HW2

(1) [**20 points**] Bayesian Chicken Farmer

The Cost Table: $\lambda(a|\omega)$

Action a \ State ω	*healthy*	*sick*
sell	-$1	$500
kill	$0.5	$0.5

Probabilities:

$P(\omega_{Healthy}) = 0.99, \quad P(\omega_{Sick}) = 0.01$

$P(t_{Normal} \mid \omega_{Healthy}) = 0.99, \quad P(t_{High} \mid \omega_{Healthy}) = 0.01$

$P(t_{High} \mid \omega_{Sick}) = 0.99, \quad P(t_{Normal} \mid \omega_{Sick}) = 0.01$

(1.1) [**10 points**] Compute the total risks of the following three decision functions

1. *Sell* regardless of t: $f_1(t) = sell$
2. *Kill* regardless of t: $f_2(t) = kill$
3. $f_3(t_{Normal}) = sell$, $f_3(t_{High}) = kill$

(1.2) [**10 points**) Assuming you are a policy maker and the above f_3 is chosen, to allow this industry to be profitable, what is the upper limit of $\lambda(a{=}sell \mid \omega{=}sick)$ if the rest costs remain the same? Explain your choice.

(2) [**20 points**] Consider the following decision rule for a two-category one-dimensional problem: Classify $\vec{x}$ into class 1 if $x > \tau_0$; otherwise classify x into class 2.

(2.1) [**5 points**] Show that the probability of error for this rule is given by:

$$P(error) = P(\omega_1)\int_{-\infty}^{\tau_0} p(x|\omega_1)\,dx + P(\omega_2)\int_{\tau_0}^{\infty} p(x|\omega_2)\,dx$$

(2.2) [**10 points**] Show that a necessary condition to minimize $P(error)$ is that τ_0 satisfies:

$$P(\omega_1)p(\tau_0|\omega_1) = p(\tau_0|\omega_2)P(\omega_2)$$

(2.3) [**5 points**] Give an example where a value of τ_0 satisfying the condition defined in (2.2) actually maximizes $P(error)$.

(3) [**40 points**] In many pattern classification problems, one has the option either to assign the pattern to one of c classes, or to reject it as being unrecognizable. Let

$$\lambda(a_i|\omega_j) = \begin{cases} 0 & i = j;\, 1 \le i,j \le c \\ \lambda_0 & reject \\ \lambda_s & otherwise \end{cases}$$

(3.1) [**10 points**] Show the minimal risk is obtained if we classify x into the i-th class if $P(\omega_i|x) \ge P(\omega_j|x)$ for all j and if $P(\omega_i|x) \ge 1 - \lambda_0/\lambda_s$, and reject otherwise.

(3.2) **[15 points]** Show the following discriminant functions are optimal for such problems:

$$g_i(x) = \begin{cases} p(x|\omega_i)P(\omega_i) & i = 1, \dots, c \\ \dfrac{\lambda_s - \lambda_r}{\lambda_s}\displaystyle\sum_{j=1}^{c} p(x|\omega_j)P(\omega_j) & reject \end{cases}$$

(3.3) [**5 points**] What happens if $\lambda_r = 0$?

(3.4) [**5 points**] What happen if $\lambda_0 > \lambda_s$?

(3.5) **[5 points]** Describe qualitatively what happens as λ_0/λ_s is increase from 0 to 1.

(4) [20 points] Calculate the entropy of the following distribution

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(4.1) [10 points] $$p(x)=\frac{1}{(2\pi)^{1/2}\sigma}\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

(4.2) [10 points] $$p(\vec{x})=\frac{1}{(2\pi)^{D/2}\,|\Lambda|^{1/2}}\exp\left[-\frac{1}{2}(\vec{x}-\vec{\mu})^T\Lambda^{-1}(\vec{x}-\vec{\mu})\right]$$