



# Trigonometry Formulas

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## Right triangle trigonometry (SOH CAH TOA)

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\csc \theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\cot \theta = \frac{\text{adjacent}}{\text{opposite}}$$

## Trigonometric functions

$$\sin \theta = \frac{y}{r}$$

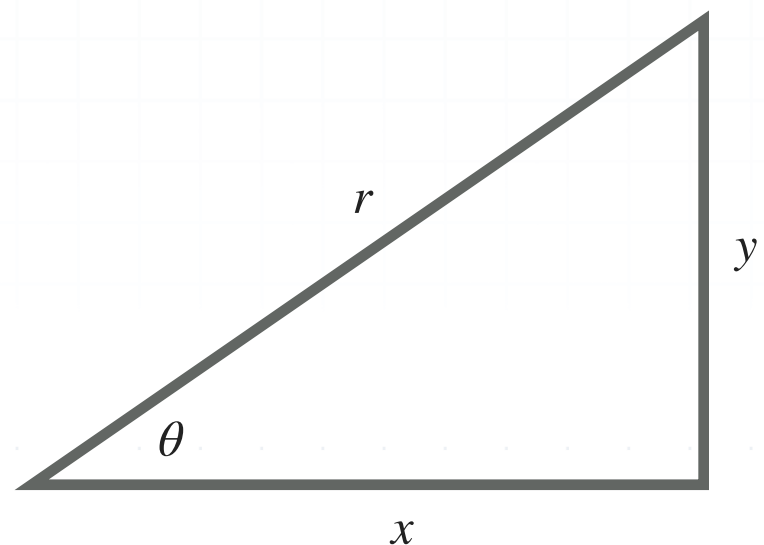
$$\csc \theta = \frac{r}{y}$$

$$\cos \theta = \frac{x}{r}$$

$$\sec \theta = \frac{r}{x}$$

$$\tan \theta = \frac{y}{x}$$

$$\cot \theta = \frac{x}{y}$$



## Transformations

### Vertical and horizontal shifting

Assume  $c > 0$

to shift a function  $c$  units upward, find  $y = f(x) + c$

to shift a function  $c$  units downward, find  $y = f(x) - c$



to shift a function  $c$  units to the right, find  $y = f(x - c)$

to shift a function  $c$  units to the left, find  $y = f(x + c)$

## Vertical and horizontal stretching and reflecting

Assume  $c > 1$

to stretch vertically by a factor of  $c$ , find  $y = cf(x)$

to shrink vertically by a factor of  $c$ , find  $y = (1/c)f(x)$

to stretch horizontally by a factor of  $c$ , find  $y = f(x/c)$

to shrink horizontally by a factor of  $c$ , find  $y = f(cx)$

to reflect about the  $x$ -axis, find  $y = -f(x)$

to reflect about the  $y$ -axis, find  $y = f(-x)$

## Reciprocal identities

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$



## Quotient identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

## Pythagorean identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

## Even-odd identities

$$\sin(-\theta) = -\sin \theta$$

$$\csc(-\theta) = -\csc \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\sec(-\theta) = \sec \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\cot(-\theta) = -\cot \theta$$

## Co-function identities

$$\sin \left( \frac{\pi}{2} - \theta \right) = \cos \theta$$

$$\cos \left( \frac{\pi}{2} - \theta \right) = \sin \theta$$



$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta$$

## Sum-difference identities

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

## Double-angle identities

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x = 2 \cos^2 x - 1$$

$$\cos 2x = 1 - 2 \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

## Half-angle (power reducing) identities

$$\sin^2 x = \frac{1 - \cos 2x}{2} = \frac{1}{2}(1 - \cos 2x)$$



$$\cos^2 x = \frac{1 + \cos 2x}{2} = \frac{1}{2}(1 + \cos 2x)$$

$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

## Product-to-sum identities

$$\sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)]$$

$$\sin x \cos y = \frac{1}{2} [\sin(x + y) + \sin(x - y)]$$

$$\cos x \cos y = \frac{1}{2} [\cos(x - y) + \cos(x + y)]$$

$$\cos x \sin y = \frac{1}{2} [\sin(x + y) - \sin(x - y)]$$

## Sum-to-product identities

$$\sin x + \sin y = 2 \sin \left( \frac{x + y}{2} \right) \cos \left( \frac{x - y}{2} \right)$$

$$\sin x - \sin y = 2 \cos \left( \frac{x + y}{2} \right) \sin \left( \frac{x - y}{2} \right)$$

$$\cos x + \cos y = 2 \cos \left( \frac{x + y}{2} \right) \cos \left( \frac{x - y}{2} \right)$$



$$\cos x - \cos y = -2 \sin \left( \frac{x+y}{2} \right) \sin \left( \frac{x-y}{2} \right)$$

## Law of sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

## Law of cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$



# The Unit Circle

