MMAE 443 System Analysis and Control

Final Report - Adaptive Cruise Control

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Model Description

Adaptive cruise control was selected as the final project topic. Adaptive cruise control is a driver assistance system which is made to enhance the convenience by reducing the fatigue for the driver in long drive. Unlike the traditional cruise control, adaptive cruise control also considers for the safe distance from the vehicle ahead. It works by setting the desired speed, like the traditional cruise control, but also detects the distance using radar sensors.

The applied force along the horizontal axis of car, or *plant*, is shown in the fig 1. In fig 1, the car is mainly faced with two driving forces: drag force and vehicle power. Assuming drag force to be proportional to the square of velocity with the coefficient d, we get equation of motion for the plant as in eq. 1.

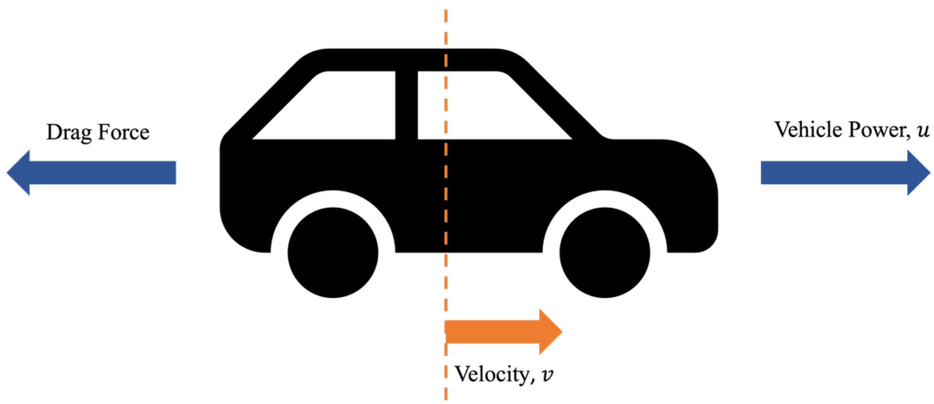

Figure 1. Schematic drawing of the applied forces on the plant

$$m\dot{v} + dv^2 = u \qquad \text{... (1)}$$

The coefficient of drag (d) is obtained from eq. 2, where ρ is the density of air, A is the frontal area of the vehicle, and C_d is the parasitic drag coefficient. The specific value for each parameter will be discussed in the next section of this article. Furthermore, the drag force, which is second order of velocity, was linearized to remain simplicity of the model. So, the drag force term dv^2 was substituted into a damping term bv instead, giving the simplified plant equation of motion shown in eq. 3.

$$d = 0.5\rho A C_d v^2 \qquad \text{... (2)}$$

$$m\dot{v} + bv = u \qquad \text{... (3)}$$

Given the plant equation of motion, Laplace transform was performed on eq. 3. The obtained transfer function $G_p(s)$ is shown in eq. 4. The block diagram of adaptive cruise control was constructed as in fig. 2, where v_r is the reference speed, which is the speed of the car ahead, e is the error of the current speed and the reference speed, and H block is a part that accounts for the measurement of reference speed: if there is car ahead, there will be a sensor used on car to measure reference speed, i.e. radar, or it will be the speed that driver set. We assumed perfect sensor later, meaning that H was neglected.

$$G_p(s) = \frac{1}{ms+b} \qquad \text{... (4)}$$

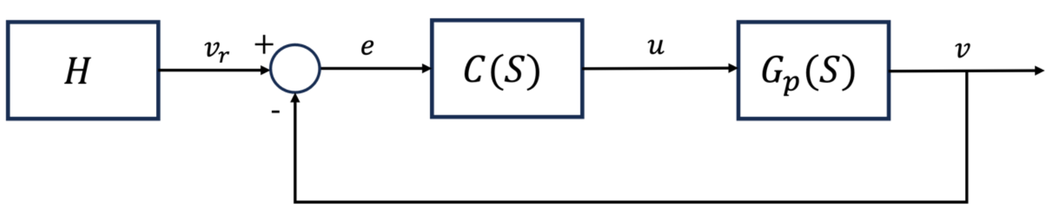

Figure 2. The box diagram of adaptive cruise closed loop control

Calculating the closed loop transfer function, we get eq. 5, assuming PID control. Note that the controller C(s) = $K_P + \frac{K_I}{S} + K_D S$.

$$CLTF = \frac{K_D S^2 + K_P S + K_I}{(K_D + m)S^2 + (K_P + b)S + K_I} \quad \dots (5)$$

Assumptions

1. Mass of the vehicle

Due to the long hours of driving, the adaptive cruise control will be applied to trucks first. Therefore, we focus on a truck, Volvo FH16. According to the official Volvo site, the mass of truck is 12000kg.

2. Dimension of the vehicle

According to the official Volvo site, the dimensions of Volvo FH16 is as shown in fig. 3. Assuming that the front of Volvo FH16 is rectangle, the frontal area A is 8.356 m^2.

Figure 3. The Dimension of Volvo FH16 (unit : mm)

3. Parasitic drag coefficient & Density of air

In general, C_d is 0.25 - 0.3. Average value was taken, making $C_d = 0.275$. We assumed the density of air to be 1.225kg/m^3.

4. **Linearization**

To simplify the drag force in eq. 2 to be linear value, we first explored using the Talyor series expansion: $d = 0.5\rho S C_d v_0^2 + \rho S C_d v_0(v - v_0)$ with $v_0 = 13.89$m/s (=50km/h=31mph) which was assumed to be the average velocity of truck. However, the constant term, acts as an additional term when Laplace transformed, making the analysis much more complicated . Therefore, we approximated the quadratic equation by setting constant term as 0. It is valid only very small partial portion as in fig. 4. In this region, drag force was linearized into $19.54v$.

