Your name goes here

Assignment 1

Due BEFORE 8:00AM on Monday 9/10/2018

On time / 20% off / no credit

Total points: 85

This is an **individual** assignment. You must work alone and submit your own work.

This assignment will test your knowledge from CS 212 and CS 271. You must write up your solutions to this assignment IN THIS FILE using LaTeX by filling in all of the boxes below. If your submitted `.tex` file does not compile, then you will receive 0 points.

All of the proofs that you write for this and subsequent assignments should be **as detailed as possible**. You must write EACH step on its own line so that it is very easy to see how each step follows directly from the previous step. Furthermore, be sure to explain/justify each step in your proof. For example, state that you added 5 to each side, that you simplified a fraction, etc. Do NOT skip a bunch of algebraic steps in your proofs. For example, do not simply say $\frac{k(k+1)}{2} + (k+1) = \frac{(k+1)(k+2)}{2}$. It is a true statement, but it is not trivial. It is not obvious that the left hand side equals the right hand side. If you are unsure whether a step is needed or not, ask me first before omitting it. Use the symbol $\square$ (i.e., `\qed` in your `.tex` file) to denote the end of each proof.

Here is an example of proper justification of algebraic steps and formatting of a sequence of equations:

$$\begin{aligned}
\frac{k(k+1)}{2} + (k+1) &= \frac{k(k+1)}{2} + \frac{2(k+1)}{2} && \text{unified denominators} \\
&= \frac{k(k+1) + 2(k+1)}{2} && \text{added fractions} \\
&= \frac{(k+1)(k+2)}{2} && \text{factored out common factor}
\end{aligned}$$

Note how this list of equations is aligned vertically into three columns around the equal signs and with the justifications on the right. This is the REQUIRED format for your submissions whenever applicable.

You should NOT add any LaTeX packages to your `.tex` file. Note that I included the `qtree` package and included its documentation in the handout. You must use this package to produce your trees for problem 7.

Submission procedure:

1. Complete this file, called `a1.tex`, with your full name and answers typed up below.
2. Compile this file to produce a file called `a1.pdf`. Make sure that this file compiles properly and that its contents and appearance meet the requirements described in this handout.
3. Create a directory called `a1` and copy exactly two files into this directory, namely:
   - `a1.tex` (this file with all of your answers added)
   - `a1.pdf` (the compiled version of the file above)

4. Zip up this directory to yield a file called `a1.zip`
5. Submit this zip file to the D2L dropbox for A1 before the deadline above.
6. Submit a single-sided, hard copy of your `a1.pdf` file BEFORE the beginning of class on the due date above.

Problem statements

1. **(10 points) Write a direct proof of the following statement:**
   for all $n \in \mathbb{N}$, if n is odd, then n^3 is odd

   Direct proof:

   Your proof goes here

2. **(10 points) State and disprove the contrapositive of the following statement:**
   for all $m, n \in \mathbb{N}$, if $mn < 361$, then $m < 19$ and $n < 19$

   Contrapositive:

   Write the contrapositive here

   Proof:

   Your proof goes here

3. **(10 points) Write a proof by contradiction of the following statement:**
   for all $m, n, p \in \mathbb{N}$, if $m^2 + n^2 = p^2$, then at least one of m and n is even

   Proof by contradiction:

   Your proof goes here

4. **(10 points) Prove by induction that $P(n)$ holds for $n \geq 2$, where $P(n)$ is:**

$$\prod_{i=2}^{n} \left(1 - \frac{1}{i^2}\right) = \frac{n+1}{2n}$$

   Proof:

   Your proof goes here

5. **(10 points) If F_n is defined as follows:**

$$F_n = \begin{cases} 1 & n = 1 \\ 1 & n = 2 \\ F_{n-1} + F_{n-2} & n \geq 3 \end{cases}$$

prove by induction that $P(n)$ holds for $n \geq 1$, where $P(n)$ is:

$$\sum_{i=1}^{n} F_i^2 = F_n \cdot F_{n+1}$$

Proof:

Your proof goes here

6. **(10 points) Let S be the subset of $\mathbb{Z} \times \mathbb{Z}$ defined recursively/inductively by the following rules:**

R_0: $(0, 0)$ **is in** S

R_1: **If** (a, b) **is in** S**, then** $(a, b + 1)$ **is also in** S

R_2: **If** (a, b) **is in** S**, then** $(a + 1, b + 1)$ **is also in** S

R_3: **If** (a, b) **is in** S**, then** $(a + 2, b + 1)$ **is also in** S

R_4: S **only contains elements that are generated from rules** R_0 **through** R_3 **above**

Prove, using structural induction, that $\forall (a, b) \in S$, $a \leq 2b$.

Proof:

Your proof goes here

7. **(10 points) Insert the following elements, in this order, into an AVL tree: 5, 20, 2, 7, 13, 4, 22 to be able to answer the following questions.**

(7 points) [Part a] Show the tree after each insertion. For each step, if a rotation is needed, you must show the tree both before and after the rotation. In all other steps, only one tree must be included.

- Insert 5:

  Your tree (or trees) goes here

- Insert 20:

  Your tree (or trees) goes here

- Insert 2:

  Your tree (or trees) goes here

- Insert 7:

  Your tree (or trees) goes here

- Insert 13:

  Your tree (or trees) goes here

- Insert 4:

  Your tree (or trees) goes here

- Insert 22:

  Your tree (or trees) goes here

(1 point) [Part b] What is the depth of node 4 in the final tree?

The depth of node 4 is :

your answer goes here

(1 point) [Part c] Write down, on one line, all of the node values in the preorder traversal of this tree.

Preorder traversal:

your answer goes here

(1 point) [Part d] Write down, on one line, all of the node values in the postorder traversal of this tree.

Postorder traversal:

your answer goes here

8. **(5 points) Insert the following elements, in this order, into a max heap:**
5, 20, 2, 7, 13, 4, 22
Show all of the intermediate steps, i.e., show the heap after each element is inserted, by filling in the following table, in which each row represents the 1-indexed heap array after an additional insertion.

Write down your complete answer in the following table making sure that empty array locations remain empty cells in the table.

	1	2	3	4	5	6	7
insert 5							
insert 20							
insert 2							
insert 7							
insert 13							
insert 4							
insert 22							

9. **(5 points) Let $G = (V, E, w)$ be an undirected graph, with vertex set $V = \{1, 2, 3, 4, 5, 6, 7\}$, edge set $E = \{(1, 2), (1, 4), (1, 6), (2, 3), (2, 4), (3, 4), (3, 5), (4, 5), (4, 6), (4, 7), (5, 7), (6, 7)\}$ and weight function $w : E \to \mathbb{N}$, such that $w(1, 4) = 6$, $w(1, 6) = 8$, $w(2, 1) = 1$, $w(2, 3) = 4$, $w(2, 4) = 5$, $w(3, 4) = 5$, $w(3, 5) = 2$, $w(4, 5) = 9$, $w(4, 6) = 6$, $w(4, 7) = 6$, $w(5, 7) = 3$, and $w(6, 7) = 7$. What is the total edge weight of a minimal spanning tree of G?**

Your answer goes here

10. **(5 points) Let $G = (V, E)$ be a DIRECTED graph, where V and E are from the previous problem and the direction of the edges is given by the order of the vertices in the weight function in the previous problem (e.g., $w(2, 1)$ means that the edge between vertices 1 and 2 is directed from 2 to 1). Does G have a topological ordering? If so, provide such an ordering. If not, explain why not.**

Your answer goes here