in which the n-disks in tower of Hanoi puzzle can be solved! Also solve the obtained recurrence relation through an iterative method.

Ans

The Tower of Hanoi puzzle consists of three pegs and n disks of different sizes. The disks are initially stacked on one peg in order of decreasing size, with the largest disk at the bottom. The objective is to transfer the entire stack to another peg, following these rules:

1.Only one disk can be moved at a time.

2.Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack or on an empty peg.

3.No disk may be placed on top of a smaller disk.

Let H(n) be the minimum number of moves required to solve the Tower of Hanoi puzzle with n disks.

For n = 1, it takes only one move to solve the puzzle. Therefore, H(1) = 1.

Now, suppose we have n disks. To move the largest disk to the destination peg, we must first move the n-1 smaller disks to the spare peg. This requires H(n-1) moves. Once we have moved the smaller disks, we can move the largest disk to the destination peg with one move. Finally, we need to move the n-1 smaller disks from the spare peg to the destination peg, which requires H(n-1) moves. Therefore, the total number of moves required is:

$H(n) = 2H(n-1) + 1$

with the base case $H(1) = 1$.

We can solve this recurrence relation iteratively by starting with the base case and using the formula to find the solution for larger values of n.

$H(1) = 1$ $H(2) = 2H(1) + 1 = 3$ $H(3) = 2H(2) + 1 = 7$ $H(4) = 2H(3) + 1 = 15$ $H(5) = 2H(4) + 1 = 31$

and so on.

Therefore, the minimum number of moves required to solve the Tower of Hanoi puzzle with n disks is given by the formula:

$H(n) = 2^n - 1$

This can be proved by mathematical induction using the recurrence relation.

j) Find the solution of the recurrences relation an = an-1 + 2an-1, n > 2 with a0 = 0, a1=1

Ans

Let us find the solution of recurrence relation

$a_n = a_{n-1} + 2a_{n-1}$

$a_0 = 1$

$a_1 = 1$

$a_2 = a_1 + 2a_1$

$= 1 + 2$

$= 3$

$a_3 = a_2 + 2a_2$

$= 3 + 6$

$= 9$

$a_4 = a_3 + 2a_3$

$= 9 + 18 = 27$

$a_5 = a_4 + 2a_4 = 27 + 54 = 81$

we notice that each term is 3 times the previous term. So we can express the nth term as

