Inference and Simulation

EC 607, Set 04

Edward Rubin

Prologue

Schedule

Last time

The *CEF* and least-squares regression

Today

Inference

Read *MHE* 3.1

Upcoming

Lab: TBD

Problem set 002 coming soon.

Project 1, step 1 due on May 9.

Inference

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Why?

Q What's the big deal with inference?

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A We rarely know the CEF or the population (and its regression vector).

We *can* draw statistical inferences about the population using samples.

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A We rarely know the CEF or the population (and its regression vector).

We *can* draw statistical inferences about the population using samples.

Important The issue/topic of *statistical inference* is separate from *causality*.

Separate questions

1. How do we interpret the estimated coefficient $\hat{\beta}$?
2. What is the sampling distribution of $\hat{\beta}$?

Moving from population to sample

Recall The population-regression function gives us the best linear approximation to the CEF.

Inference

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We're interested in the (unknown) population-regression vector

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Moving from population to sample

Recall The population-regression function gives us the best linear approximation to the CEF.

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$$\beta = \boldsymbol{E}\left[\mathbf{X}_i\mathbf{X}_i'\right]^{-1}\boldsymbol{E}[\mathbf{X}_i\mathbf{Y}_i]$$

which we estimate via the ordinary least squares (OLS) estimator[†]

$$\hat{\beta} = \left(\sum_i \mathbf{X}_i\mathbf{X}_i'\right)^{-1}\left(\sum_i \mathbf{X}_i\mathbf{Y}_i\right)$$

† *MHE* presents a method-of-moments motivation for this derivation, where $\dfrac{1}{n}\sum_i \mathbf{X}_i\mathbf{X}_i'$ is our sample-based estimated for $\boldsymbol{E}\left[\mathbf{X}_i\mathbf{X}_i'\right]$. You've also seen others, *e.g.*, minimizing MSE of $\mathbf{Y}_i$ given $\mathbf{X}_i$.

A classic

However you write it, this OLS estimator

$$
\begin{aligned}
\hat{\beta} &= (\mathrm{X}'\mathrm{X})^{-1}\mathrm{X}'\mathrm{y} \\
&= \left(\sum_i \mathrm{X}_i \mathrm{X}_i'\right)^{-1} \left(\sum_i \mathrm{X}_i \mathrm{Y}_i\right) \\
&= \beta + \left[\sum_i \mathrm{X}_i \mathrm{X}_i'\right]^{-1} \sum_i \mathrm{X}_i e_i
\end{aligned}
$$

is the same estimator you've been using since undergrad.

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Note I'm following *MHE* in defining $e_i = \mathbf{Y}_i - \mathbf{X}_i'\beta$.

Inference

A classic

As you've learned, the OLS estimator

$$\hat{\beta} = \left(\sum_i \mathrm{X}_i \mathrm{X}_i'\right)^{-1} \left(\sum_i \mathrm{X}_i \mathrm{Y}_i\right) = \beta + \left[\sum_i \mathrm{X}_i \mathrm{X}_i'\right]^{-1} \sum_i \mathrm{X}_i e_i$$

has asymptotic covariance

$$\boldsymbol{E}\left[\mathrm{X}_i \mathrm{X}_i'\right]^{-1} \boldsymbol{E}\left[\mathrm{X}_i \mathrm{X}_i' e_i^2\right] \boldsymbol{E}\left[\mathrm{X}_i \mathrm{X}_i'\right]^{-1}$$

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which we estimate by (1) replacing e_i with $\hat{e}_i = \mathbf{Y}_i - \mathbf{X}_i' \hat{\beta}$ and (2) replacing expectations with sample means, *e.g.*, $\boldsymbol{E}\left[\mathrm{X}_i \mathrm{X}_i' e_i^2 \right]$ becomes $\dfrac{1}{n} \sum \left[\mathrm{X}_i \mathrm{X}_i' \hat{e}_i^2 \right]$.

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Standard errors of this flavor are known as heteroskedasticity-consistent (or -robust) standard errors (or Eicker-Huber-White).

Inference

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Now, returning to to the asym. covariance matrix of $\hat{\beta}$,

$$\begin{aligned} E\left[\mathrm{X}_i \mathrm{X}_i'\right]^{-1} E\left[\mathrm{X}_i \mathrm{X}_i' e_i^2\right] E\left[\mathrm{X}_i \mathrm{X}_i'\right]^{-1} &= E\left[\mathrm{X}_i \mathrm{X}_i'\right]^{-1} \sigma^2 \, E\left[\mathrm{X}_i \mathrm{X}_i'\right] E\left[\mathrm{X}_i \mathrm{X}_i'\right]^{-1} \\ &= \sigma^2 \, E\left[\mathrm{X}_i \mathrm{X}_i'\right]^{-1} \end{aligned}$$

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$$\begin{aligned} &E\Big[\left(\mathrm{Y}_i - \mathrm{X}_i'\beta\right)^2 \mid \mathrm{X}_i\Big] \\ &\quad = E\left[\left(\left\{\mathrm{Y}_i - E[\mathrm{Y}_i \mid \mathrm{X}_i]\right\} + \left\{E[\mathrm{Y}_i \mid \mathrm{X}_i] - \mathrm{X}_i'\beta\right\}\right)^2 \middle| \mathrm{X}_i\right] \end{aligned}$$

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&\quad = \mathrm{Var}(\mathrm{Y}_i \mid \mathrm{X}_i) + \left(E[\mathrm{Y}_i \mid \mathrm{X}_i] - \mathrm{X}_i'\beta\right)^2
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Thus, even if $\mathrm{Y}_i \mid \mathrm{X}_i$ has contant variance, $e_i \mid \mathrm{X}_i$ is heteroskedastic.

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Thus, even if $\mathrm{Y}_i \mid \mathrm{X}_i$ has contant variance, $e_i \mid \mathrm{X}_i$ is heteroskedastic. Unless you want to assume the CEF is *linear*.

Inference

Two notes

1. Heteroskedasticity is **not our biggest concern** in inference.

> ...as an empirical matter, heteroskedasticity may matter very little... If heteroskedasticity matters a lot, say, more than a 30 percent increase or any marked decrease in standard errors, you should worry about possible programming errors or other problems. (*MHE*, p.47)

2. Notice that we've **avoided "standard" stronger assumptions**, *e.g.*, normality, fixed regressors, linear CEF, homoskedasticity.

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1. Heteroskedasticity is **not our biggest concern** in inference.

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2. Notice that we've **avoided "standard" stronger assumptions**, *e.g.*, normality, fixed regressors, linear CEF, homoskedasticity.

Following (2): We only have large-sample, asymptotic results (consistency) rather than finite-sample results (unbiasedness).

Inference

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One practical way we can study the behavior of an estimator: **simulation**.

Note You need to make sure your simulation can actually test/respond to the question you are asking (*e.g.*, bias *vs.* consistency).