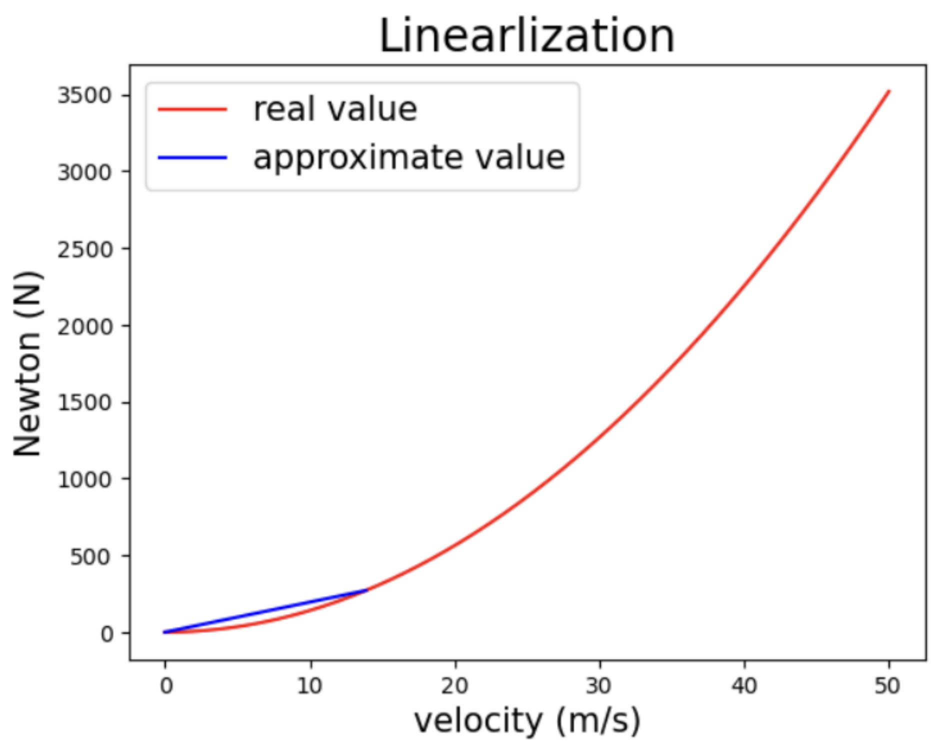

Figure 4. Linearization graph

Analysis of Step Response for Plant

1. Step Response

From the model discussed, the step response of the plant was examined, focusing on the 2% settling time, static gain, and overshoot. For the plant model shown in eq. 4, settling time was calculated to be 2403.24s, static gain was 0.05118, and there was no overshoot. We could see here that the settling time was too long, and static gain was too small. So, we focused on reducing settling time while increasing the static gain.

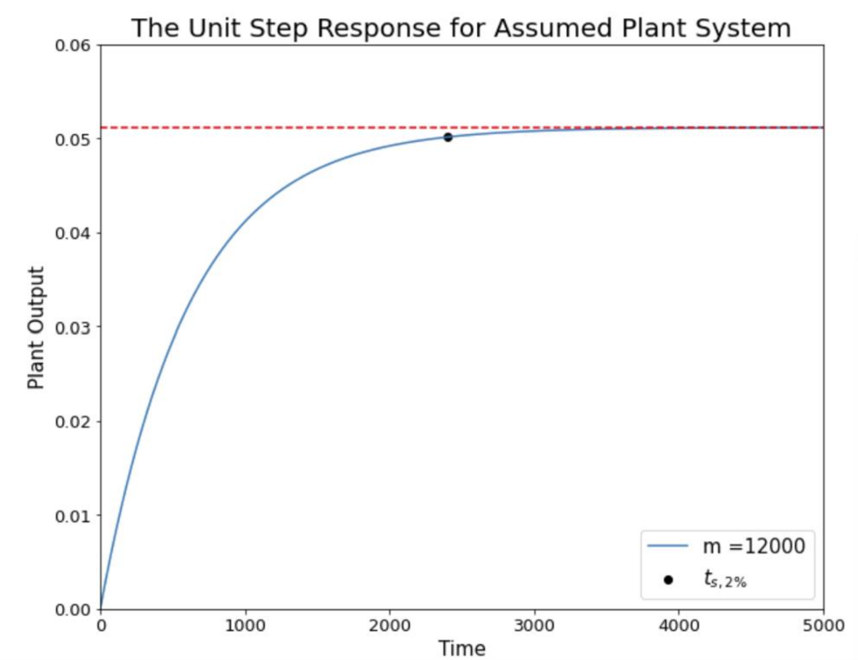

Figure 5. Unit Step Response for Assumed Plant System

2. The effect of mass and damping coefficient

We experimented with different mass (m) and damping coefficient (b) to see how it affects the system performance. First, we set the mass m to vary from 10000 to 18000kg. The resulting plot in fig. 6 show that as m increases, the settling time increases. Static gain and overshoot remained the same.