Step2 – Expressing general term

$a_n = 3^{(n-1)}$

Step 3 Checking the base cases

We can check the cases to make sure formula works

$a_0 = 3^\wedge(-1) = 0$

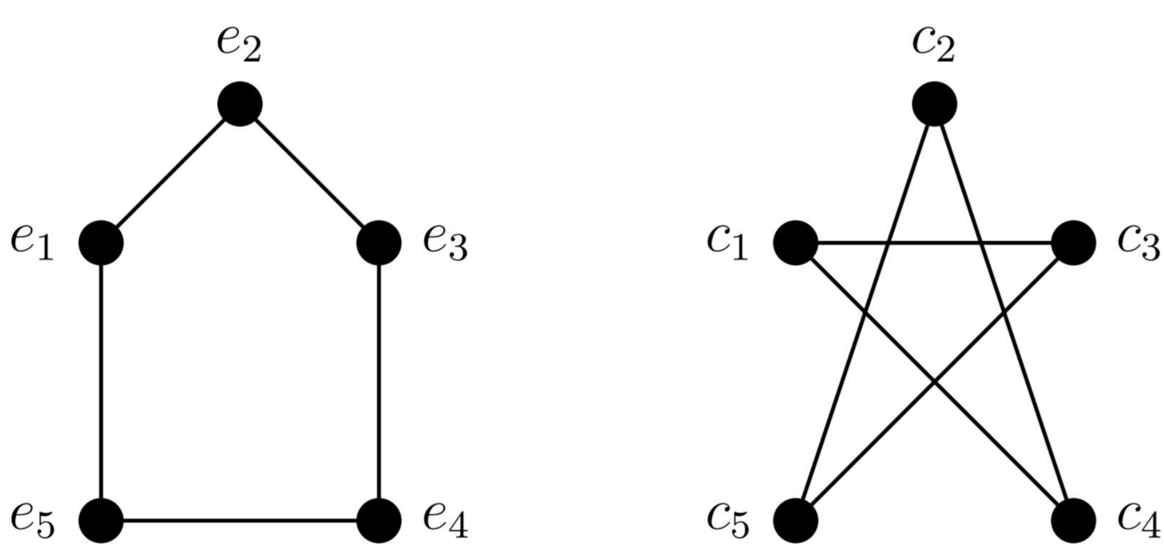
$a_1 = 3^{n-1} = 3^0 = 1$

So solution to recurrence relation is

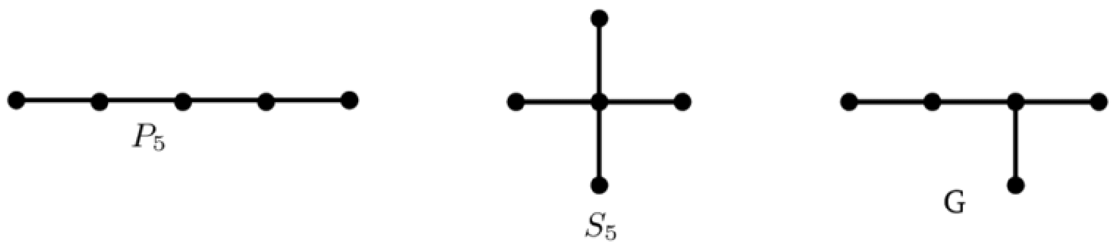
$a_n = 3^{(n-1)}$

Q4: Attempt the following, all questions are compulsory, and each question carries 2 marks for each

a)Draw 2-isomorphic graphs and 3 non- isomorphic graphs on five vertices.



isomorphic graph



Non -isomorphic graph

b) Prove that the complement of G Bar is G

Ans

Let G be a simple undirected graph with vertex set V(G) and edge set E(G), and let G bar be the complement of G, with vertex set V(G) and edge set E(G bar) consisting of all edges not in E(G).

To prove that the complement of G bar is G, we need to show that:

1.Every edge in G is not in G bar, and

2.Every edge not in G is in G bar.

Proof:

1.Suppose that (u, v) is an edge in G. Then (u, v) is not in E(G bar) because G bar contains all edges not in G. Therefore, (u, v) is not in the complement of G bar, which is G.

2.Suppose that (u, v) is not an edge in G. Then (u, v) is in E(G bar) because G bar contains all edges not in G. Therefore, (u, v) is in the complement of G bar, which is G.

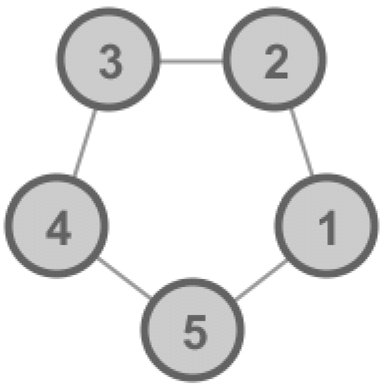
Thus, we have shown that every edge in G is not in G bar, and every edge not in G is in G bar. Therefore, the complement of G bar is G.

c) Draw the following graphs and state which of following graph is a regular graph?

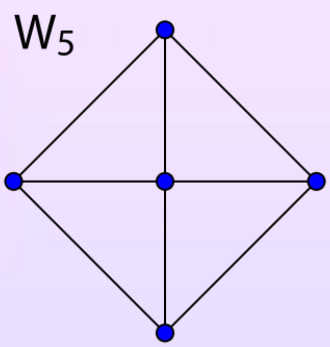
(i)C5 (ii) W5 (iii) Q4 (iv) K5,5

Here are the drawings of the graphs:

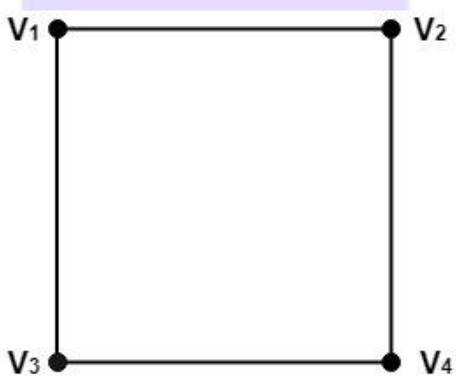
(i) C5:



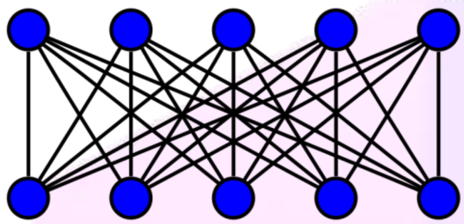
(ii) W5:



(iii) Q4:



(iv) K5,5



A graph is said to be regular if all of its vertices have the same degree.