Simulation

Let's compare false- and true-positive rates[†] for

1. **Homoskedasticity-assuming standard errors** $\left(\mathrm{Var}[e_i|\mathrm{X}_i] = \sigma^2\right)$
2. **Heteroskedasticity-robust standard errors**

† The false-positive rate goes by many names; another common name: *type-I error rate*.

Simulation

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Simulation outline

```
1. Define data–generating process (DGP).
2. Choose sample size n.
3. Set seed.
4. Run 10,000 iterations of
     a. Draw sample of size n from DGP.
     b. Conduct inference.
     c. Record inferences' outcomes.
```

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Let's keep the disturbances well behaved.

$$\mathbf{Y}_i = 1 + e^{0.5\mathbf{X}_i} + \varepsilon_i$$

where $\mathbf{X}_i \sim \text{Uniform}(0, 10)$ and $\varepsilon_i \sim N(0, 1)$.

Simulation

Data-generating process

$$\mathrm{Y}_i = 1 + e^{0.5\mathrm{X}_i} + \varepsilon_i$$

where $\mathrm{X}_i \sim \mathrm{Uniform}(0, 10)$ and $\varepsilon_i \sim N(0, 15^2)$.

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where $\text{X}_i \sim \text{Uniform}(0, 10)$ and $\varepsilon_i \sim N(0, 15^2)$.

```
library(pacman)
p_load(dplyr)
# Choose a size
n = 1000
# Generate data
dgp_df = tibble(
  ε = rnorm(n, sd = 15),
  x = runif(n, min = 0, max = 10),
  y = 1 + exp(0.5 * x) + ε
)
```

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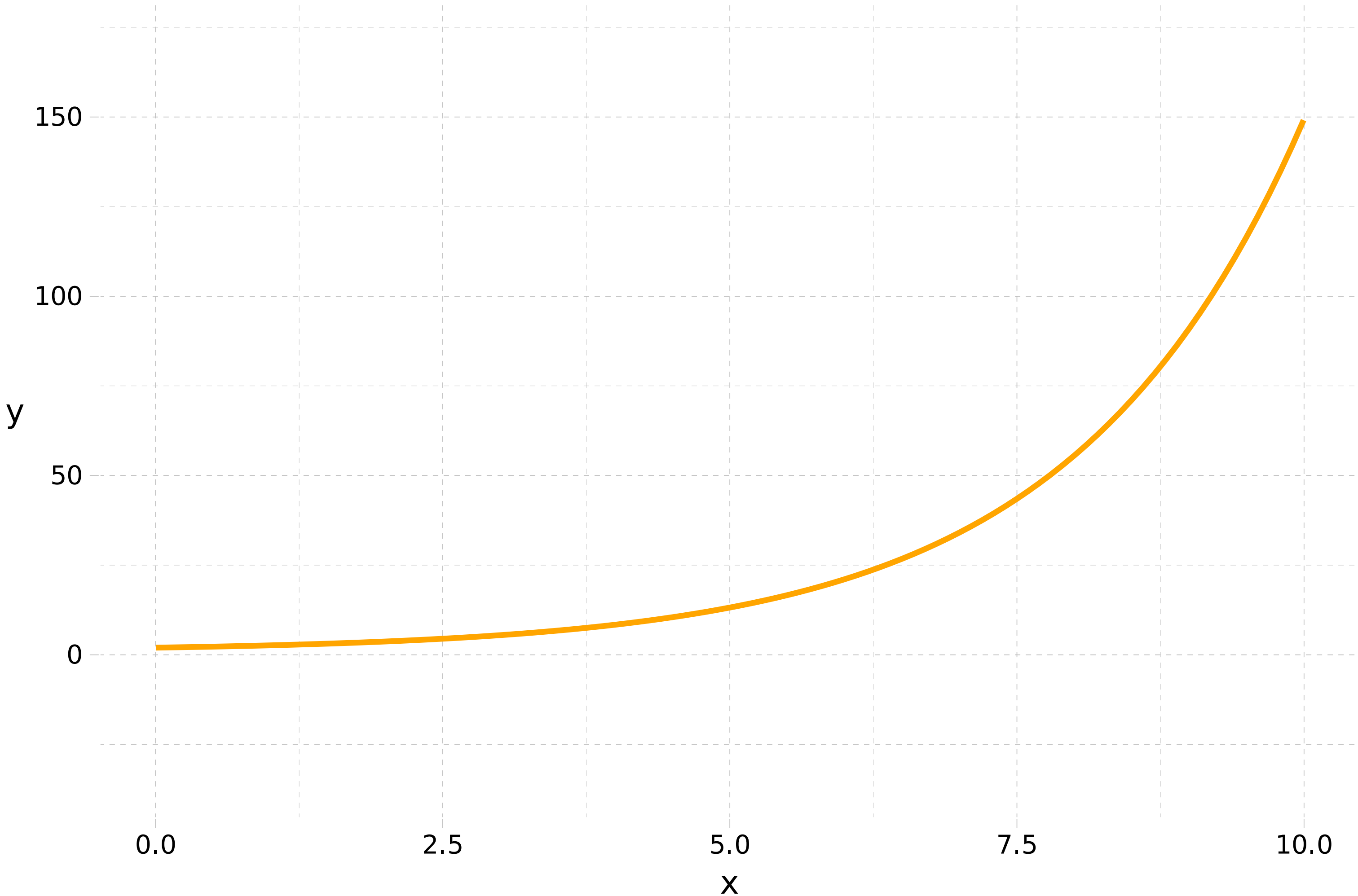
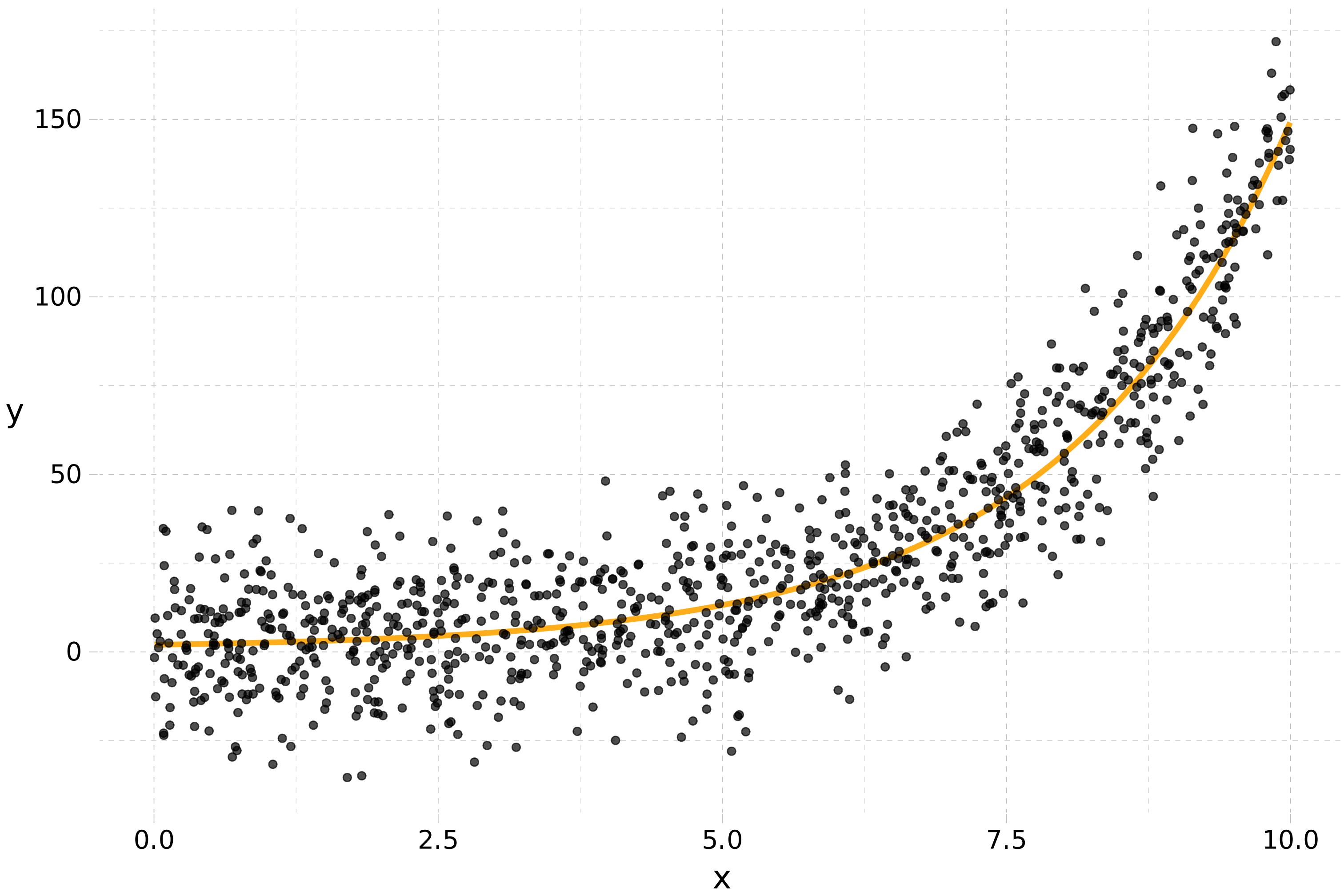
)
```