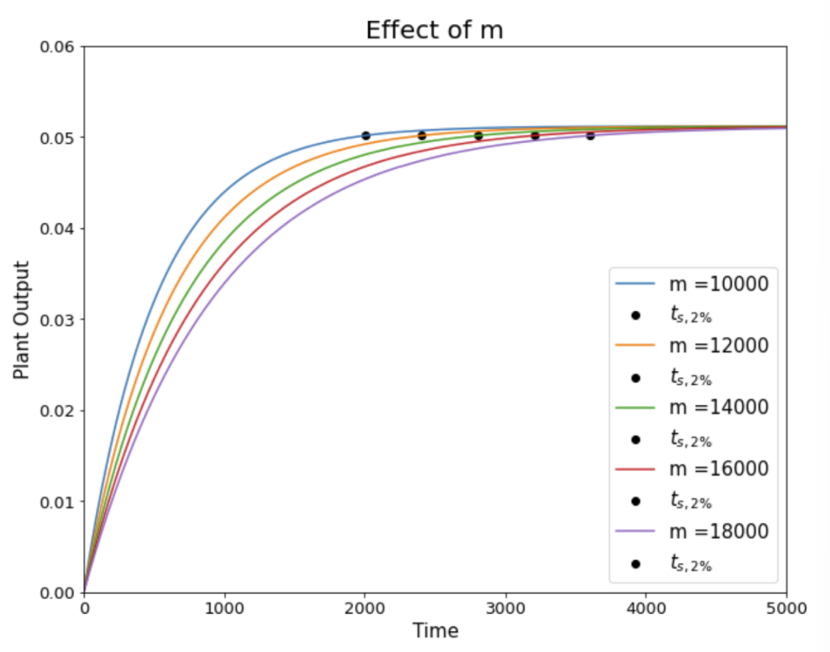

Figure 6. Effect of m on the Step Response

Then, we varied damping coefficient b from 20 to 100. As b increases, both the settling time and the static gain decreased.

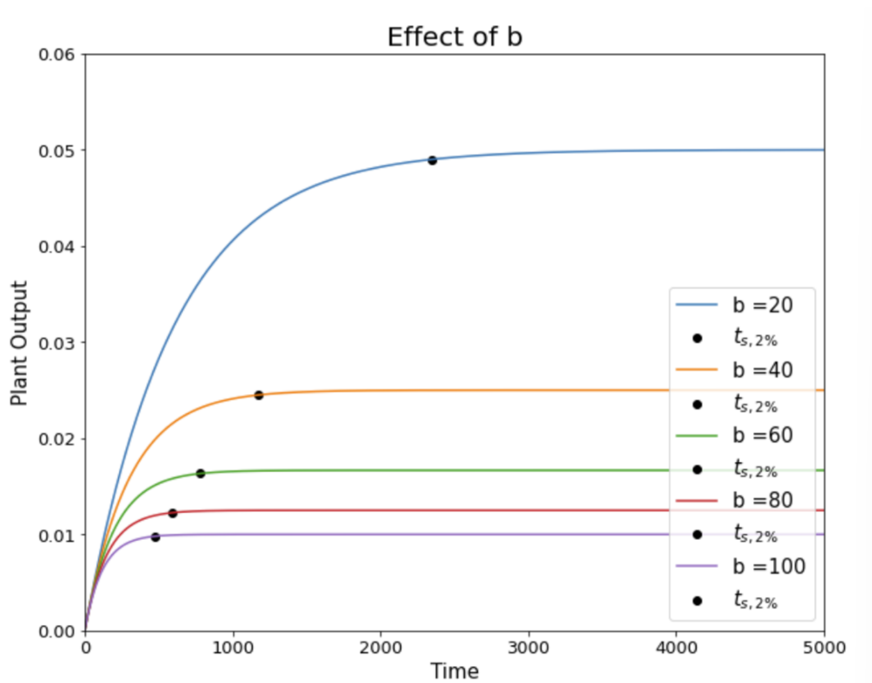

Figure 7. Effect of b on the Step Response

PID Controller Design

In this section, we show the selection of desired closed loop system performance requirements and justification. Then, PID controller designed according to the requirements using pole placement will be introduced along with how control gains impact the performance by leveraging root locus plots.

1. Desired closed loop system performance requirements and justification

First, overshoot of 2.2% was selected. The goal of the percentage value was to make overshoot be less than 1km/h which is one tick in the velocimeter. Assuming a truck to be driving at average velocity of 50km/h, 2.2% is the appropriate value.

According to FreightVikingcan accelerate from 0-60 miles per hour in 15 seconds. To make the car to be reactive enough but not too numb, we set 2% settling time to be over 5 seconds and less 10 seconds.

Lastly, the static gain was designed to 0.99, meaning that the steady state error is 1%. The driver may feel unsafe when vehicle velocity is far from the target velocity, bringing skepticism of driver toward the vehicle.

2. Designing a PID controller using pole placement

* Note that the equations apply for the standard form of second order transfer function. Although our closed loop transfer function is not the standard form, we assumed it to be standard to get idea from the theory we learned in class.

To decide the design region, we used overshoot and settling time requirement. Setting % overshoot to be less than 2.2%, we get eq. 6. Solving it, we get $\zeta = 0.772$. Using the relation $\zeta = cos\theta$, we get $\theta = 39.47°$. Setting the 2% settling time to be between 5 and 10 seconds, we get eq. 7. Solving, $\zeta\omega_n$ needs to be between 0.4 and 0.8. Drawing the requirements obtained, we get fig. 8.

$$\% \text{ overshoot} = 100\, e^{-\frac{\pi\xi}{\sqrt{1-\zeta^2}}} < 2.2 \qquad \text{... (6)}$$

