1.

a)

$\exists \neg n P(2n^2+3n+1)$	Prove the negation
$\neg \exists n P(2n^2+3n+1)$	
$\forall n \neg P(2n^2+3n+1)$	De Morgan's
$x =$ an arbitrary integer	Assumption
$\neg P((x)^2+3(x)+1$	Universal instantiation
$\neg P((2x+1)(x+1)$	Distribution
$\therefore \forall \neg P(2n^2+3n+1)$	Is not prime (valid)
$\therefore \exists P(2n^2+3n+1)$	Invalid (Proof by negation)

Because the negation is proved to be true the hypothesis is invalid.

b)

$x = 0$	assumption
$P(2(0)^2+3(0)+2)$	
$P(2)$	
2 is prime	
$\therefore \exists P(2n^2+3n+1)$	Valid (Definition of $\exists$)

2.

$\frac{a}{b}$ =some rational number where a and b are integers	assumption/definition of a rational number
$\frac{n}{m}$ =some rational number where a and b are integers	assumption/definition of a rational number
$x =$ some irrational number	assumption
$\frac{a}{b} - x \;=$	$\frac{n}{m}$
$-x \;=$	$\frac{n}{m} - \frac{a}{b}$
$x \;=$	$\frac{a}{b} - \frac{n}{m}$
$x \;=$	$\frac{am-nb}{bm}$
$\frac{am-nb}{bm}$	by definition is a rational number
$\therefore \frac{a}{b} - x$	cannot be irrational via contradiction above

3.

$\|x\| + \|y\| \geq \|x+y\|$	assumption
$\|n\| \;=$	$\binom{n \geq 0 \rightarrow n}{n<0 \rightarrow (-n)}$

Case 1.

$x, y \geq 0 \Rightarrow \|x\| + \|y\| : x+y$	definition of absolute value
$x, y \geq 0 \Rightarrow x+y \geq 0$	
$\therefore x+y = \|x+y\|$	definition of absolute value

Case 2.

$x, y < 0 \Rightarrow \|x\| + \|y\| : (-x)+(-y)$	definition of absolute value
$(-x)+(-y) = -(x+y)$	
$x, y < 0 \Rightarrow -(x+y) < 0$	
$\therefore -(x+y) = \|x+y\|$	definition of absolute value

Case 3.

$x \geq 0, y < 0 \Rightarrow x + (-y)$	definition of absolute value
$y < 0 \Rightarrow x + -(-y) \Rightarrow x+y > 0$	
$x+y > 0 \Rightarrow \|x+y\|$	definition of absolute value
$\|x+y\| \Rightarrow x+y \geq 0$ or $-(x+y) < 0$	definition of absolute value
$\because \|x+y\| < 0$ or ≥ 0 and $\|x\| + \|y\| = \|x+y\|$ and $\|x\| + \|y\| > 0$	
$\therefore x + (-y) \geq \|x+y\|$	

Case 4.

$x < 0, y \geq 0 \Rightarrow (-x) + y$	definition of absolute value
$x < 0 \Rightarrow -(-x) + y \Rightarrow x + y > 0$	
$x + y > 0 \Rightarrow \|x + y\|$	definition of absolute value
$\|x + y\| \Rightarrow x + y \geq 0$ or $-(x + y) < 0$	definition of absolute value
$\because \|x + y\| < 0$ or ≥ 0 and $\|x\| + \|y\| = \|x + y\|$ and $\|x\| + \|y\| > 0$	
$\therefore (-x) + y \geq \|x + y\|$	

Because all the cases of $|x| + |y|$ are either $\geq$ or $=$, $|x| + |y| \geq |x + y|$

4.

$$
\begin{array}{rcl}
E(x) & = & x \text{ is even} \\
O(x) & = & x \text{ is odd} \\
E(x) & = & \neg O(x) \\
O(x) & = & \neg E(x) \\
E(x^2 + x + 1) \rightarrow O(x) & & \\
\neg O(x) \rightarrow \neg E(x^2 + x + 1) & & \text{start proof by contrapositive} \\
E(x) \rightarrow O(x^2 + x + 1) & & \text{definition} \\
k = \text{some arbitrary int} & & \text{assumption} \\
2k = \text{even } 2k + 1 = \text{odd} & & \text{definition of odd and even} \\
2k^2 + 2k + 1 = (2k^2 + 2k) + 1 & & \\
2k^2 + 2k = \text{some integer } +1 = 2k + 1 & & \text{definition of odd} \\
\hline
\therefore E(x^2 + x + 1) \rightarrow O(x) & & \text{proof by contrapositive}
\end{array}
$$

5.

$3a + 5b = 7k$	$3c + 5d = 7k$
$3(a + c) + 5(b + d) = 7k$	$k =$some integer
$3a + 3c + 5b + 5d = 7k$	distribute
$3a + 3b + 3c + 5d = 7k$	
$7k + 7k = 7k$	definition
$7(k + k) = 7k$	distribution
$k + k = someinteger$	definition
$\therefore 3(a + c) + 5(b + d) = 7k$	valid