```
#> # A tibble: 1,000 × 3
#>          ε     x      y
#>      <dbl> <dbl>  <dbl>
#>  1   8.78   9.53 127.
#>  2  10.6    6.22  34.0
#>  3  -1.64   5.32  13.6
#>  4  -6.80   8.92  80.7
#>  5   9.09   1.96  12.8
#>  6 -27.3    8.84  57.0
#>  7   9.45   2.18  13.4
#>  8  -4.14   3.78   3.47
#>  9  -4.26   3.52   2.54
#> 10 -13.8    9.88 127.
#> # ℹ 990 more rows
```

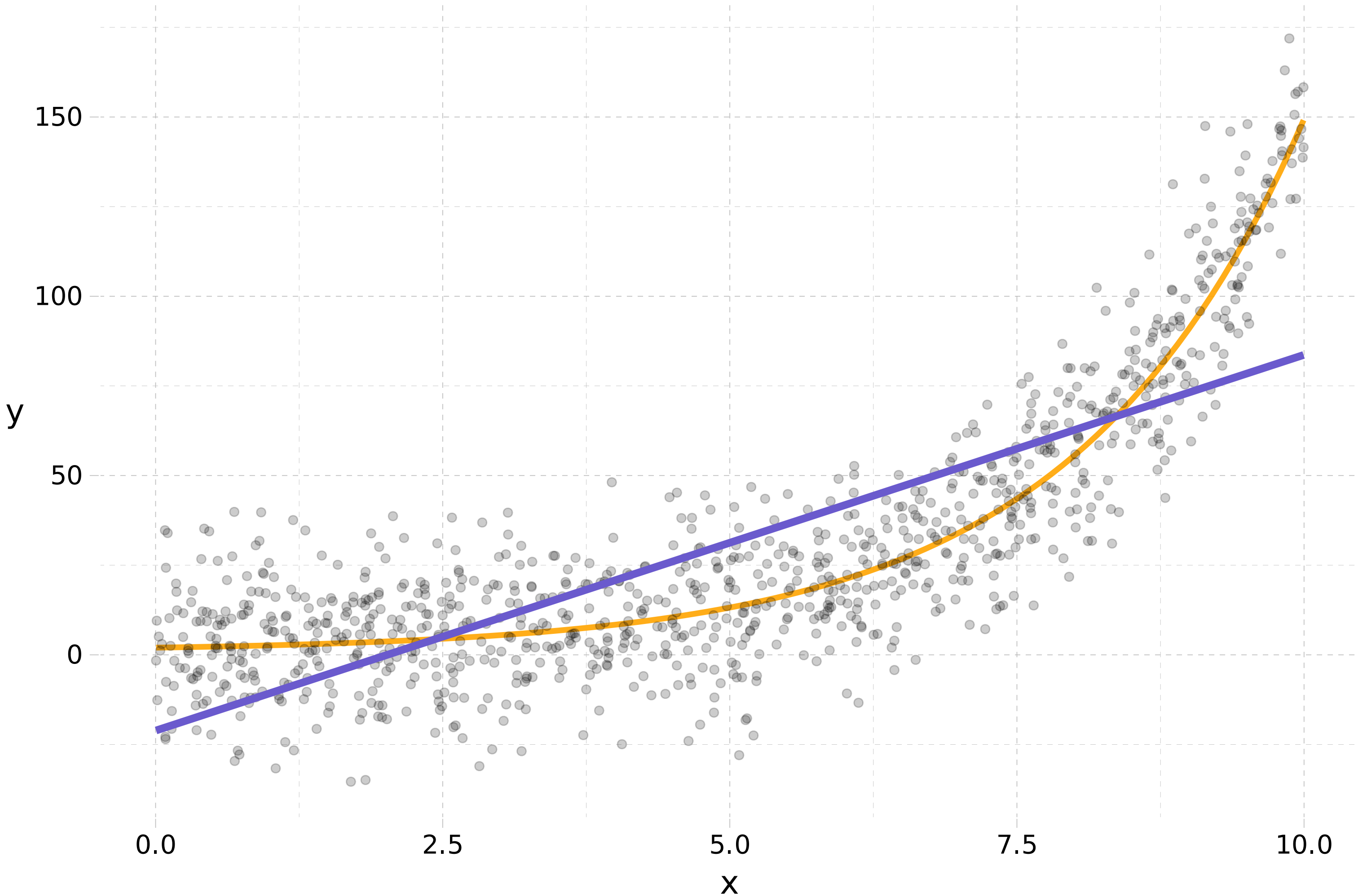
Our CEF



Our population



The population least-squares regression line



Simulation

Iterating

To make iterating easier, let's wrap our DGP in a function.