$$5 < 2\%\ Settling\ time = \frac{4}{\zeta\omega_n} < 10 \qquad \text{... (7)}$$

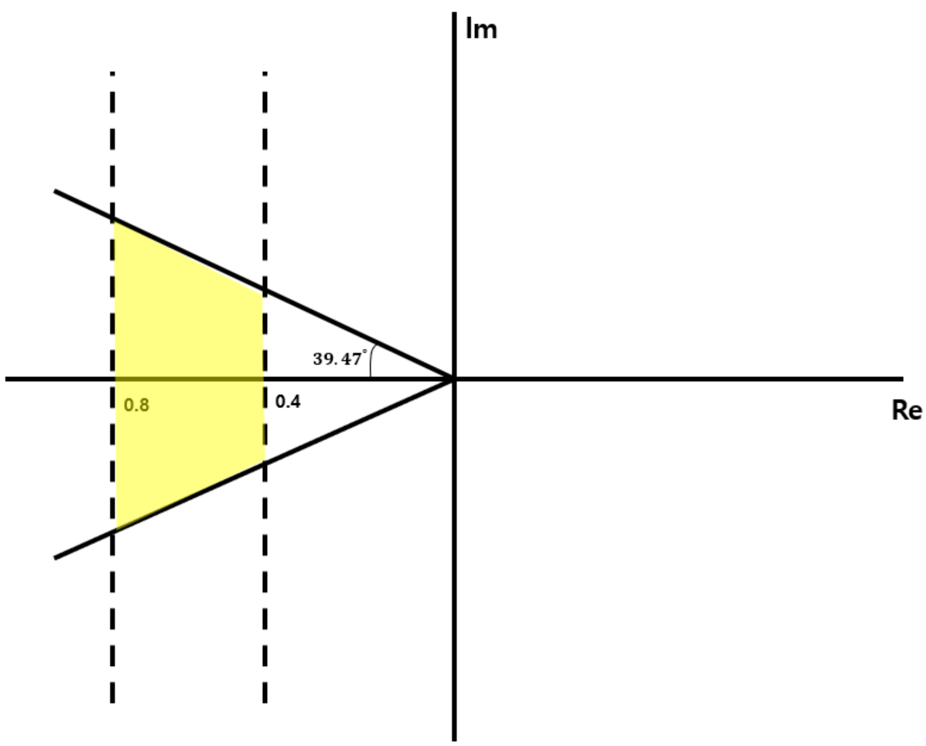

Figure 8. pole placement

From fig. 8, we chose a set of proper values $\{K_P,\ K_D,\ K_I\}$. We could confirm that the unit step response for CLTF does follow the requirement forces, as in fig. 9 and table 1.

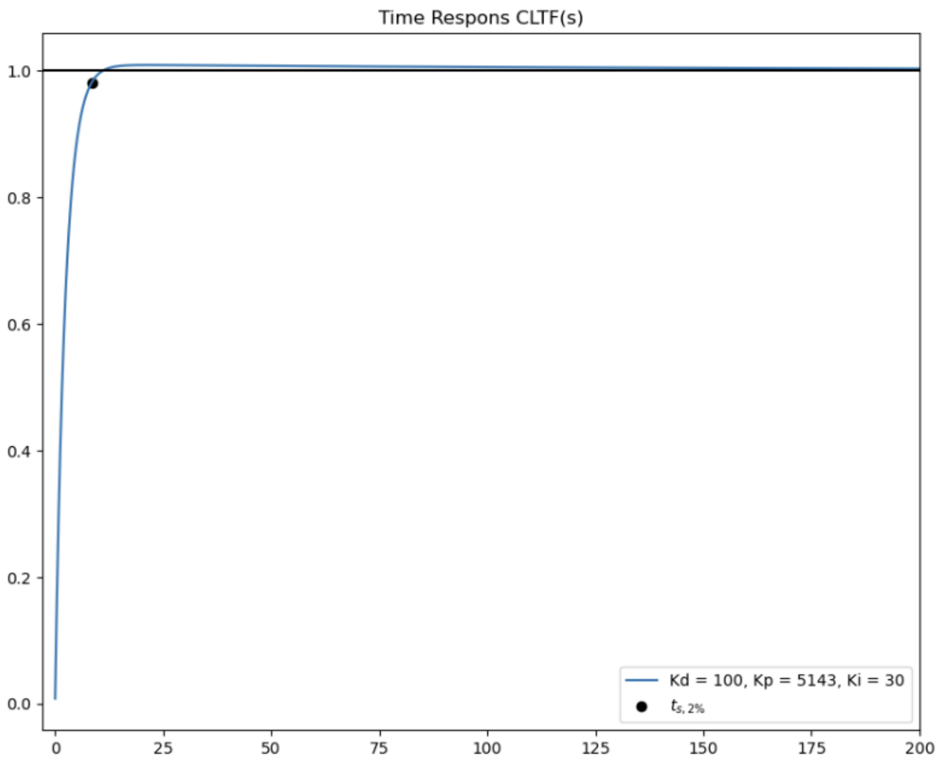

Figure 9. the unit step response for CLTF

$K_P = 5143,\ K_D = 100,\ K_I = 30$	
Settling time	*8.451 seconds*
Overshoot (%)	*1.0005*
Static Gain	*0.8323%*

Table 1. the closed loop system performance given $K_P = 5143,\ K_D = 100,\ K_I = 30$

3. Root locus plots: Effect of different control gains

<u>Effect of Proportional Control</u>

$$C(s) = Kp \qquad G_p(s) = \frac{1}{ms + b} \qquad D_{CL} = 1 + K_p \frac{1}{ms + b}$$

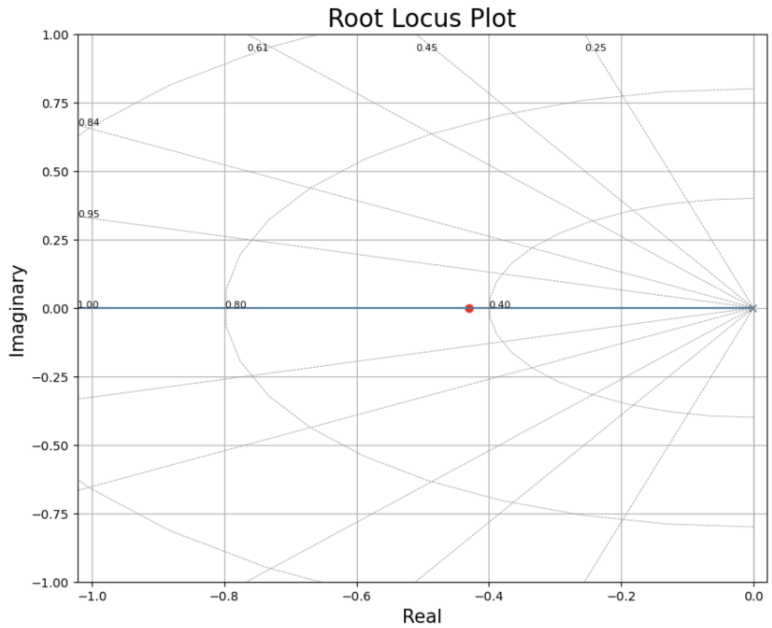

Figure 10. Root locus plot for K_P

As in fig. 10, the system converged faster as K_p increased, meaning that settling time was shorter. Also, the steady-state error should be smaller as there is a multiplier. To confirm it, we drew the graphs with increasing K_P in fig. 11.

