Inf 674: Epidemic Competition

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Until now, we have dealt with contamination with only one type of "virus". The only bottleneck was the ability of a node to contaminate another one. For this session, we propose a selection of competition models, where multiple "viruses" compete for a same resource.

Question 1

We begin with the Mitochondrial Eve model seen during the first session. We remind the principle: there is a first generation of n individuals (the potential eves). At each next generation, there are n individuals as well. The parent of each individual is chosen uniformly i.i.d. among the individuals of the previous generation. The first generation ancestor of any individual is its Eve. We propose to study the age of Eve, i.e. the number of generation required for all individuals to share the same Eve.

- Propose a function that returns for a given n the age of Eve computed on one instance.
- Compute and display the age of Eve as a function of n averaged over a few trials (recommended values: 100:100:2000, 10 trials).
- Mitochondrial Eve is estimated to have lived between 99,000 and 200,000 years ago. Population size at that time was about 500,000. Comment the Mitochondrial Eve model we exposed in this session.

Question 2

We now propose a more general approach: the Voter model. We consider an undirected, connected graph $G = (V, E)$ with self-loops on all nodes. At time $t = 0$, each node has a given opinion (distinct nodes can share the same opinion, so there are between 1 and $n = |V|$ distinct opinions). At each time $t > 1$, each node picks up a neighbor at random and adopts its opinion at time $t - 1$. We want to study how the opinions evolve with time.

- Tell how the Voter model generalizes the Mitochondrial Eve model. Can you guess something about the evolution of opinions over time?
- Let us do some math. Let P_t^a a vector of size n that tells for each node the probability that it has opinion a at time t. P_0^a is 1 over all nodes that have initially opinion a, 0 elsewhere.

  - Give an exact expression for P_t^a. Does it remind something?
  - Tell what happens for $t \to \infty$.
  - Optional if time: verify your results with simulations (for instance with some Erdös=Rényi graph and 2 opinions).

- You are a high manager from the company Lodi (Lodi is a sort of Apple). You have the budget to select 1000 customers and offer them the brand new Lodi anvil, hoping that this helps you dominate the strategic market of smart anvils against your competitors. You bought from the social network company FriendFace their database. According to the Voter model, what should you do?

Question 3

We propose now a completely different framework: P2P Live streaming. There are n peers that want to watch a Live event. The Live stream is produced by a source that cuts it into chunks of 1 second and can deliver one chunk per second. Each peer has the capacity to deliver 1 chunk per second as well.

- Is it possible for all peers to receive all chunks of the stream in a reasonable time? If no, say why. If yes, propose a scheme to do that and give a bound to the diffusion delay (time between the introduction of a chunk by the source and the completion of its broadcast).
- In order to limit the complexity of the diffusion, we consider the use of a fully randomize distribution scheme. Each second: the source delivers the new chunk to a random uniform peer; each peer chooses the chunk it possesses (if any) with the most recent timestamp and delivers it to a random uniform peer. This strategy is called *random peer, latest blind*.
  - Using a simulation over T seconds (recommendation: $n = T = 1000$, a single run is enough): display for each chunk its percentage of diffusion; display the average percentage of diffusion of a chunk as a function of its age. Comment the results.
  - Propose a value for the asymptotic eventual percentage of diffusion when n is large enough. What becomes that value if all peers can deliver k chunks per second?
  - Discuss the potential interest of such a distribution scheme.