```
fun_iter = function(iter, n = 30) {
  # Generate data
  iter_df = tibble(
    ε = rnorm(n, sd = 15),
    x = runif(n, min = 0, max = 10),
    y = 1 + exp(0.5 * x) + ε
  )
}
```

We still need to run a regression and draw some inferences.

Note We're defaulting to size-30 samples.

Simulation

We will use `lm_robust()` from the `estimatr` package for OLS and inference.†

- `se_type = "classical"` provides homoskedasticity-assuming SEs
- `se_type = "HC2"` provides heteroskedasticity-robust SEs

```
lm_robust(y ~ x, data = dgp_df, se_type = "classical") %>% tidy() %>% select(1:5)
```

```
#>          term  estimate std.error statistic       p.value
#> 1 (Intercept) -21.14183  1.473496 -14.34807  1.383951e-42
#> 2           x  10.48074  0.257810  40.65294 6.560626e-214
```

```
lm_robust(y ~ x, data = dgp_df, se_type = "HC2") %>% tidy() %>% select(1:5)
```

```
#>          term  estimate std.error statistic       p.value
#> 1 (Intercept) -21.14183 1.4335274 -14.74812  1.112039e-44
#> 2           x  10.48074 0.3097606  33.83495 8.788638e-168
```

† `lm()` works for "spherical" standard errors but cannot calculate het.-robust standard errors.

Inference

Now add these estimators to our iteration function...

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fun_iter = function(iter, n = 30) {
  # Generate data
  iter_df = tibble(
    ε = rnorm(n, sd = 15),
    x = runif(n, min = 0, max = 10),
    y = 1 + exp(0.5 * x) + ε
  )
  # Estimate models
  lm1 = lm_robust(y ~ x, data = iter_df, se_type = "classical")
  lm2 = lm_robust(y ~ x, data = iter_df, se_type = "HC2")
  # Stack and return results
  bind_rows(tidy(lm1), tidy(lm2)) %>%
    select(1:5) %>% filter(term == "x") %>%
    mutate(se_type = c("classical", "HC2"), i = iter)
}
```

Simulation

Run it

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There are a lot of ways to run a single function over a list/vector of values.

- `lapply()`, *e.g.*, `lapply(X = 1:3, FUN = sqrt)`
- `for()`, *e.g.*, `for (x in 1:3) sqrt(x)`
- `map()` from `purrr`, *e.g.*, `map(1:3, sqrt)`

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We're going to go with `map()` from the `purrr` package because it easily parallelizes across platforms using the `furrr` package.

Simulation

Run it!

Run our function 10,000 times

```
# Packages
p_load(purrr)
# Set seed
set.seed(12345)
# Run 10,000 iterations
sim_list = map(1:1e4, fun_iter)
```

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Parallelized 10,000 iterations

```
# Packages
p_load(purrr, furrr)
# Set options
set.seed(123)
# Tell R to parallelize
plan(multisession)
# Run 10,000 iterations
sim_list = future_map(
  1:1e4, fun_iter,
  .options = furrr_options(seed = T)
)
```

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Note Use `multisession` or `multicore` instead of `multiprocess`.

Simulation

Run it!!

Our `fun_iter()` function returns a `data.frame`, and `future_map()` returns a `list` (of the returned objects).