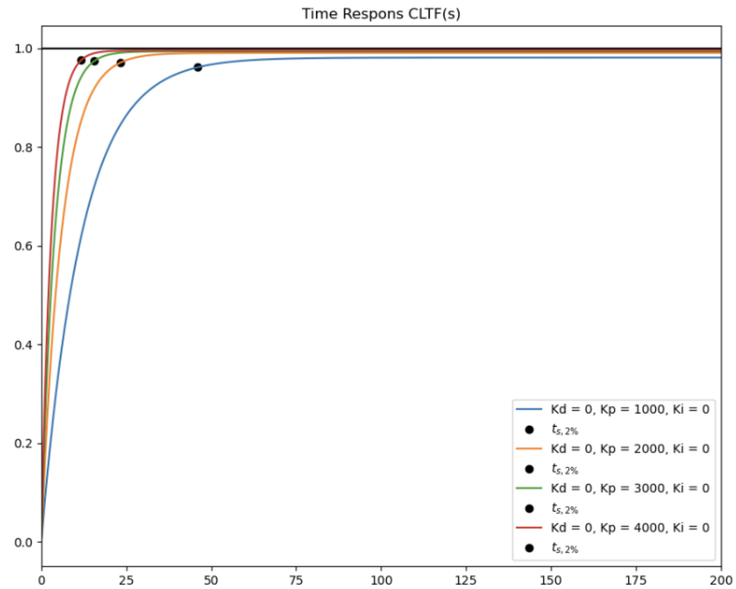

Figure 11. Closed Loop Unit Step Response: Change Kp in proportional control

Effect of Integral Control

$$C(s) = K_p + \frac{K_I}{s} \qquad G_p(s) = \frac{1}{ms+b} \qquad D_{CL} = 1 + K_I \frac{1}{ms^2 + (b+K_p)s}$$

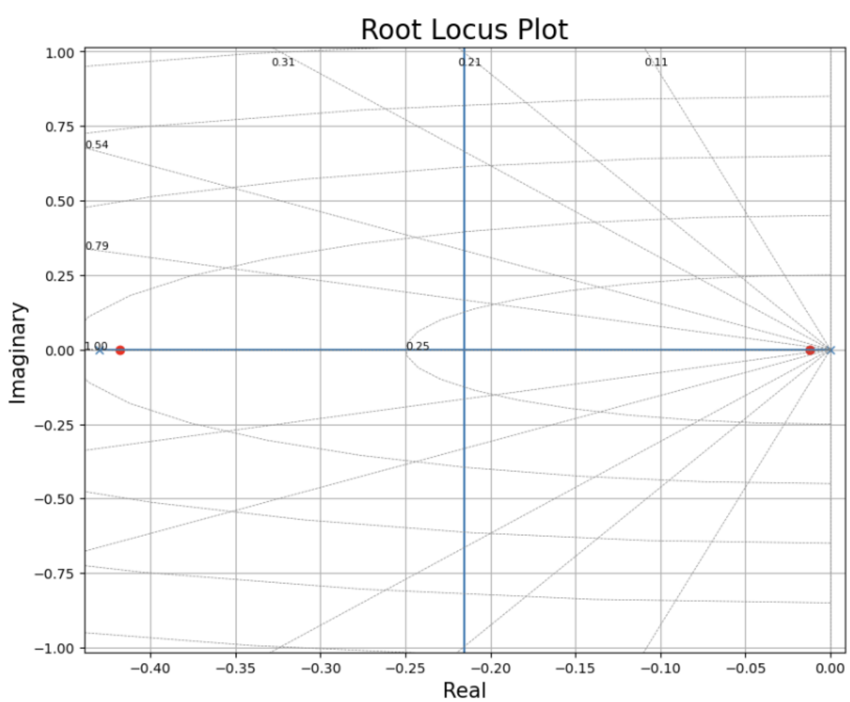

Figure 12. Root locus plot for K_I

K_P was fixed to 5143 to check the effect of K_I. The plot is shown in fig. 12. From D_{CL}, the intersection points of root locus plot, or break-away point, can be calculated as well:

$$(b + K_P)^2 - 4mK_I = 0 \longrightarrow K_I = 555.25$$

K_I rises, the magnitude of overshooting becomes larger, and the frequency of oscillations increases like fig. 12. Also, the settling time decrease as K_I increases until K_I reaches 555.25.

