1 Definition of *and*, *or*, and *not*

- $p \wedge q$ is true when both p is true and q is true, and in no other case.
- $p \vee q$ is true when either p is true, or q is true, or both p and q are true, and in no other case.
- $\neg p$ is true when p is false, and in no other case.

2 Definition of implication, bi-implication and exclusive or

For any propositions p and q, we define the propositions $p \to q$, $p \leftrightarrow q$, and $p \oplus q$ according to the truth table:

p	q	$p \to q$	$p \leftrightarrow q$	$p \oplus q$
false	false	true	true	false
false	true	true	false	true
true	false	false	false	true
true	true	true	true	false

3 Here's a nice proof, with a reason given for each step

$$\begin{aligned} p \wedge (p \to q) &\equiv p \wedge (\neg p \vee q) && \text{definition of } p \to q \\ &\equiv (p \wedge \neg p) \vee (p \wedge q) && \text{Distributive Law} \\ &\equiv \mathbf{F} \vee (p \wedge q) && \text{Law of Contradiction} \\ &\equiv (p \wedge q) && \text{Identity Law} \end{aligned}$$

4 Some examples of set operations

Suppose that $A = \{a, b, c\}$, that $B = \{b, d\}$, and that $C = \{d, e, f\}$. Then:

$$\begin{array}{llllll} A \cup B & = \{a, b, c, d\} & A \cap B & = \{b\} & A - B & = \{a, c\} \\ A \cup C & = \{a, b, c, d, e, f\} & A \cap C & = \emptyset & A - C & = \{a, b, c\} \end{array}$$

5 A proof in set theory

To show that $\forall x\big((x \in A \cup B) \leftrightarrow (x \in B \cup A)\big)$:

For any x,

$$\begin{aligned} x \in A \cup B &\leftrightarrow x \in A \vee x \in B && \text{(by the definition of } \cup \text{)} \\ &\leftrightarrow x \in B \vee x \in A && \text{(by commutativity of } \vee \text{)} \\ &\leftrightarrow x \in B \cup A && \text{(by the definition of } \cup \text{)} \end{aligned}$$