1

Assumptions

$$\begin{aligned} x = \text{Oliver} = f(x) &= \text{Oliver is a CS Major} \\ m(x) &= \text{Oliver is a Math Major} \\ t(x) &= \text{Oliver is required to take Math 333} \end{aligned}$$

Argument

$$\begin{array}{ll} f(x) \vee m(x) & Proposition 1 \\ m(x) \rightarrow t(x) & Proposition 2 \\ \hline \therefore f(x) \vee \neg t(x) & \end{array}$$

Proof

$$\begin{array}{lll} \text{Conclusion 1} & \neg f(x) \rightarrow m(x) & \text{Conditional Identity, Proposition 1} \\ \text{Conclusion 2} & \neg f(x) \rightarrow t(x) & \text{Hypothetical Syllogism, Proposition 2, Conclusion 1} \\ \text{Conclusion 3} & \neg\neg f(x) \vee t(x) & \text{Conditional Identity, Conclusion 2} \\ \text{Conclusion 4} & f(x) \vee t(x) & \text{Double Negation Law, Conclusion 3} \\ \hline \because f(x) \vee t(x) & \neq f(x) \vee \neg t(x) & \therefore Invalid \end{array}$$

Truth Table

$f(x)$	$m(x)$	$t(x)$	$f(x) \vee m(x)$	$m(x) \rightarrow t(x)$	$f(x) \vee \neg t(x)$
0	0	0	0	1	1
1	0	0	1	1	1
1	1	0	1	0	1
1	1	1	1	1	1
0	1	1	1	1	0
0	0	1	0	1	0
1	0	1	1	1	1

Conclusion

The truth table above shows that this is an invalid argument. Specifically rows 1, 3, and 5 show that it is invalid because: 1. the conclusion is false when the hypothesis is true 2. the conclusion is true when the hypothesis is false.

2

a)

$$\exists y \forall x Triangle(y) \wedge \big(Square(x) \rightarrow \neg ToTheRightOf(y,x)\big)$$

b)

$$\begin{array}{ll} \neg\exists y \forall x \Big(Triangle(y) \wedge \big(Square(x) \rightarrow \neg ToTheRightOf(y,x)\big)\Big) & \text{Premise} \\ \forall y \neg\forall x \Big(Triangle(y) \wedge \big(Square(x) \rightarrow \neg ToTheRightOf(y,x)\big)\Big) & \text{De Morgan's} \\ \forall y \exists x \neg \Big(Triangle(y) \wedge \big(Square(x) \rightarrow \neg ToTheRightOf(y,x)\big)\Big) & \text{De Morgan's} \\ \forall y \exists x \Big(\neg Triangle(y) \vee \neg\big(Square(x) \rightarrow \neg ToTheRightOf(y,x)\big)\Big) & \text{De Morgan's} \\ \forall y \exists x \Big(\neg Triangle(y) \vee \neg\big(\neg Square(x) \vee \neg ToTheRightOf(y,x)\big)\Big) & \text{Conditional Identity} \\ \forall y \exists x \Big(\neg Triangle(y) \vee \big(\neg\neg Square(x) \vee \neg\neg ToTheRightOf(y,x)\big)\Big) & \text{De Morgan's} \\ \forall y \exists x \Big(\neg Triangle(y) \vee \big(Square(x) \vee ToTheRightOf(y,x)\big)\Big) & \text{Double Negation} \end{array}$$

c)

All polygons are not triangles or there exists a square or all polygons are to the right of a polygon.

3

a)

$$\begin{array}{rclrcl} t(x) &=& \text{trust } x & a(x) &=& x \text{ is admitted to study} \\ o(x) &=& \text{own } x & b(x) &=& x \text{ begs} \\ d(x) &=& x \text{ is a dog} & y(x) &=& x \text{ is in the yard} \\ g(x) &=& x \text{ gnaws bones} & c(x) &=& x \text{ is told to beg} \end{array}$$

b)

$$\forall x\big(g(x) \land y(x)\big)$$

c)
Hypothesis

$$\begin{array}{ll} \forall x\big(o(x) \rightarrow t(x)\big) & \text{Proposition 1} \\ \forall x\big(d(x) \rightarrow g(x)\big) & \text{Proposition 2} \\ \forall x\Big(a(x) \rightarrow \big(c(x) \land b(x)\big)\Big) & \text{Proposition 3} \\ \forall x\big(y(x) \rightarrow o(x)\big) & \text{Proposition 4} \\ \forall x\big(t(x) \rightarrow a(x)\big) & \text{Proposition 5} \\ \forall x\Big(\big(c(x) \land b(x)\big) \rightarrow d(x)\Big) & \text{Proposition 6} \\ \hline \therefore \forall x\big(y(x) \rightarrow g(x)\big) & \end{array}$$

Proof

Conclusion 1	x is an arbitrary element, all animals	Assumption
Conclusion 2	$o(x) \rightarrow t(x)$	Universal Instansiation, Con 1, Prop 1
Conclusion 3	$d(x) \rightarrow g(x)$	Universal Instansiation, Con 1, Prop 2
Conclusion 4	$a(x) \rightarrow \big(c(x) \land b(x)\big)$	Universal Instansiation, Con 1, Prop 3
Conclusion 5	$y(x) \rightarrow o(x)$	Universal Instansiation, Con 1, Prop 4
Conlcusion 6	$t(x) \rightarrow a(x)$	Universal Instansiation, Con 1, Prop 5
Conclusion 7	$\big(c(x) \land b(x)\big) \rightarrow d(x)$	Universal Instansiation, Conc 1, Prop 6
Conclusion 8	$y(x) \rightarrow t(x)$	Hypothetical Syllogism, Con 5, Con 2
Conclusion 9	$y(x) \rightarrow a(x)$	Hypothetical Syllogism, Con 8, Con 6
Conclusion 10	$y(x) \rightarrow \big(c(x) \land b(x)\big)$	Hypothetical Syllogism, Con 9, Con 4
Conclusion 11	$y(x) \rightarrow d(x)$	Hypothetical Syllogism, Con 10, Con 7
Conclusion 12	$y(x) \rightarrow g(x)$	Hypothetical Syllogism, Con 11, Con 3
$\because y(x) \rightarrow g(x) = y(x) \rightarrow g(x)$	$\therefore Valid$	

4

a)
No deductive errors
b)

P	Q	$P \rightarrow Q$	Q	P
0	0	1	0	0
1	0	0	0	1
1	1	1	1	1
0	1	1	1	0

It is invalid because: 1. there is a Hypothesis where all of the Proposition are false but the Conclusion is true 2. there is a hypothesis where all the Propositions are true but the Conclusion is false.
c)

P	Q	R	$P \rightarrow R$	$Q \rightarrow R$	$P \rightarrow Q$
0	0	0	1	1	1
1	0	0	0	1	0
1	1	0	0	0	1
1	1	1	1	1	1
0	1	1	1	1	1
0	0	1	1	1	1
1	0	1	1	1	0

It is invalid because: 1. there is a Hypothesis where all of the Proposition are false but the Conclusion is true 2. there is a hypothesis where all the Propositions are true but the Conclusion is false.