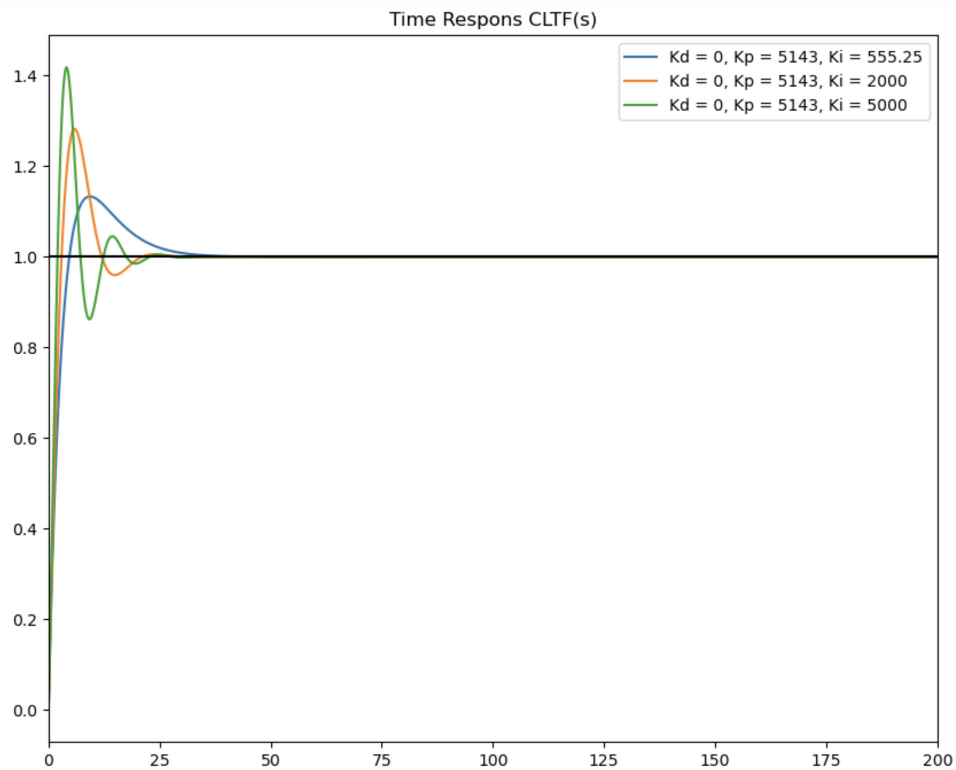

Figure 13. Closed Loop Unit Step Response: Change K_I in PI control

Effect of Derivative Control

$$C(s) = K_P + \frac{K_I}{s} + K_D s \qquad G_p(s) = \frac{1}{ms + b} \qquad D_{CL} = 1 + K_D \frac{s^2}{ms^2 + (b + K_p)s + K_I}$$

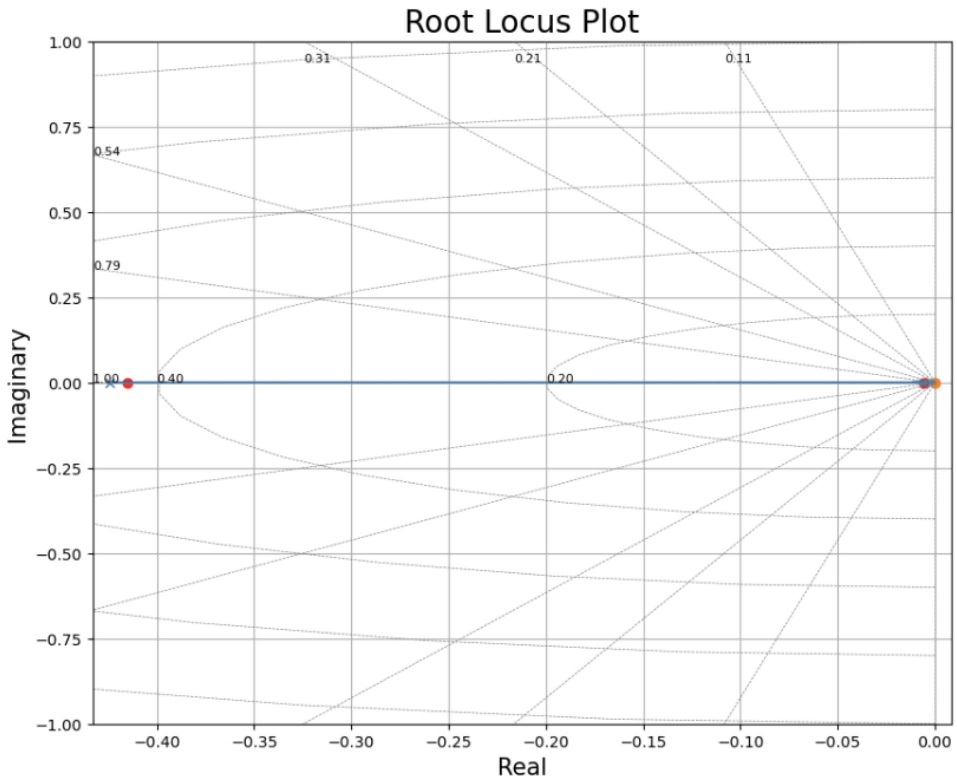

Figure 13. Root locus plot for K_D

K_P, K_I were fixed to 5143, 30, isolating the derivative control. The root locus plot is shown in fig. 14. Similar to fig. 13, the system converged faster as K_D increased, meaning that the settling time was shorter.

Analysis of Frequency Response

In this section, we explore the frequency response of the system using Bode plot. Bode plot lets you break down multiplication terms into addition term, by taking logarithm. This allows for the analysis to be additional effects of 4 different kinds of terms: constant multiplier, real root, pole or zero at origin, and complex poles or zeros. From previous part where the PID controller was discussed, the optimal closed loop transfer function was decided, where $K_P = 5143,\ K_I = 30,\ K_D = 100$. Plugging these control values and b, m values discussed in the assumption section, we get eq. 10, and rearranging terms to obtain break frequency w_b, we get eq. 11.

$$CLTF = \frac{100S^2 + 5143S + 30}{1.21\times10^4 S^2 + 5163S + 30} \quad \text{... (10)}$$

$$CLTF = \frac{100(S + 51.424)(S + 5.834\times10^{-3})}{1.21\times10^4 (S + 0.4208)(S + 5.892\times10^{-3})} \quad \text{... (11)}$$