(i) C5 is a regular graph because each vertex has degree 2. (ii) W5 is not a regular graph because all vertices don't have same degree (iii) Q4 is a regular graph because each vertex has degree 2. (iv) K5,5 is not a regular graph because the vertices on one side have degree 5 while the vertices on the other side have degree 4.

d) What is a chromatic number of a graph? What is a chromatic number of the following graph?

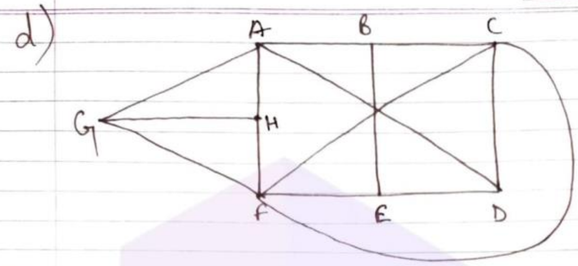
Ans.

The chromatic number of a graph is the smallest number of colors needed to color the vertices of the graph in such a way that no adjacent vertices have the same color. In other words, it is the minimum number of colors required to color all the vertices of the graph in such a way that no two adjacent vertices have the same color.

Formally, let G be a graph. A coloring of the vertices of G is an assignment of a color to each vertex such that no two adjacent vertices have the same color. The chromatic number of G, denoted by $\chi(G)$, is the smallest number of colors required for a proper coloring of G.

The concept of chromatic number is important in graph theory, as it provides a measure of the "colorability" of a graph. The chromatic number of a graph is related to many other properties of the graph, such as its planarity, its maximum degree, and its clique number. The chromatic number is also used in many practical applications, such as scheduling problems and register allocation in compilers.

d)



vertex	A	B	C	D	E	F	G	H
colour	c_1	c_2	c_1	c_2	c_1	c_2	c_3	c_4

Applying Greedy Algorithm, we have —

From here,

Minimum no. of colours used to color the given graph are 4.

Therefore, Chromatic Number of given graph is 4.

e) Determine whether the above graph has a Hamiltonian circuit. If it has, find such a circuit. If it does not have, justify it

Ans.

In graph theory, a Hamiltonian circuit is a path that starts from a vertex in a graph and visits every other vertex exactly once before returning to the starting vertex. A graph that has a Hamiltonian circuit is called a Hamiltonian graph.

Above Graph is Hamiltonian Circuit because starting from vertex G we can visit every other vertex exactly once before returning to stating vertex.

Hamiltonian Circuit - GABCDEFHG

f) Explain and prove the Handshaking Theorem, with suitable example.

The Handshaking Theorem, also known as the degree sum formula, is a fundamental theorem in graph theory that states that the sum of the degrees of all the vertices in a graph is equal to twice the number of edges. In other words, if G is a simple undirected graph with n vertices and m edges, then

∑deg(v) = 2m,

where the summation is taken over all vertices v in G.

Proof: Let G be a simple undirected graph with n vertices and m edges. We can count the number of edges in two ways: we can count the edges by summing the degrees of all vertices (each edge contributes 2 to the sum, one for each endpoint), or we can count the edges directly as m. Thus, we have:

∑deg(v) = 2m

To understand the theorem, let us consider a simple example of a graph:

Example: Consider the following graph G with 4 vertices and 5 edges:

1 -- 2 | | 3 -- 4

We can calculate the degree of each vertex as follows:

deg(1) = 2, deg(2) = 2, deg(3) = 2, deg(4) = 2

Therefore, by the Handshaking Theorem, we have:

∑deg(v) = deg(1) + deg(2) + deg(3) + deg(4) = 2 + 2 + 2 + 2 = 8

On the other hand, we can count the edges directly and see that there are 5 edges in the graph. Since each edge contributes 2 to the sum of degrees, we have:

2m = 2*5 = 10

Thus, the Handshaking Theorem holds for this example.