So `sim_list` is going to be a `list` of `data.frame` objects. We can bind them into one `data.frame` with `bind_rows()`.

```
# Bind list together
sim_df = bind_rows(sim_list)
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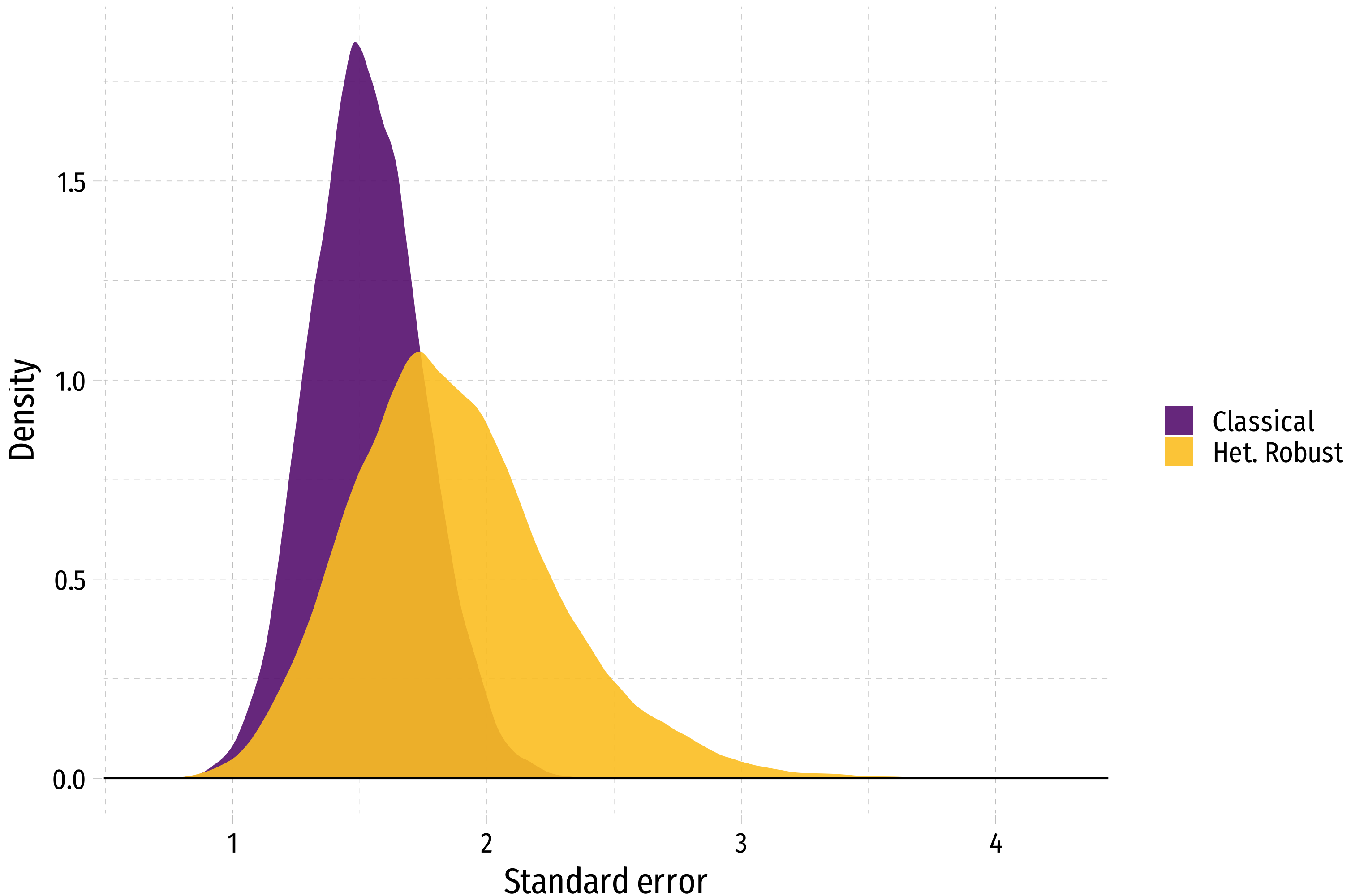
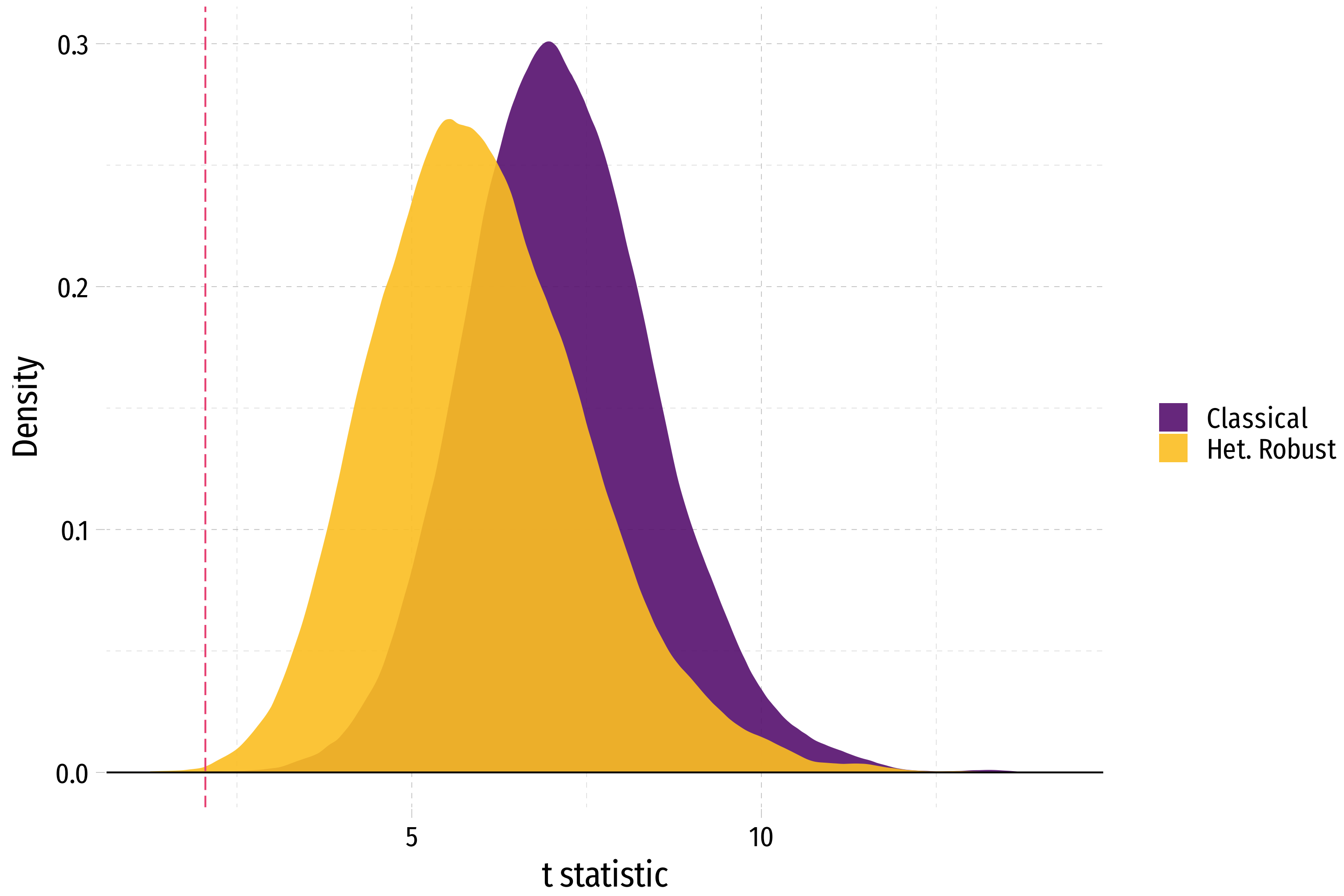
So what are the results?

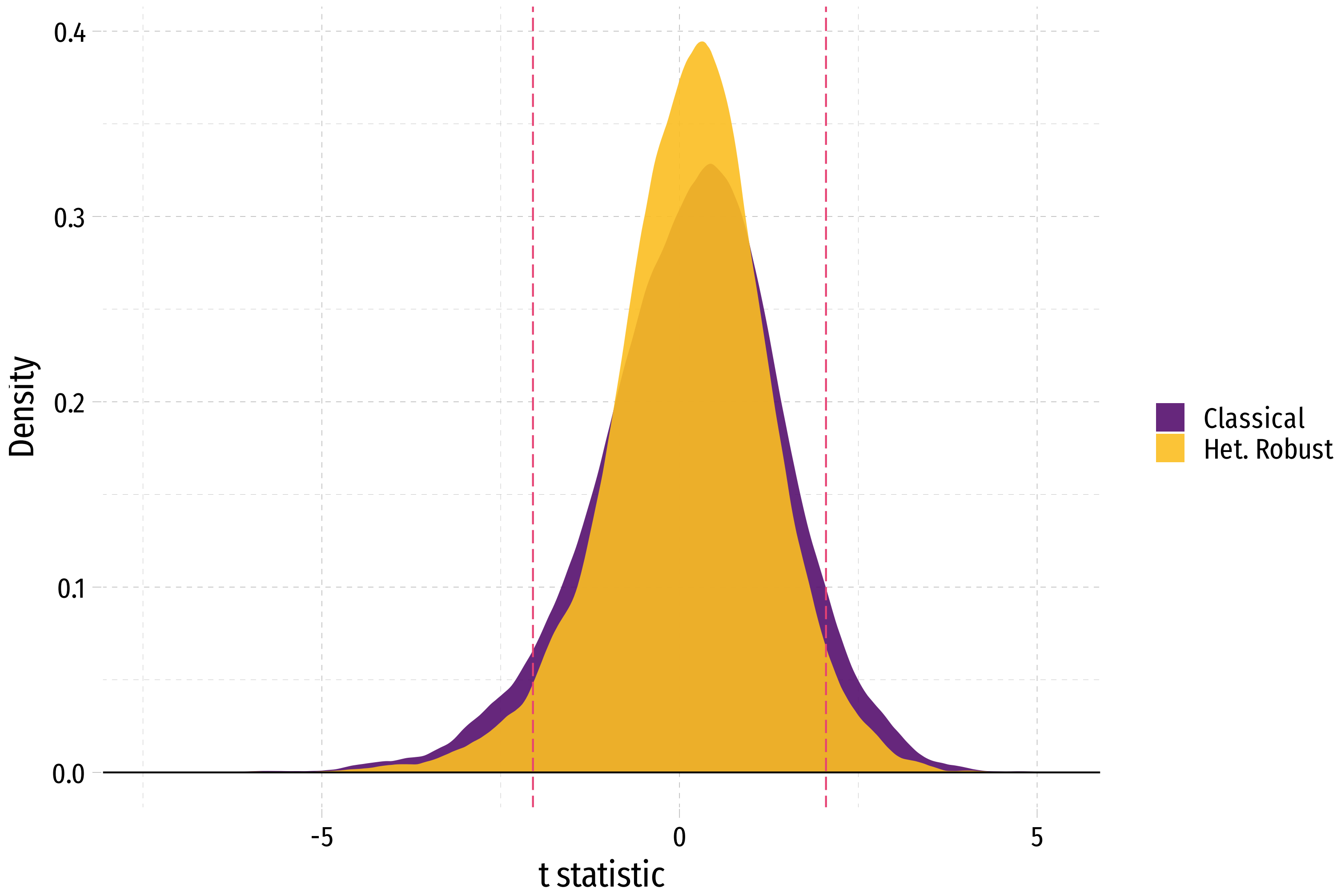
Comparing the distributions of standard errors for the coefficient on x



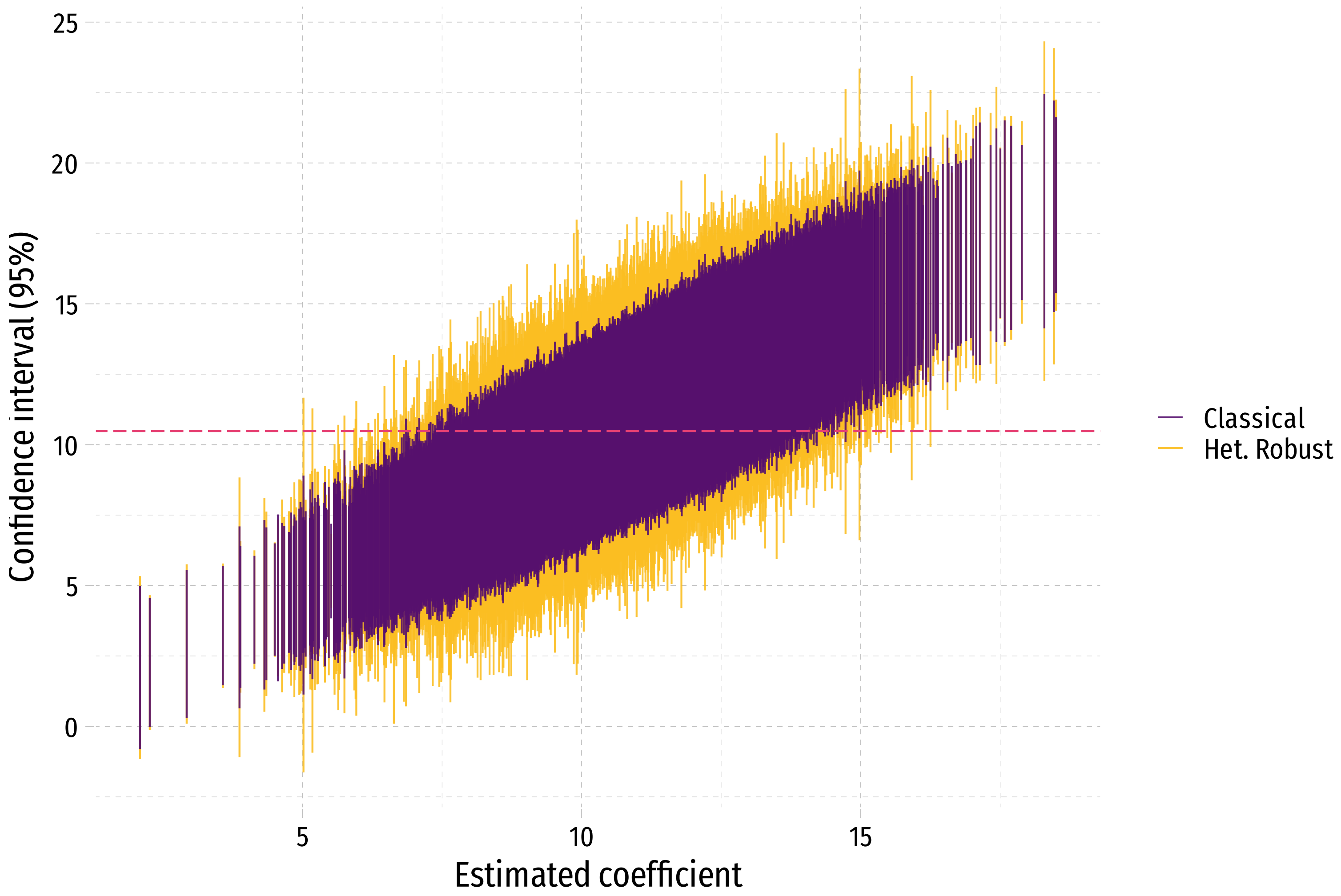
Comparing distributions of t stats for the coefficient on x $(H_o:\ \beta_1 = 0)$



Comparing distributions of t stats for the coefficient on x ($H_o: \ \beta_1 = \beta$)



Comparing the confidence intervals for the coefficient on x



Simulation

How did it go?

For a 5% test the **classical** SEs

