1.

a)

$\exists\neg nP(2n^2+3n+1)$	Prove the negation
$\neg\exists nP(2n^2+3n+1)$	
$\forall n\neg P(2n^2+3n+1)$	De Morgan's
$x =$ an arbitrary integer	Assumption
$\neg P((x)^2+3(x)+1$	Universal instantiation
$\neg P((2x+1)(x+1)$	Distribution
$\therefore \forall\neg P(2n^2+3n+1)$	Is not prime (valid)
$\therefore \exists P(2n^2+3n+1)$	Invalid (Proof by negation)

Because the negation is proved to be true the hypothesis is invalid.

b)

$x = 0$	assumption
$P(2(0)^2+3(0)+2)$	
$P(2)$	
$2isprime$	
$\therefore \exists P(2n^2+3n+1)$	Valid (Definition of $\exists$)

2.

$\frac{a}{b}$ =some rational number where a and b are integers	assumption/definition of a rational number
$\frac{n}{m}$ =some rational number where a and b are integers	assumption/definition of a rational number
$x = someirrationalnumber$	assumption
$\frac{a}{b}-x = \frac{n}{m}$	
$-x = \frac{n}{m}-\frac{a}{b}$	
$x = \frac{a}{b}-\frac{n}{m}$	
$x = \frac{am-nb}{bm}$	
$\frac{am-nb}{bm}$	by definition is a rational number
$\therefore \frac{a}{b}-x$	cannot be irrational via contradiction above

3.

$\|x\|+\|y\| \geq \|x+y\|$	assumption
$\|n\| =$	$\binom{n\geq 0\to n}{n<0\to(-n)}$

Case 1.

$x, y \geq 0 \to \|x\|+\|y\| : x+y$	
$x, y \geq 0 \to x+y \geq 0$	
$\therefore x+y = \|x+y\|$	definition of absolute value

Case 2.

$x, y < 0 \to \|x\|+\|y\| : x+y$	
$x, y < 0 \to x+y < 0$	
$\therefore x+y = \|x+y\|$	definition of absolute value

Case 3.

$x \geq 0, y < 0 \to \|x+y\| : \|x-y\|$	
$\|x-y\| \to x-y$ or $-(x-y)$	definition of absolute value
$x-y < -(x-y)$	w/ assumption $x \geq 0$ or $y < 0$
$\|x\|+\|y\| : x-y$	
$\therefore \|x\|+\|y\| \geq \|x+y\|$	w/ assumption $x \geq 0$ or $y < 0$

Case 4.

$$\begin{array}{ll}
y \geq 0, x < 0 \rightarrow |x+y| : \big|-(x)+y\big| & \\
\big|-(x)+y\big| \rightarrow (-x)+y \text{ or } -(-(x)+y) & \text{definition of absolute value} \\
(-x)+y < -((-x)+y) & \text{w assumption } y \geq 0 \text{ or } x < 0 \\
|x| + |y| : -(x)+y & \\
\hline
\therefore |x| + |y| \geq |x+y| & \text{w/ assumption } y \geq 0 \text{ or } x < 0
\end{array}$$

4.

$$\begin{array}{rcl}
E(x) & = & x \text{ is even} \\
O(x) & = & x \text{ is odd} \\
E(x) & = & \neg O(x) \\
O(x) & = & \neg E(x) \\
E(x^2+x+1) \rightarrow O(x) & & \\
\neg O(x) \rightarrow \neg E(x^2+x+1) & & \text{start proof by contrapositive} \\
E(x) \rightarrow O(x^2+x+1) & & \text{definition} \\
k = \text{some arbitrary int} & & \text{assumption} \\
2k = \text{even } 2k+1 = \text{odd} & & \text{definition of odd and even} \\
2k^2 + 2k + 1 = (2k^2+2k)+1 & & \\
2k^2 + 2k = \text{some integer } +1 = 2k+1 & & \text{definition of odd} \\
\hline
\therefore E(x^2+x+1) \rightarrow O(x) & & \text{proof by contrapositive}
\end{array}$$

5.

$$\begin{array}{ll}
3a + 5b = 7k & 3c + 5d = 7k \\
3(a+c) + 5(b+d) = 7k & k = someinteger \\
3a + 3c + 5b + 5d = 7k & \text{distribute} \\
3a + 3b + 3c + 5d = 7k & \\
7k + 7k = 7k & \text{definition} \\
7(k+k) = 7k & \text{distribution} \\
k + k = someinteger & \text{definition} \\
\hline
\therefore 3(a+c) + 5(b+d) = 7k & \text{valid}
\end{array}$$