Inf 674: Erdös-Rényi Graphs

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Reminder: an Erdös-Rényi graph $G(n,p)$ is a random graph with n nodes where each possible edge exists with probability p (i.i.d.). We focus here on undirected, loop-free graphs.

Question 1

We first need to produce Erdös-Rényi graphs $G(n,p)$.

- Propose a function that computes an ER graph. The graph can be returned for instance as a list of adjacency lists.
- Propose a function that takes as input a symmetric graph and return the sizes of its connected components.
- For a fixed n (say between 1000 and 10000), compute and display the average size of the largest component as a function of p. Choose n, the range of p (display critical values) and the number of trials wisely according to your machine capabilities. Discuss the results.

Question 2

We consider a Reed-Frost-like epidemic propagation with infection probability i that occurs on a $G(n,p)$ graph (contamination can only occur on graph edges). Give/Remind the critical condition. Verify your result with some simulations (you may adapt functions from previous question).

Question 3

We now consider heterogenous $G(n_1, p_1, n_2, p_2, p)$ graphs as follows:

- The graph has n_1 nodes of type 1 and n_2 nodes of type 2.
- Two distinct nodes of type 1 are connected with probability p_1.
- Two distinct nodes of type 2 are connected with probability p_1.
- A type 1 node and a type 2 node are connected with probability p.

The questions to address:

- Propose a function that computes a $G(n_1, p_1, n_2, p_2, p)$ graph. The graph can be returned for instance as a list of adjacency lists.

- In the case where $p_1 = p_2 = 0$, following the Galton-Watson approach, try to guess where the critical regime occurs.
- Verify your guess with simulations.
- Bonus: generalize for arbitrary p_1, p_2.