- reject the **true value** in 11.38% of samples
- reject **zero** in 99.98% of samples

For a 5% test the **het.-robust** SEs

- reject the **true value** in 6.97% of samples
- reject **zero** in 99.93% of samples

All of these test are for a false H_0.

Q How would the simulation change to enforce a *true* null hypothesis?

Updating to enforce the null

Let's update our simulation function to take arguments `γ` and `δ` such that

$$\mathrm{Y}_i = 1 + e^{\gamma \mathrm{X}_i} + \varepsilon_i$$

where $\varepsilon_i \sim \mathrm{N}(0, \sigma^2 \mathrm{X}_i^\delta)$.

Simulation

Updating to enforce the null

Let's update our simulation function to take arguments `γ` and `δ` such that

$$\mathrm{Y}_i = 1 + e^{\gamma \mathrm{X}_i} + \varepsilon_i$$

where $\varepsilon_i \sim \mathrm{N}(0, \sigma^2 \mathrm{X}_i^\delta)$.

In other words,

- $\gamma = 0$ implies no relationship between Y_i and X_i.
- $\delta = 0$ implies homoskedasticity.

Updating to enforce the null

Updating the function...

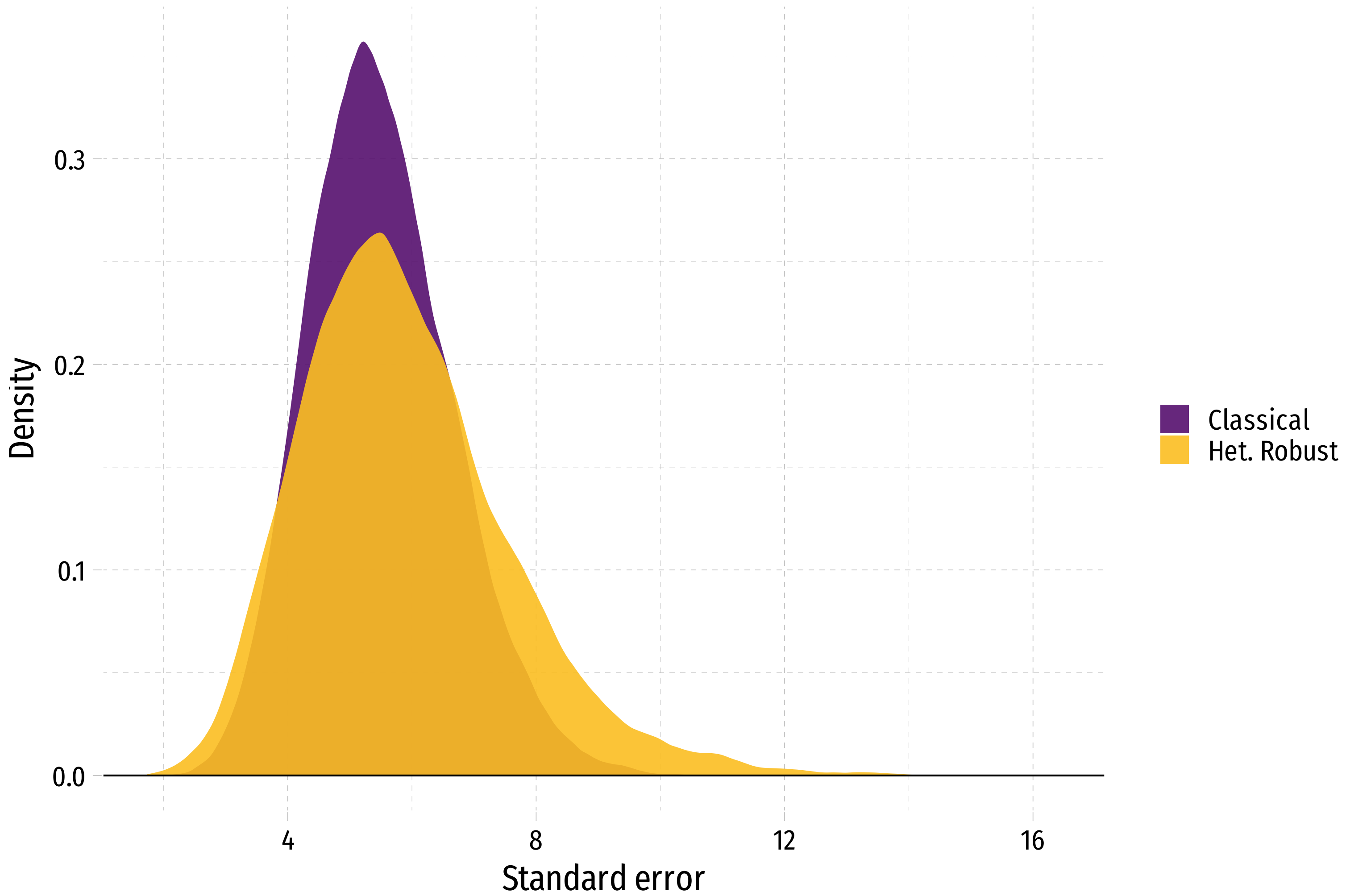
```
flex_iter = function(iter, γ = 0, δ = 1, n = 30) {
  # Generate data
  iter_df = tibble(
    x = runif(n, min = 0, max = 10),
    ε = rnorm(n, sd = 15 * x^δ),
    y = 1 + exp(γ * x) + ε
  )
  # Estimate models
  lm1 = lm_robust(y ~ x, data = iter_df, se_type = "classical")
  lm2 = lm_robust(y ~ x, data = iter_df, se_type = "HC2")
  # Stack and return results
  bind_rows(tidy(lm1), tidy(lm2)) %>%
    select(1:5) %>% filter(term == "x") %>%
    mutate(se_type = c("classical", "HC2"), i = iter)
}
```

Run again!

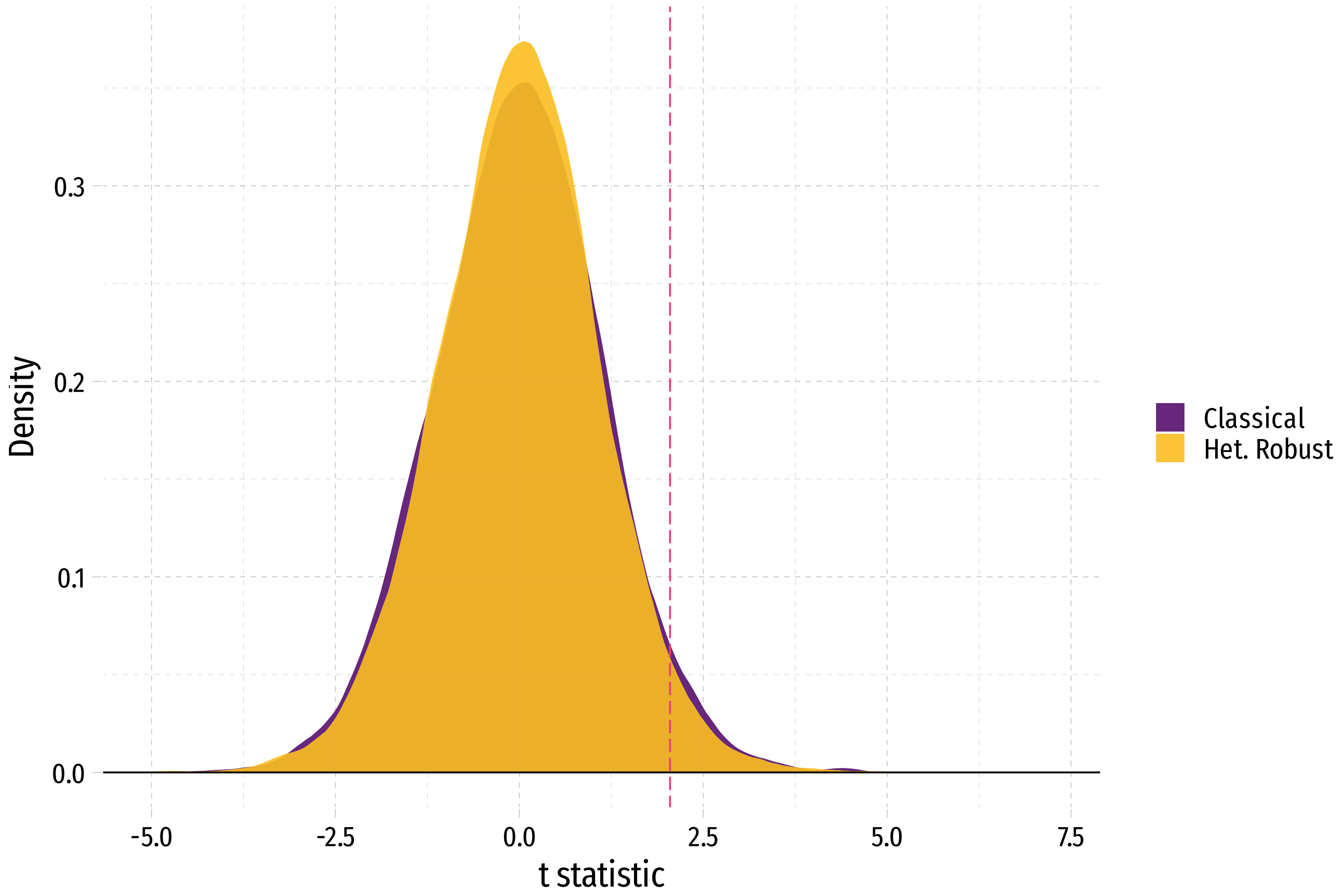
Now we run our new function `flex_iter()` 10,000 times

