Controls

EC 607, Set 06

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Prologue

Schedule

Last time

The conditional independence assumption: $\{Y_{0i}, Y_{1i}\} \perp\!\!\!\perp D_i | X_i$
I.e., conditional on some controls (X_i), treatment is as-good-as random.

Today

- Omitted variable bias
- Good and bad controls

Upcoming

- DAGS
- Matching estimators

Omitted-variable bias

Omitted-variable bias

Revisiting an old friend

Let's start where we left off: Returns to schooling.

We have two linear, population models

$$Y_i = \alpha + \rho s_i + \eta_i \tag{1}$$
$$Y_i = \alpha + \rho s_i + X_i'\gamma + \nu_i \tag{2}$$

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For model (2), we can interpret $\hat{\rho}$ causally ***if*** $\mathrm{Y}_{si} \perp\!\!\!\perp \mathrm{s}_i | \mathrm{X}_i$ (CIA).

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For model (2), we can interpret $\hat{\rho}$ causally **if** $Y_{si} \perp\!\!\!\perp s_i | X_i$ (CIA).

In other words, the CIA says that our **observable vector X_i must explain all of correlation between s_i and η_i**.

Omitted-variable bias

The OVB formula

We can use the omitted-variable bias (OVB) formula to compare regression estimates from **models with different sets of control variables**.

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We're concerned about selection and want to use a set of control variables to account for ability $(\mathbf{A}_i)$—family background, motivation, intelligence.

$$\mathbf{Y}_i = \alpha + \beta \mathbf{s}_i + v_i \qquad (1)$$
$$\mathbf{Y}_i = \pi + \rho \mathbf{s}_i + \mathbf{A}_i'\gamma + e_i \qquad (2)$$

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What happens if we can't get data on A_i and opt for (1)?

$$\frac{\text{Cov}(Y_i,\, s_i)}{\text{Var}(s_i)} = \rho + \gamma' \delta_{As}$$

where δ_{As} are coefficients from regressing A_i on s_i.

Omitted-variable bias

Interpretation

Our two regressions

$$\mathrm{Y}_i = \alpha + \beta \mathrm{s}_i + v_i \tag{1}$$
$$\mathrm{Y}_i = \pi + \rho \mathrm{s}_i + \mathrm{A}'_i \gamma + e_i \tag{2}$$

will yield the same estimates for the returns to schooling

$$\frac{\mathrm{Cov}(\mathrm{Y}_i,\, \mathrm{s}_i)}{\mathrm{Var}(\mathrm{s}_i)} = \rho + \gamma' \delta_{As}$$

if (**a**) schooling is uncorrelated with ability ($\delta_{As} = 0$) *or* (**b**) ability is uncorrelated with earnings, conditional on schooling ($\gamma = 0$).

Omitted-variable bias

Example

Table 3.2.1, The returns to schooling

	1	2	3	4
Schooling	0.132	0.131	0.114	0.087
	(0.007)	(0.007)	(0.007)	(0.009)
Controls	None	Age Dum.	2 + Add'l	3 + AFQT

Here we have four specifications of controls for a regression of log wages on years of schooling (from the NLSY).

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Column 1 (no control variables) suggests a 13.2% increase in wages for an additional year of schooling.

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Column 2 (age dummies) suggests a 13.1% increase in wages for an additional year of schooling.

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Column 3 (column 2 controls plus parents' ed. and self demographics) suggests a 11.4% increase in wages for an additional year of schooling.

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	1	**2**	**3**	**4**
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Column 4 (column 3 controls plus AFQT[†] score) suggests a 8.7% increase in wages for an additional year of schooling.

† AFQT is *Armed Forces Qualification Test.*

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As we ratchet up controls, the estimated returns to schooling drop by 4.5 percentage points (34% drop in the coefficient) from **Column 1** to **Column 4**.

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If we think **ability positively affects wages**, then it looks like we also have **positive selection into schooling**.

