CLASSMATE

Date

Page

Name- Juhi Dugar Roll no- 2301010053

Linear Algebra

Q1. Find the rank of the matrix A by reducing in Row reduced echelon form.

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

Applying elementary row operations toget the matrix into row echelon form and then reduce it to Row Reduced Echelon form

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - 6R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -5 & 5 \end{bmatrix}$$

$R_3 \to R_3 - 4/3 R_2$

$R_4 \to R_4 - 4/3 R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & 0 & -1 \\ 0 & -4 & 0 & 7/3 \end{bmatrix}$$

Date
Page

Name - Juhi Dugar

Roll no: 2301010053

$R_4 \to R_4 - R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & 0 & -1 \\ 0 & 0 & 0 & 13/3 \end{bmatrix}$$

This is row echelon form

Now, converting into Row Reduced Echelon Form.

$R_3 \to R_3 + 4/3 R_2$
$R_1 \to R_1 - 2R_2$

$$\begin{bmatrix} 1 & 2 & 0 & -4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & 0 & -1 \\ 0 & 0 & 0 & 13/3 \end{bmatrix}$$

Dividing third row by -4
$R_3 \to -1/4 R_3$

$$\begin{bmatrix} 1 & 2 & 0 & -4 \\ 0 & 0 & -3 & 2 \\ 0 & 1 & 0 & 1/4 \\ 0 & 0 & 0 & 13/3 \end{bmatrix}$$

This is Row Reduced Echelon Form

Rank of the matrix is 3.

Name - Juhi Dugar Roll no: 2301010053

Q2. Let W be the vector space of all symmetric 2×2 matrices and let $T: W \to P_2$ be the linear transformation defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b)+(b-c)x+(c-a)x^2$. Find the rank and nullity of T.

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

By rank-nullity theorem,
rank(T) + nullity(T) = dim(W)

Finding the images of the basic vectors of W under T.

1) For the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$:

$$T\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = (1-0)+(0-0)x+(0-1)x^2 = 1-x^2$$

2) For the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$:

$$T\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = (0-1)+(1-1)x+(1-0)x^2 = -1+x$$

Date
Page

Name - Juhi Dugar   Roll no : 2301010053

3) for the matrix $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$:

$$T\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = (0-0) + (0-0)x + (0-0)x^2 = 0$$

The images of the basis vectors $\{1-x^2, -1+x, 0\}$ are linearly independent.
They span the image of T. So, the rank of T is 3.

Using rank-nullity theorem,

$$\text{rank}(T) + \text{nullity}(T) = \dim(W)$$
$$3 + \text{nullity}(T) = 3$$
$$\text{nullity}(T) = 0$$

rank of T is 3 and nullity of T is 0.

Q3. Let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. Find the eigen values of eigen vectors of A^{-1} and $A+4I$.

1) Eigen values and Eigen vectors of A.

~~def~~ $\det(A-\lambda I) = 0$

$$\det(A-\lambda I) = \begin{vmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix}$$

$$= (2-\lambda)^2 - (-1)(-1)$$
$$= \lambda^2 - 4\lambda + 4 - 1$$

Name-Juhi Dugar Roll no: 2301010053

$= \lambda^2 - 4\lambda + 3$

$= (\lambda-1)(\lambda-3) = 0$

So $\lambda_1 = 1$ and $\lambda_2 = 3$

$(A - \lambda I) r = 0$

Solve r we get,

for $\lambda_1 = 1$

$$(A - \lambda_1 I) r_1 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} r_1 = 0$$

we get $r_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

for $\lambda_2 = 3$

$$(A - \lambda_2 I) r_2 = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} r_2 = 0$$

we get

$$r_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Date
Page

Roll no: 2301010053

Name - Juhi Dugar

2) Eigen values and Eigen vectors of A^{-1}

If λ is eigen value of A, then λ^{-1} is for A^{-1}.

Same for Eigen vectors.

eigen values of A^{-1} and λ_1^{-1} and λ_2^{-1}

eigen values of A^{-1}: $\frac{1}{\lambda_1}, \frac{1}{\lambda_2} = \frac{1}{1}, \frac{1}{3} = 1, \frac{1}{3}$

Eigen vectors remains the same.

3) Eigenvalues and Eigen vectors of $A+4I$:

If μ is eigen values of A, then $\mu + 4$ is an eigen value of $A+4I$, with same eigen vectors.

So, eigenvalues of $A+4I$ are $\mu_1 + 4$ and $\mu_2 + 4$

Eigen values of $A+4I$: $\lambda_1 + 4, \lambda_2 + 4 = 1 + 4 . 3 + 4$

$= 5 . 7$

Eigen vectors remains the same.