```
# Packages
p_load(purrr, furrr)
# Set options
set.seed(123)
# Tell R to parallelize
plan(multisession)
# Run 10,000 iterations
null_df = future_map(
  1:1e4, flex_iter,
  # Enforce the null hypothesis
  γ = 0,
  # Specify heteroskedasticity
  δ = 1,
  .options = furrr_options(seed = T)
) %>% bind_rows()
```

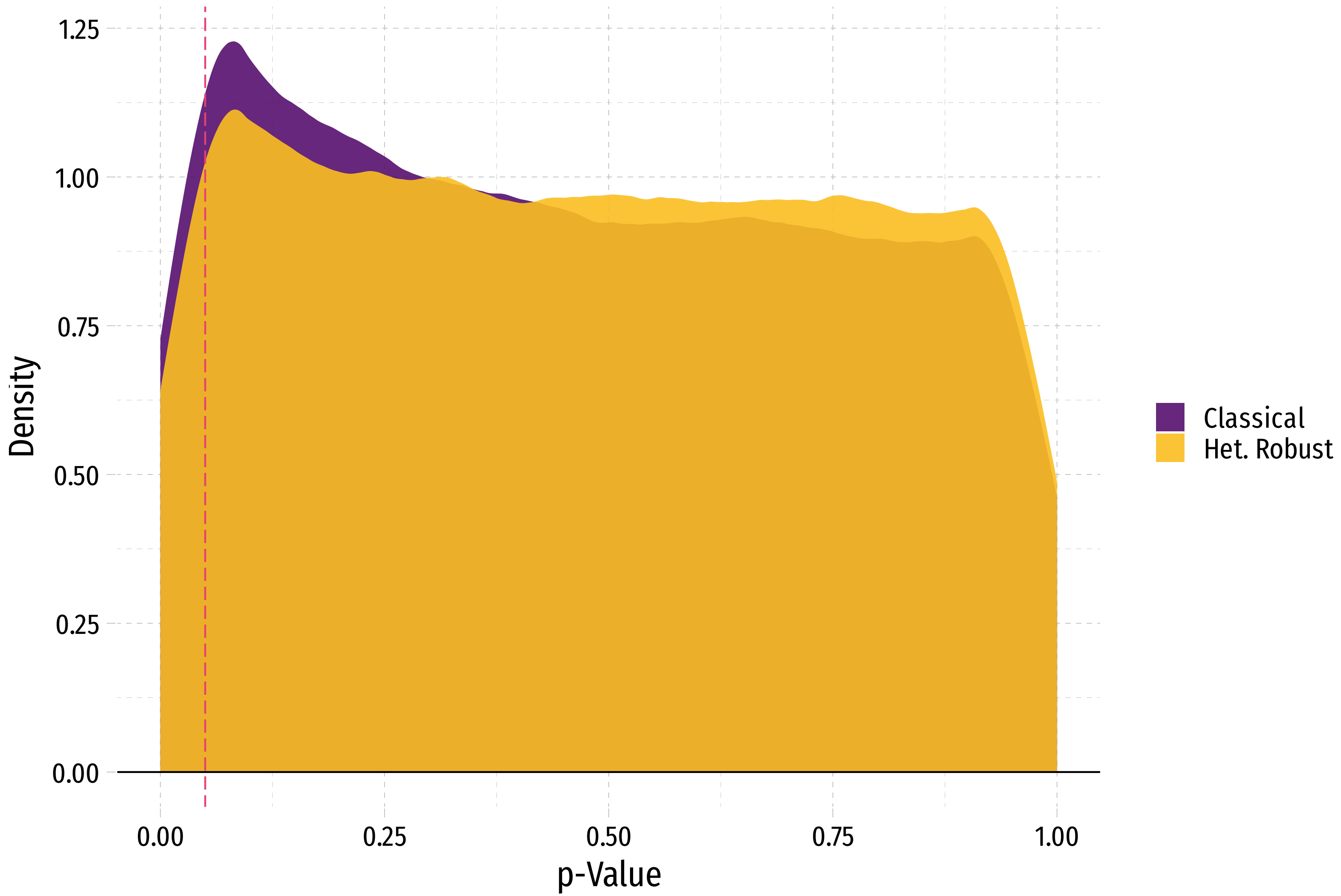
Comparing the distributions of standard errors for the coefficient on x



Comparing the distributions of t statistics for the coefficient on x



Distributions of *p*-values: both methods slightly over-reject the (true) null



Simulation

How did it go? (The sequel)

For a 5% test

- the **classical** SEs reject the **true value (zero)** in 7.73% of samples;
- the **het.-robust** SEs reject the **true value (zero)** in 6.68% of samples.

In this setting,

- **over-rejection** of the **true null** is a bit worse with IID SE estimator;
- **false precision** is *much worse*.

Summary

Wrapping up

While research often ignores it,
inference is just as important as identification.

Without understanding our **uncertainty** and the **population** onto which we draw inference, how can we learn anything from point estimates?

Wrapping up

While research often ignores it,
inference is just as important as identification.

Without understanding our **uncertainty** and the **population** onto which we draw inference, how can we learn anything from point estimates?

(Enter simulation)

Simulation is a fantastic tool for understanding estimators' behaviors.

Keep in mind: Simulation results impose (more) assumptions.

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