투빅스 18기 정규세션 3주차

Neural Network Basic Assignment

이름: 국주현

1. Sigmoid Function을 z에 대해 미분하세요.

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

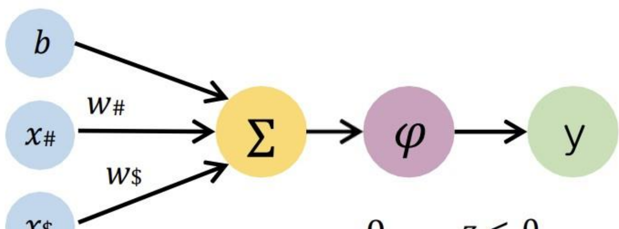
$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1+e^{-z})^{-1}$$

$$= (-1)\frac{1}{(1+e^{-z})^2} \times \frac{d}{dz}(1+e^{-z})$$

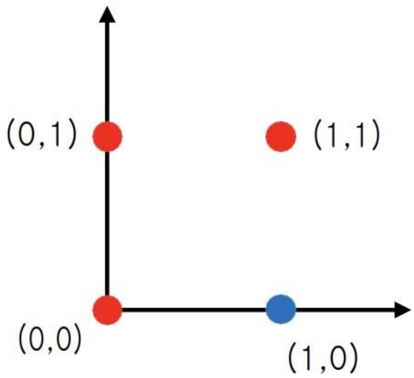
$$= (-1)\frac{1}{(1+e^{-z})^2} \times e^{-z} \times (-1)$$

$$= \frac{e^{-z}}{(1+e^{-z})^2} = \frac{1+e^{-z}-1}{(1+e^{-z})^2} = \frac{(1+e^{-z})}{(1+e^{-z})^2} - \frac{1}{(1+e^{-z})^2} = \frac{1}{(1+e^{-z})} - \frac{1}{(1+e^{-z})^2} = \frac{1}{(1+e^{-z})}\left(1 - \frac{1}{1+e^{-z}}\right)$$

2. 다음과 같은 구조의 Perceptron과 ●(=1), ●(=0)을 평면좌표상에 나타낸 그림이 있습니다.



$$\varphi(z) = \begin{cases} 0, & z < 0 \\ 1, & z \geq 0 \end{cases}$$



2-1. ●, ●을 분류하는 임의의 b, w를 선정하고 분류해보세요.

if $b=1$, $w_\# = 1$, $w_\$ = 2$

: $(0,1) = 0\times1 + 2\times1 + 1 = 3 \Rightarrow 3 \geq 0$ ●

$(0,0) = 0\times1 + 2\times0 + 1 = 1 \Rightarrow 1 \geq 0$ ●

$(1,1) = 1\times1 + 2\times1 + 1 = 4 \Rightarrow 4 \geq 0$ ●

$(1,0) = 1\times1 + 2\times0 + 1 = 2 \Rightarrow 2 \geq 0$ ●

2-2. Perceptron 학습 규칙에 따라 임의의 학습률을 정하고 b, w를 1회 업데이트 해주세요.