So, $w_{b,1} = 0.4208, w_{b,2} = 5.892 \times 10^{-3}, w_{b,3} = 51.424, w_{b,4} = 5.892 \times 10^{-3}$. The first two terms, $w_{b,1}$ & $w_{b,2}$ is real poles, and the last two terms, $w_{b,3}$&$w_{b,4}$ is real zeros. For the magnitude bode plot, real poles create a downward pitching movement at break frequency, and zeros create an opposite effect. For phase bode plot, real poles create a downward pitching movement at one tenth of break frequency, and it flattens at the break frequency. Zeros do the opposite here as well. So, analyzing with the actual values, we can predict how bode plot will look like:

1. The value of $w_{b,2}$&$w_{b,4}$ is very near, and they will produce opposite effect as they are pole and zero, respectively. So, at break frequency of term 2 and 4, and additionally one tenth of break frequency, there will be no effect.
2. Near $w_{b,1}$, there will be downward pitching in magnitude plot, and there will be inflection point in phase plot.
3. Near 0.1 $w_{b,1}$ in phase plot, there will be downward pitching.
4. Near $w_{b,3}$, there will be upward pitching in magnitude plot, and there will be inflection point in phase plot.
5. Near 0.1 $w_{b,3}$ in phase, there will be upward pitching.
6. Due to constant multiplier effect, the magnitude plot will be shifted down.

Plotting the values using Python Control library, we get fig. 14. The prediction made can be clearly seen in the bode plot.

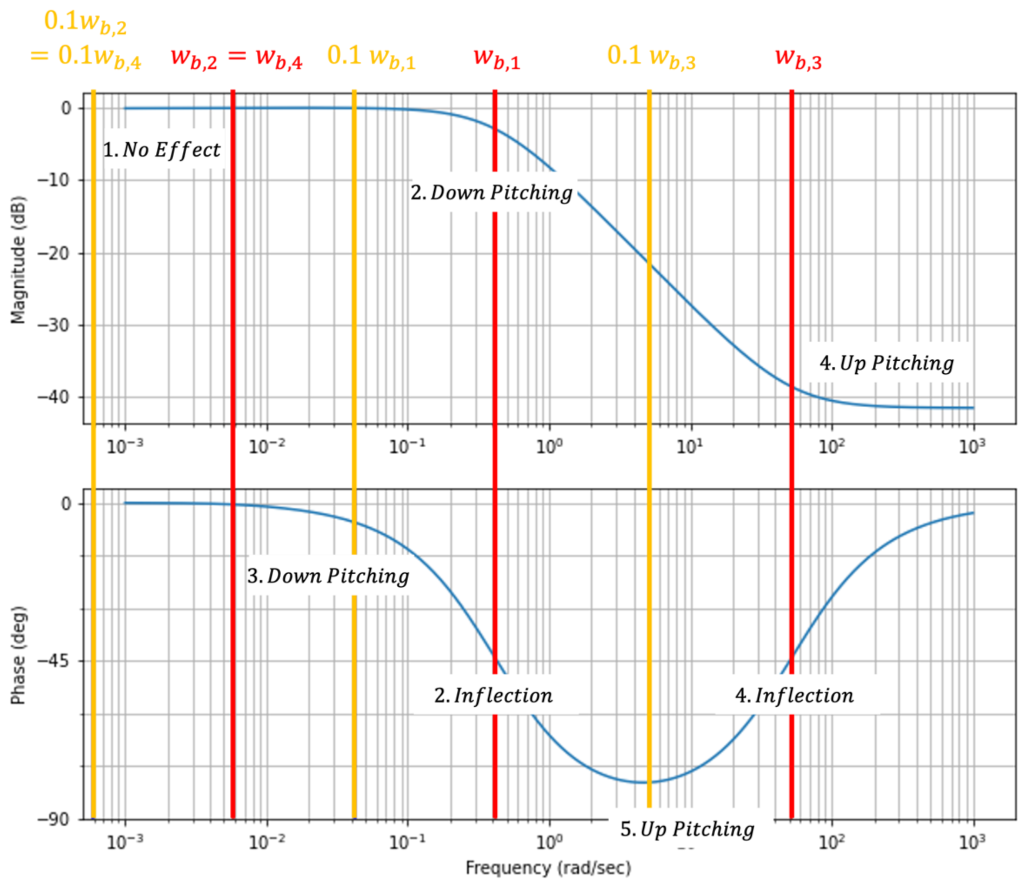

Figure 14. Bode plot of the system with optimized PID controller

The effect of changing the value of m and b can be easily predicted. The damping coefficient, b, will cause the first order s term in the denominator to increase in eq. 10. This will shift the pole values. However, considering that the optimized K_p value is 5143, and assumed b value is 19.54, it will have a very small effect in the frequency response change. The plot in fig. 15 shows the bode plot for different values of b in {1, 19.54, 100}, but the difference could not be seen. If the b value was increase more dramatically, this may cause difference, but not practical. Adaptive cruise control is optimized into different cars, and the drag force can be calculated via experiments such as wind tunnel, meaning that the designers have the value of expected b. Increasing b into extreme value is not realistic: unless you are using adaptive cruise control in a hurricane.

