Topic: Converting between degrees and DMS

Question: What is the measure, in DMS (degrees, minutes, and seconds), of the angle?

$$38.27°$$

Answer choices:

A $38°0'0.27''$

B $38°16'12''$

C $38°0.27'0''$

D $38°12'16''$

Solution: B

First, note that

$$38.27° = 38° + 0.27° = 38° + \left(\frac{27}{100}\right)°$$

Since $38°$ is the integer part of $38.27°$, in DMS the degrees part of the measure of this angle is $38°$. What remains is to convert the non-integer part of $38.27°$, which is $(27/100)°$, to minutes and seconds.

We'll first convert $(27/100)°$ from degrees to minutes, so we'll multiply the angle measure by 1, written as the conversion factor $(60')/(1°)$:

$$\left(\frac{27}{100}\right)° = \left(\frac{27}{100}\right)° (1)$$

$$\left(\frac{27}{100}\right)° = \left(\frac{27}{100}\right)° \left(\frac{60'}{1°}\right)$$

On the right-hand side of this equation, the $°$ in the numerator cancels against the $°$ in the denominator, so we get

$$\left(\frac{27}{100}\right)° = \left[\frac{27(60)}{100}\right]'$$

$$\left(\frac{27}{100}\right)° = \left(\frac{1{,}620}{100}\right)'$$

$$\left(\frac{27}{100}\right)° = \left(\frac{81}{5}\right)'$$

Note that

$$\frac{81}{5} = \frac{80+1}{5} = \frac{80}{5} + \frac{1}{5} = 16 + \frac{1}{5} = 16\frac{1}{5}$$

Substituting this result, we obtain

$$\left(\frac{27}{100}\right)^\circ = \left(16\frac{1}{5}\right)'$$

Thus the minutes part of an angle of measure $(27/100)^\circ$ is equal to the integer part of

$$16\frac{1}{5}'$$

which is $16'$. To get the seconds part of that angle, we need to convert the non-integer part, namely $(1/5)'$, to seconds. To do that, we'll multiply by 1, written as the conversion factor $(60'')/(1')$:

$$\left(\frac{1}{5}\right)' = \left(\frac{1}{5}\right)'(1)$$

$$\left(\frac{1}{5}\right)' = \left(\frac{1}{5}\right)'\left(\frac{60''}{1'}\right)$$

On the right-hand side of this equation, the $'$ in the numerator cancels against the $'$ in the denominator, so we get

$$\left(\frac{1}{5}\right)' = \left(\frac{60}{5}\right)''$$

$$\left(\frac{1}{5}\right)' = 12''$$

Substituting this result, we find that

$$\left(\frac{27}{100}\right)^{\circ} = \left(16\frac{1}{5}\right)' = 16' + \left(\frac{1}{5}\right)' = 16' + 12''$$

Thus

$$38.27° = 38°16'12''$$

Topic: Converting between degrees and DMS

Question: If the measure of an angle in DMS is given below, what is its measure in decimal degrees?

$$55°36'18''$$

Answer choices:

A 55.9°

B 55.54°

C 55.5°

D 55.605°

Solution: D

The degrees part of $55°36'18''$ is $55°$, so (in decimal degrees) the integer part of its measure is $55°$. What we still need to do is convert the measures of the minutes and seconds parts of the measure of the given angle to decimal degrees. We'll do that by converting those two parts separately, and then adding the results to get the non-integer part of the measure of the given angle.

Since there are 60 minutes in one degree, we can convert the minutes part of the given angle (i.e., $36'$) to decimal degrees by multiplying it by 1, written as the conversion factor $(1°)/(60')$:

$$36' = 36'(1)$$

$$36' = 36'\left(\frac{1°}{60'}\right)$$

$$36' = \left(\frac{36}{60}\right)^°$$

$$36' = \left(\frac{3}{5}\right)^°$$

Note that $(3/5) = 0.6$, so $36' = 0.6°$.

Since there are 60 minutes in one degree, and 60 seconds in one minute, we will use a product of two conversion factors, $(1°)/(60')$ and $(1')/(60'')$, to convert the seconds part of the given angle (i.e., $18''$) to decimal degrees:

$$18'' = 18''(1)(1)$$

$$18'' = 18'' \left(\frac{1°}{60'} \right) \left(\frac{1'}{60''} \right)$$

On the right-hand side of this equation, the ″ in the numerator cancels against the ″ in the denominator, and the ′ in the numerator cancels against the ′ in the denominator, so we are left with the following:

$$18'' = \left[\frac{18}{60(60)} \right]°$$

$$18'' = \left(\frac{18}{3,600} \right)°$$

$$18'' = \left(\frac{1}{200} \right)°$$

Note that $(1/200) = 0.005$, so $18'' = 0.005°$.

Adding the measures of the minutes and seconds parts of the given angle, each of which is now expressed in units of decimal degrees, we have

$$36'18'' = 36' + 18'' = 0.6° + 0.005° = 0.605°$$

When we include the integer part of the given angle, we obtain

$$55°36'18'' = 55° + 36'18'' = 55° + 0.605° = 55.605°$$