if) $lr = 0.1$

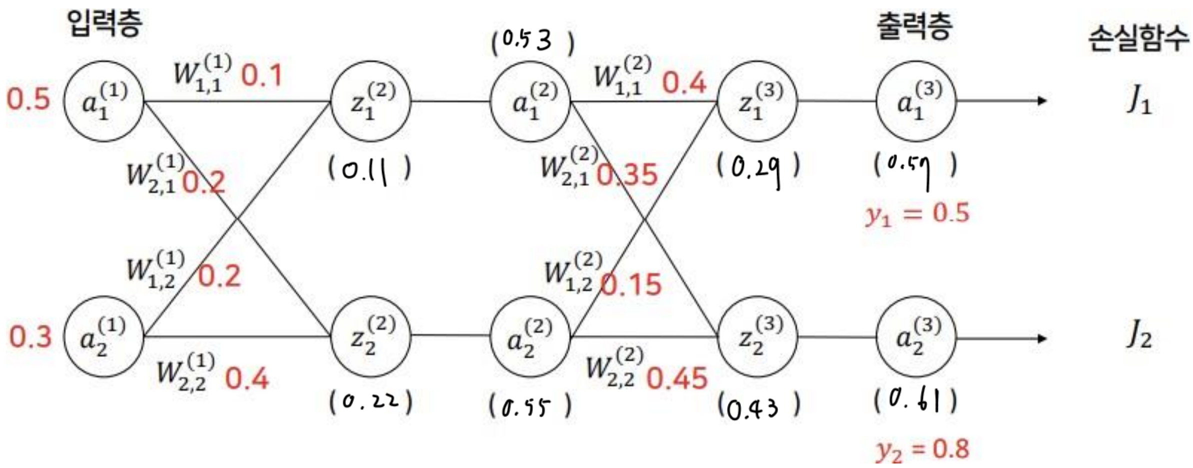
(0,1)의 실제값 : 1, 예측값 : 0

$\Rightarrow w_\# = w_\# + 0.1(1-0)\times 0 = 1+0 = 1$

$w_\$ = w_\$ + 0.1(1-0)\times 1 = 2+0.1 = 2.1$

$b = b + 0.1(1-0)\times 1 = 1+0.1 = 1.1$

3. **다음과 같이 입력과 가중치가 주어진 퍼셉트론이 있을 때, 아래의 물음에 답해주세요. 모든 문제는 풀이과정을 자세하게 적어주세요! (3-3까지 있습니다.)**



3-1. FeedForward가 일어날 때, 각 노드가 갖는 값을 빈칸에 써주세요. 단, 활성화함수는 sigmoid 함수입니다. (모든 계산의 결과는 소수점 셋째자리에서 반올림하여 둘째자리까지만 써주세요.)

$Z_1^{(2)} = 0.5 \times 0.1 + 0.3 \times 0.2 = 0.11$

$Z_2^{(2)} = 0.5 \times 0.2 + 0.3 \times 0.4 = 0.22$

$a_1^{(2)} = \text{sigmoid}(0.11) = 0.53$

$a_2^{(2)} = \text{sigmoid}(0.22) = 0.55$

$z_1^{(3)} = 0.53 \times 0.4 + 0.55 \times 0.15 = 0.29$

$Z_2^{(3)} = 0.53 \times 0.35 + 0.55 \times 0.45 = 0.43$

$a_1^{(3)} = \text{sigmoid}(0.29) = 0.57$

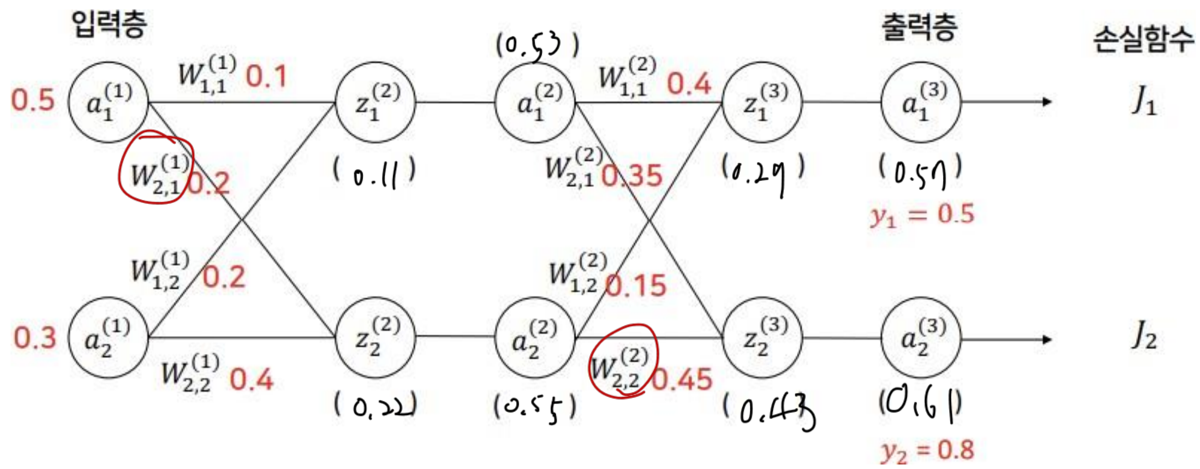
$a_2^{(3)} = \text{sigmoid}(0.43) = 0.61$

3-2. 3-1에서 구한 값을 이용하여 손실함수 J_1과 J_2의 값을 구해주세요. (J_1과 J_2는 반올림하지 말고 써주세요.)

$$MSE = \frac{1}{2N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2 \text{ 라 하면}$$

$$J_1 = \frac{1}{2}(0.57 - 0.5)^2 = 0.00245$$

$$J_2 = \frac{1}{2}(0.61 - 0.8)^2 = 0.01805$$



3-3. 위에서 구한 값을 토대로, BackPropagation이 일어날 때 $W_{2,2}^{(2)}$과 $W_{2,1}^{(1)}$의 조정된 값을 구해주세요. 단, learning rate는 0.1입니다. (계산 과정에서 소수점 넷째자리에서 반올림하여 셋째자리까지만 써주시고, 마지막 결과인 $W_{2,1}^{(1)}$과 $W_{2,2}^{(2)}$의 값만 반올림하지 말고 써주세요.)