For A^{-1}

Eigen value $\lambda_1 = 1$, Eigen vector $V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Eigen value $\lambda_2 = 3$, Eigen vector $V_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Name - Juhi Dugar Rollno: 2301010053

for A+4I

Eigen value $\lambda_1 + 4 = 5$, Eigen vector $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Eigen value $\lambda_2 + 4 = 7$, Eigen vector $v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Q4. Solve by Gauss-Seidel Method (Take three iterations)

$$3x - 0.1y - 0.2z = 7.85$$
$$0.1x + 7y - 0.3z = -19.3$$
$$0.3x - 0.2y + 10z = 71.4$$

with initial values $x(0) = 0$, $y(0) = 0$, $z(0) = 0$.

~~8~~ Equations are:

$$3x - 0.1y - 0.2z = 7.85$$
$$0.1x + 7y - 0.3z = -19.3$$
$$0.3x - 0.2y + 10z = 71.4$$

can be written as:

1) $x = \dfrac{(7.85 + 0.1y + 0.2z)}{3}$

2) $y = \dfrac{(-19.3 - 0.1x + 0.3z)}{7}$

3) $z = \dfrac{(71.4 - 0.3x + 0.2y)}{10}$

Date
Page

Name: Juhi Dugar   Roll no: 2301010053

Iteration 1:

Using initial values $x(0)=0, y(0)=0, z(0)=0$

1) $x(1) = \dfrac{7.85+0.1(0)+0.2(0)}{3} = 2.61667$

2) $y(1) = \dfrac{-19.3-0.1(2.61667)+0.3(0)}{7} = -2.77295$

3) $z(1) = \dfrac{71.4-0.3(2.61667)+0.2(-2.77295)}{10} = 7.18943$

Iteration 2:

Using $x(1)=2.61667, y(1)=-2.77295, z(1)=7.18943$

1) $x(2) = \dfrac{7.85+0.1(-2.77295)+0.2(7.18943)}{3} = 3.00056$

2) $y(2) = \dfrac{-19.3-0.1(3.00056)+0.3(7.18943)}{7} = -2.99984$

3) $z(2) = \dfrac{71.4-0.3(3.00056)+0.2(-2.99984)}{10} = 7.00004$

classmate
Date
Page

Name : Juhi Dugar Rollno : 2301010053

Iteration 3:

using $x(2) = 3.00056$, $y(2) = -2.99984$, $z(2) = 7.00004$

1) $x(3) = \dfrac{7.85 + 0.1(-2.99984) + 0.2(7.00004)}{3} = 3.00002$

2) $y(3) = \dfrac{-19.3 - 0.1(3.00002) + 0.3(7.00004)}{7} = -3$

3) $z(3) = \dfrac{71.4 - 0.3(3.00002) + 0.2(-3)}{10} = 7$

After 3 iterations:

$x = 3$, $y = -3$, $z = 7$

Q5. Define consistent and inconsistent system of equations. Hence solve the following system of equations if consistent
$x + 3y + 2z = 0$, $2x - y + 3z = 0$, $3x - 5y + 4z = 0$, $x + 17y + 4z = 0$.

Consistent system of equations: A system of linear equations is considered consistent if it has at least one solution, meaning there exists a set of values for the variables that satisfies all equations simultaneously. In other words, the system either has a unique solution, infinitely many solutions, or a single

classmate
Date
Page

Name: Juhi Dugar   Roll no: 2301010053

consistent solution.

<u>Inconsistent System of Equations:</u> A system of linear equations is considered inconsistent if it has no solution, meaning there are no values for the variables that satisfy all equations simultaneously. Geometrically, this situation corresponds to parallel lines.

Writing equations in matrix form:

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} : \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Using row ~~reduction~~ operations to get matrix into row-echelon form

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2R_1$
$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{array}\right]$$

classmate
Date
Page

Name : Juhi Dugar   Roll no : 2301010053

$R_3 \rightarrow R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 1 & 17 & 4 & 0 \end{array}\right]$$

$R_4 \rightarrow R_4 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{array}\right]$$

$R_3 \rightarrow R_3 - 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{array}\right]$$

$R_4 \rightarrow R_4 + \cancel{R_2}\, 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Now, the matrix is in row-echelon form.

Date
Page

Name: Juhi Dugar Roll no: 2801010053

We can see that the last row is 0 = 0. This means that it is dependent equation. So, there are infinitely many solutions.

We can write the equations in parametric form. Let's denote the free variable as t.

Let

$z = t$

from second-row:

$-7y - z = 0$

$-7y - t = 0$

$y = \frac{-t}{7}$

from first row:

$x + 3y + 2z = 0$

$x + 3\left(\frac{-t}{7}\right) + 2t = 0$

$x = \frac{3t}{7} - 2t$

So, the solution in parametric form is:

$x = \frac{3t}{7} - 2t$

$y = \frac{-t}{7}$ and $z = t$

Name: Juhi Dugar Roll no: 2301010053

where t can take any real value.