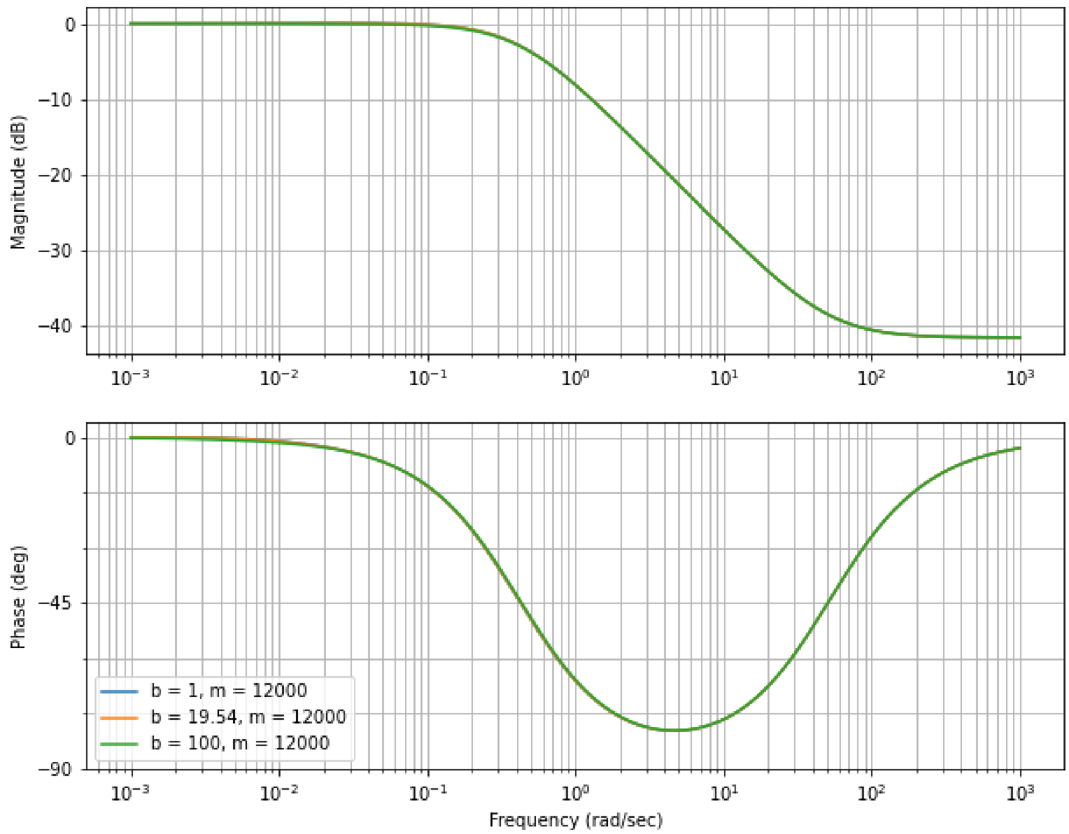

Figure 15. Effect of varying b on bode plot of the system with optimized PID controller

The mass of car, m, will cause the second order term, s^2, to increase. Unlike b, m will have significant effect as m value is much larger compared to K_D. The changing m will have no effect on the nominator and shift the breaking frequency at denominator. This will cause the downward pitching movement to happen earlier when the m is larger, and it to happen later when the m is smaller compared to the basic case of m = 12000 kg. Also, as the absolute value of constant multiplier effect varies; when m is smaller, the absolute value of multiplier decreases, and vice versa when m is larger. This will make the magnitude plot to change shifting the plot downward when m value is larger. This can be seen in fig. 16.

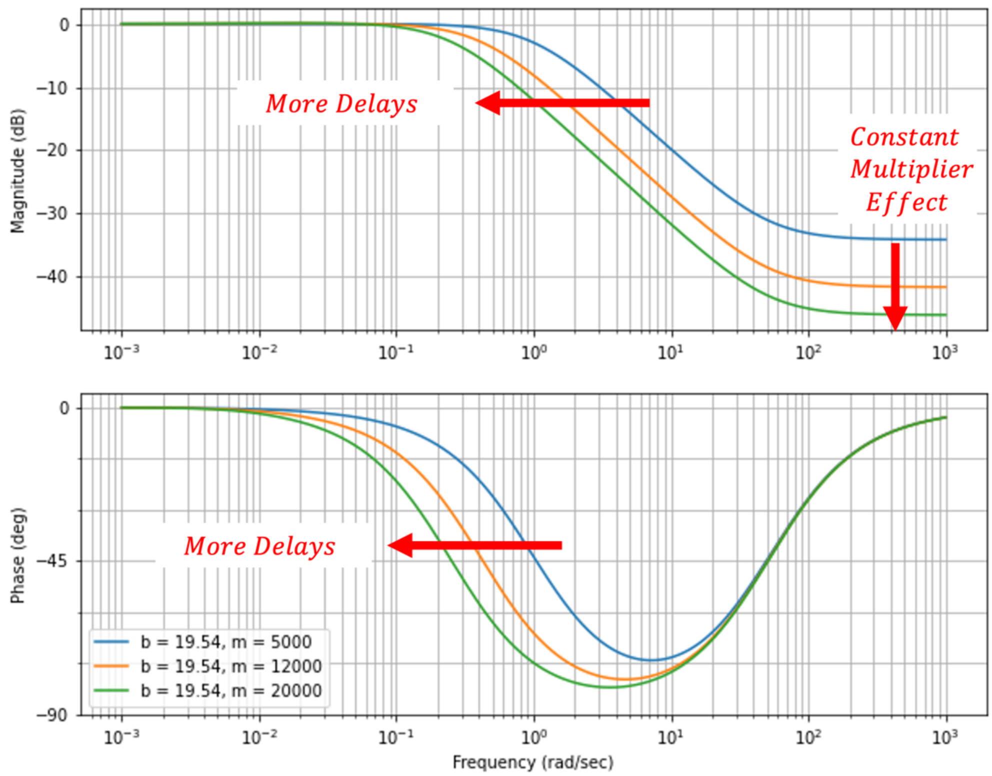

Figure 16. Effect of varying m on bode plot of the system with optimized PID controller

Reference

[1] Thomas, Rob, How Fast is a Semi Truck Without a Trailer? (9 Speedy Facts), https://freightviking.com/fast-semi-truck-without-trailer/

[2] Volvo, Cab specifications for Volvo FH16, https://www.volvotrucks.co.uk/en-gb/trucks/trucks/volvo-fh16/specifications/cab.html