$J_2 = 0.01805$

$$\frac{\partial J_2}{\partial W_{22}^{(3)}} = \frac{\partial J_2}{\partial a_2^{(3)}} \times \frac{\partial a_2^{(3)}}{\partial z_2^{(3)}} \times \frac{\partial z_2^{(3)}}{\partial W_{22}^{(2)}} = -0.19 \times 0.238 \times 0.55$$
$$= -0.02487$$

$$\boxed{\begin{aligned} W_{2,2}^{(2)} &= W_{2,2}^{(2)} + 0.02487 \times 0.1 \\ &= 0.45 + 0.002487 \\ &= 0.452487 \end{aligned}}$$

$$\frac{\partial J_2}{\partial a_2^{(3)}} = \frac{1}{2} \times \frac{\partial}{\partial a_2^{(3)}} (a_2^{(3)} - y_2)^2$$
$$= \frac{1}{2} \times \frac{\partial}{\partial a_2^{(3)}} \{(a_2^{(3)})^2 - 2a_2^{(3)} \times y_2 + (y_2)^2\}$$
$$= a_2^{(3)} - y_2$$
$$= 0.61 - 0.8$$
$$= -0.19$$

$$\frac{\partial z_2^{(3)}}{\partial W_{22}^{(2)}} = \frac{\partial}{\partial W_{22}^{(2)}} (W_{2,1}^{(2)} \times a_1^{(2)} + W_{2,2}^{(2)} \times a_2^{(2)})$$
$$= a_2^{(2)} = 0.55$$

$$\frac{\partial a_2^{(3)}}{\partial z_2^{(3)}} = \frac{\partial}{\partial x} \frac{1}{1+e^{-x}} = \frac{e^{-x}}{(1+e^{-x})^2}$$

$Z_2^{(3)}$을 x로 치환

$$= \frac{1}{1+e^{-x}} \times \frac{e^{-x}}{1+e^{-x}}$$
$$= \frac{1}{1+e^{-x}} \left(1 - \frac{1}{1+e^{-x}}\right)$$
$$= 0.61 \times (1 - 0.61) = 0.238$$

165000

$$\frac{\partial J}{\partial w_{2,1}^{(1)}} = \underbrace{\frac{\partial J}{\partial a_2^{(2)}}} \times \frac{\partial a_2^{(2)}}{\partial z_2^{(2)}} \times \frac{\partial z_2^{(2)}}{\partial w_{2,1}^{(1)}}$$

$$\frac{\partial J}{\partial a_2^{(2)}} = \underbrace{\frac{\partial J_1}{\partial a_2^{(2)}}}_{①} + \underbrace{\frac{\partial J_2}{\partial a_2^{(2)}}}_{②} = 0.0025725 - 0.03249 = -0.0299175$$

$$\fallingdotseq -0.03$$

①
$$\frac{\partial J_1}{\partial a_2} = \frac{\partial J_1}{\partial a_1^{(3)}} \times \frac{\partial a_1^{(3)}}{\partial z_1^{(3)}} \times \frac{\partial z_1^{(3)}}{\partial a_2^{(2)}}$$

$$= (a_1^{(3)} - y_1) \times (0.57 \times (1-0.57)) \times \frac{\partial}{\partial a_2^{(2)}}\left(w_{1,1}^{(2)} \times a_1^{(2)} + w_{1,2}^{(2)} \times a_2^{(2)}\right)$$

$$= 0.07 \times 0.245 \times 0.15$$

$$= 0.0025725$$

②
$$\frac{\partial J_2}{\partial a_2^{(2)}} = \frac{\partial J_2}{\partial a_2^{(3)}} \times \frac{\partial a_2^{(3)}}{\partial z_2^{(3)}} \times \frac{\partial z_2^{(3)}}{\partial a_2^{(2)}}$$

$$= -0.19 \times 0.238 \times 0.45$$

$$= -0.03249$$

$$\frac{\partial J}{\partial w_{2,1}^{(1)}} = \frac{\partial J}{\partial a_2^{(2)}} \times \underbrace{\frac{\partial a_2^{(2)}}{\partial z_2^{(2)}}} \times \underbrace{\frac{\partial z_2^{(2)}}{\partial w_{2,1}^{(1)}}}$$

$$= 0.03 \times 0.2475 \times 0.5$$

$$= 0.0037125$$

$$\frac{\partial a_2^{(2)}}{\partial z_2^{(2)}}$$

$$= 0.55(1-0.55) = 0.2475$$

$$\frac{\partial z_2^{(2)}}{\partial w_{2,1}^{(1)}} = a_1^{(1)} = 0.5$$

$$w_{2,1}^{(1)} = w_{2,1}^{(1)} - 0.1 \times 0.0037125$$

$$= 0.2 - 0.00037125$$

$$= 0.19962875$$