Q6. Determine whether the function $T: P_2 \rightarrow P_2$ is a linear transformation, where $T(a+bx+cx^2) = (a+1)+(b+1)x + (c+1)x^2$

We need to check if it satisfies two properties:

1) Additivity: $T(u+v) = T(u) + T(v)$ for all u, v in P_2.
2) Homogeneity: $T(kv) = kT(v)$ for all v in P_2 and all scalars k

On evaluating these properties:

1) Additivity:

Let $u = a_1 + b_1x + c_1x^2$ and $v = a_2 + b_2x + c_2x^2$ be arbitrary polynomials in P_2.

$$\begin{aligned} T(u+v) &= T((a_1+a_2) + (b_1+b_2)x + (c_1+c_2)x^2) \\ &= (a_1+a_2+1) + (b_1+b_2+1)x + (c_1+c_2+1)x^2 \\ &= (a_1+1) + (b_1+1)x + (c_1+1)x^2 + (a_2+1) + (b_2+1)x + (c_2+1)x^2 \\ &= T(u) + T(v) \end{aligned}$$

So, $T(u+v) = T(u) + T(v)$, which satisfies the additivity property.

classmate

Date

Page

Name: Juhi Dugar

Roll no: 2301010053

2) **Homogeneity:**

Let $v = a+bx+cx^2$ be an arbitrary polynomial in P_2, and k be an arbitrary scalar.

$$T(kv) = T(k(a+bx+cx^2))$$
$$= T(ka+kbx+kcx^2)$$
$$= (ka+1)+(kb+1)x+(kc+1)x^2$$
$$= k(a+1)+k(b+1)x+k(c+1)x^2$$
$$= kT(v)$$

So, $T(kv) = kT(v)$, which satisfies the homogeneity property.

Since T satisfies both additivity and homogeneity, it is a linear transformation.

Q7. Determine whether the set $S = \{(1,2,3),(3,1,0),(-2,1,3)\}$ is a basis of $V_3(R)$. In case S is not a basis, determine the dimension and the basis of the subspace spanned by S.

To determine whether the set $S = \{(1,2,3),(3,1,0),(-2,1,3)\}$ is a basis of $V^3(R)$, we need to check two conditions:

1. Linear independence: If the vectors in S are linearly independent.
2. Spanning: If the vectors in S span $V^3(R)$.

Name: Juhi Dugar   Roll no: 2301010053

Let's start by checking for linear independence. We'll form a matrix with the vectors in S as rows and then row reduce it to check for linear independence:

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ -2 & 1 & 3 \end{bmatrix}$$

Row reduction:

$R_2 \to R_2 - 3R_1$
$R_3 \to R_3 + 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -9 \\ 0 & 5 & 9 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -9 \\ 0 & 0 & 0 \end{bmatrix}$$

The third row being all zeros after row reduction means that the vectors are linearly dependent.

Now, for a set of vectors to be a basis, it must be ~~properly~~ linearly independent, it cannot be a basis for

Date
Page

Roll no: 2301010053

Name: Juhi Dugar

for $V^3(R)$.

To determine the dimension and find a basis for the subspace spanned by S, we can use the fact that the dimension of a subspace is equal to the number of vectors in a basis for that subspace.

To find a basis for the subspace spanned by S, we can remove any linearly dependent vectors from S to obtain a linearly independent set. In this case, we can see that the first and third vectors are linearly independent, so they form a basis for the subspace spanned by S.

So, the dimension of the subspace spanned by S is 2, and a basis for this subspace is $\{(1,2,3),(-2,1,3)\}$

Q8. Using Jacobi's method (perform 3 iterations), solve $3x-6y+2z=23$, $-4x+y-z=-15$, $x-3y+7z=16$, with initial values $x_0=1$, $y_0=1$, $z_0=1$.

Equations are:

$$3x-6y+2z=23$$
$$-4x+y-z=-15$$
$$x-3y+7z=16$$

CLASSMATE
Date
Page

Name: Juhi Dugar

Roll no: 2301010053

On rearranging the equations, we get

1) $x = \frac{1}{3}(6y - 2z + 23)$

2) $y = 4x + z - 15$

3) $z = \frac{1}{7}(3y + 16 - x)$

Iteration 1:

Using intial values $x_0 = 1, y_0 = 1, z_0 = 1$

1) $x_1 = \frac{1}{3}(6(1) - 2(1) + 23) = 9$

2) $y_1 = 4(1) + 1 - 15 = -10$

3) $z_1 = \frac{1}{7}(3(1) + 16 - 1) = 2.571$

Iteration 2:

1) $x_2 = \frac{1}{3}(6(-10) - 2(2.571) + 23) = -10.62$

2) $y_2 = 4(9) + 2.571 - 15 = 23.57$

3) $z_2 = \frac{1}{7}(3(-10)+16-9) = -3.3$

Iteration 3:

1) $x_3 = \frac{1}{3}(6(23.57)-2(-3.3)+23) \approx 57$

2) $y_3 = 4(-10.62)+(-3.3)-15 = -443.1$

3) $z_3 = \frac{1}{7}(3(23.57)+16-(-10.62)) \approx 14$

So, after 3 iterations using Jacobi's method, we have:

$x \approx 57$, $y \approx -443$, $z \approx 14$

Q9. Explain one application of matrix operations in image processing with example.

One application of matrix operations in image processing is image filtering. Image filtering involves modifying the pixels of an image to enhance certain features or remove noise. This can be achieved using convolution, a fundamental operation in image processing that involves applying a filter (also known as a kernel or mask) to an image.

Name: Juhi Dugar

Roll no: 2301010053

A common example of image filtering is applying a Gaussian blur to an image. A Gaussian blur is a type of low-pass filter that reduces high-frequency noise and sharpens edges. It is often used to smooth an image and reduce detail.

Here's how the process works using matrix operations:

1) Define the Gaussian kernel: The Gaussian blur filter is defined by a Gaussian function. The size of the kernel determines the extent of blurring. A typical Gaussian kernel might be a 3x3 or 5x5 matrix with values calculated according to the Gaussian distribution.

2) Convolution operation: The kernel is applied to each pixel of the image using convolution. This involves overlaying the kernel onto each pixel of the image and computing the weighted sum of the pixel values under the kernel.

3) Weighted Sum Calculation: For each pixel, the values of the pixels in the neighbourhood defined by the kernel are multiplied by the corresponding values in the kernel matrix. These products are then summed to produce the new value for the pixel.

Name: Juhi Dugar Roll no: 2301010053

4) Normalization: After convolution, the resulting image may need to be normalized to ensure that the pixels values remain within the valid range (e.g., 0 to 255 for grayscale images)

matrix operations, such as element-wise multiplication and summation, are fundamental to performing the convolution operation efficiently. The process can be implemented using techniques like matrix multiplication or sliding window operations.

Here's a simplified example of a 3 X 3 Gaussian kernel:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

To apply this kernel to an image, each pixel in the image is replaced by a weighted sum of its neighbouring pixels, with the weights defined by the kernel. This process effectively blurs the image, smoothing out sharp transitions into pixel values.

Overall, matrix operations play a crucial role in image processing tasks like filtering, enabling efficient manipulation of pixel values to achieve desired effects such as blurring, sharpening, edge detection, and more.

classmate
Date
Page

Name : Juhi Dugar Roll no : 2301010053

Q10. Give a brief description of linear transformations for Computer Vision for rotating 2D image.

In computer vision, linear transformations are fundamental operations used for various image processing tasks, including rotating 2D images. Linear transformations are operations that preserve the structure of an image while transforming its appearance in a certain way. When rotating a 2D image, we typically use linear transformations such as rotation matrices.

Here's a brief description of the process of rotating a 2D image using linear transformations:

1) Rotation Matrix: A rotation matrix is a 2X2 matrix that defines how points in a 2D coordinate system are transformed when the system is rotated around the origin. The rotation matrix is defined as :

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

where θ is the angle of rotation in radians.

2) Affine Transformation: Rotating a 2D image involves an affine transformation, which is linear transformation followed by a translation. The rotation matrix is used to perform the rotation, and optionally, a

Name: Juhi Dugar Roll no: 2301010053

translation vector can be applied to specify the center of rotation.

3) Transformation operation: Each pixel in the original image is mapped to a new position in the rotated image using the rotation matrix. The transformation operation applies the rotation matrix to the coordinates of each pixel in the original image to determine its new position in the rotated image.

4) Interpolation: After applying the transformation, the rotated image may contain gaps or overlapping pixels. Interpolation techniques such as bilinear interpolation or nearest-neighbour interpolation are used to estimate the pixel values at these new positions based on the values of neighbouring pixels in the original image.

5) ~~Output Image: The resulting image, such as rotating 2D images, are essential in computer vision~~

5) Output Image: The resulting image after rotation is obtained by applying the transformation and interpolation to each pixel in the original image. The rotated image will have the same dimensions as the original image but will be rotated by the specified angle.

classmate

Date ________
Page ________

Name: Juhi Dugar Roll no: 2301010053

Linear transformations such as rotating 2D images, are essential in computer vision for tasks such as image alignment, object detection, and future extraction. They provide a mathematical framework for manipulating and analyzing images, enabling algorithms to perform operations like rotation efficiently and accurately.

X X