The Handshaking Theorem is an important result in graph theory that has many applications, including in the study of planar graphs, Eulerian circuits, and the chromatic number of a graph

g) Show that C6 is bipartite and K3 is not bipartite.

To show that C6 (cycle of length 6) is bipartite, we can divide its vertices into two sets, say A and B, as follows:

A = {1, 3, 5} B = {2, 4, 6}

Here, all the vertices in set A are connected only to vertices in set B, and vice versa. Thus, C6 is bipartite.

On the other hand, to show that K3 (complete graph on 3 vertices) is not bipartite, we can assume for contradiction that K3 is bipartite. Let A and B be the two disjoint sets of vertices that form a bipartition of K3. Without loss of generality, assume that one vertex in K3 belongs to set A (say vertex 1). Then the other two vertices (2 and 3) must belong to set B. But since K3 is a complete graph, there is an edge between vertices 2 and 3, which means that they belong to the same set (B). This contradicts our assumption that K3 is bipartite, and hence K3 is not bipartite.

Therefore, we have shown that C6 is bipartite and K3 is not bipartite.

h) Explain the terms PATH, CIRCUIT and CYCLES in context of Graphs.

In graph theory, a path, circuit, and cycle are all types of sequences of vertices and edges in a graph. Here's an explanation of each term:

1.Path: A path in a graph is a sequence of vertices and edges where each vertex in the sequence is adjacent to the next vertex in the sequence. In other words, a path is a sequence of connected edges that connect two or more vertices in the graph. The length of a path is the number of edges in the path.

2.Circuit: A circuit is a path that starts and ends at the same vertex in the graph. In other words, a circuit is a closed path in a graph. The length of a circuit is the number of edges in the circuit.

3.Cycle: A cycle is a circuit in a graph where no vertex is visited more than once, except for the first and last vertex, which are the same. In other words, a cycle is a closed path in a graph where only the first and last vertices are repeated. The length of a cycle is the number of edges in the cycle.

It's important to note that all cycles are circuits, but not all circuits are cycles. Also, a path can be a cycle if it connects the same vertex at the beginning and end, but this is usually not considered a cycle with a non-zero length.

i)What is the difference between an Eulerian graph and an Eulerian circuit?

An Eulerian graph is a graph that has an Eulerian circuit, which is a path that traverses all edges of the graph exactly once and ends at the starting vertex. In other words, an Eulerian graph is a graph that has a closed path that includes all edges of the graph.

A graph is said to be Eulerian if and only if every vertex in the graph has an even degree, meaning that the number of edges incident to the vertex is even. A graph that is not Eulerian is called non-Eulerian.

An Eulerian circuit is a path that starts and ends at the same vertex and traverses all edges of the graph exactly once. This means that an Eulerian circuit is a special case of a cycle, where all vertices have even degree.

In summary, the difference between an Eulerian graph and an Eulerian circuit is that an Eulerian graph is a graph that has an Eulerian circuit, while an Eulerian circuit is a path that traverses all edges of the graph exactly once and ends at the starting vertex.

j) Explain the Dirac's Criterion and Ore's Criterion for Hamiltonian graphs.

Dirac's Criterion and Ore's Criterion are two necessary conditions for a graph to be Hamiltonian, meaning that it contains a Hamiltonian cycle, which is a cycle that visits every vertex of the graph exactly once.

Dirac's Criterion states that a simple graph with n vertices (n >= 3) is Hamiltonian if every vertex in the graph has degree n/2 or greater. In other words, if the degree of each vertex in the graph is at least half the total number of vertices, then the graph is Hamiltonian.

Ore's Criterion states that a simple graph with n vertices (n >= 3) is Hamiltonian if for every pair of non-adjacent vertices, their sum of degrees is n or greater. In other words, if the sum of degrees of any two non-adjacent vertices in the graph is n or greater, then the graph is Hamiltonian.

Both Dirac's and Ore's Criteria are necessary conditions for a graph to be Hamiltonian, but neither of them is sufficient. This means that a graph that satisfies either Dirac's or Ore's Criterion may or may not be Hamiltonian. Therefore, these criteria can be used to eliminate certain graphs as non-Hamiltonian, but they cannot guarantee that a graph is Hamiltonian.