Omitted: X_2 and X_3
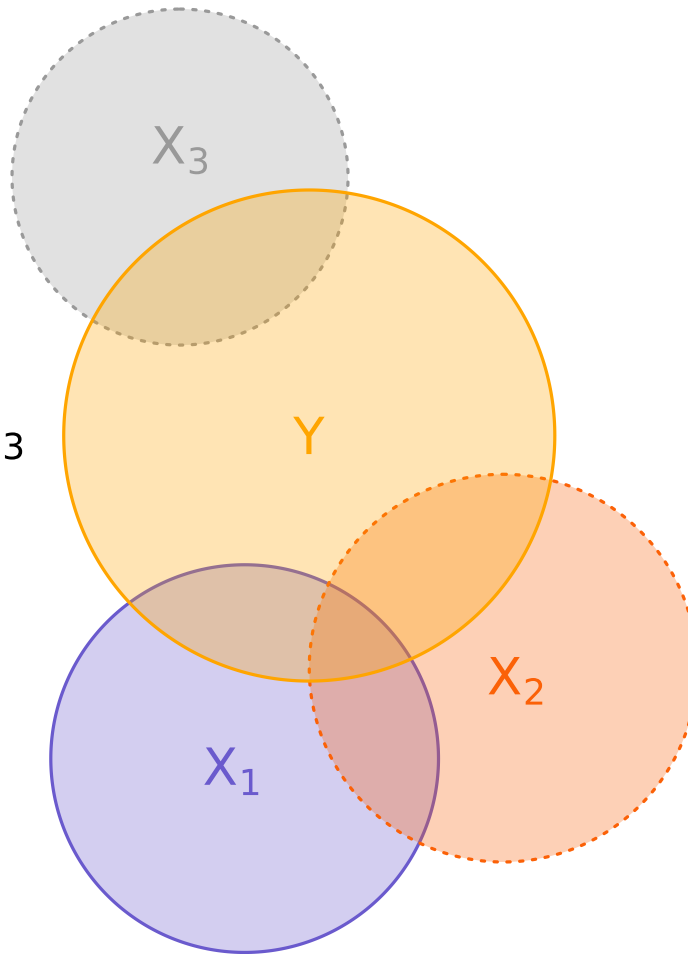
Omitted-variable bias

Note

This OVB formula **does not** require either of the models to be causal.

The formula compares the regression coefficient in a **short model** to the regression coefficient on the same variable in a **long model**.[†]

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† Here, *long model* refers to a model with more controls than the *short model*.

Omitted-variable bias

The OVB formula and the CIA^{†}

In addition to helping us think through and sign OVB, the formula

$$\frac{\mathrm{Cov}(\mathbf{Y}_i,\, \mathbf{s}_i)}{\mathrm{Var}(\mathbf{s}_i)} = \rho + \gamma' \delta_{As}$$

drives home the point that we're leaning *very* hard on the conditional independence assumption to be able to interpret our coefficients as causal.

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Q When is the CIA plausible?

A Two potential answers

   1. Randomized experiments
   2. Programs with arbitrary cutoffs/lotteries

[†] The title for my first spy novel.

Control variables play an enormous role in our quest for causality (the CIA).

Q Are "more controls" always better (or at least never worse)?

A No. There are such things as…

Bad controls

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Defined

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Let's consider an example...

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Example

Suppose we want to know the **effect of college graduation on wages**.

1. There are only two types of jobs: blue collar and white collar.
2. White-collar jobs, on average, pay more than blue-collar jobs.
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A No. Imagine college degrees are randomly assigned. When we condition on occupation, we compare degree-earners who chose blue-collar jobs to non-degree-earners who chose blue-collar jobs. Our assumption of random degrees says **nothing** about random job selection.

Bad controls can undo valid randomizations.

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Formal-ish derivation

More formally, let

- W_i be a dummy for whether i has a white-collar job
- Y_i denote i's earnings
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Because we've assumed C_i is randomly assigned, differences in means yield causal estimates, *i.e.*,

$$E[Y_i \mid C_i = 1] - E[Y_i \mid C_i = 0] = E[Y_{1i} - Y_{0i}]$$
$$E[W_i \mid C_i = 1] - E[W_i \mid C_i = 0] = E[W_{1i} - W_{0i}]$$

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$$= \underbrace{E[\mathrm{Y}_{1i} - \mathrm{Y}_{0i} \mid \mathrm{W}_{1i} = 1]}_{\text{Causal effect on white-collar workers}} + \underbrace{E[\mathrm{Y}_{0i} \mid \mathrm{W}_{1i} = 1] - E[\mathrm{Y}_{0i} \mid \mathrm{W}_{0i} = 1]}_{\text{Selection bias}}$$

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Formal-ish derivation, continued

By introducing a bad control, we introduced selection bias into a setting that did not have selection bias without controls.

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Specifically, the selection bias term

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describes how college graduation changes the composition of the pool of white-collar workers.

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Note Even if the causal effect is zero, this selection bias need not be zero.

Bad controls

A trickier example

A timely/trickier example: Wage gaps (*e.g.*, female-male or black-white).

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- What are we trying to capture?

- If we're concerned about discrimination, it seems likely that discrimination also affects occupational choice and hiring outcomes.

- Some motivate occupation controls with groups' differential preferences.

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What's the answer?

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Proxy variables

Angrist and Pischke bring up an interesting scenario that intersects omitted-variable bias and bad controls.

- We want to estimate the returns to education.
- Ability is omitted.
- We have a proxy for ability—a test taken after schooling finishes.

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We're a bit stuck.

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2. If we include our proxy, we've got a bad control.

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With some math/luck, we can bound the true effect with these estimates.

Bad controls

Example

Returning to our OVB-motivated example, we control for occupation.

<div align="center">

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	1	2	3	4	5
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Controls	None	Age Dum.	2 + Add'l	3 + AFQT	4 + Occupation

</div>

Schooling likely affects occupation; how do we interpret the new results?

Bad controls

Conclusion

Timing matters.

The right controls can help tremendously, but bad controls hurt.

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