1.

a)

$\exists\neg nP(2n^2+3n+1)$	Prove the negation
$\neg\exists nP(2n^2+3n+1)$	
$\forall n\neg P(2n^2+3n+1)$	De Morgan's
$x =$ an arbitrary integer	Assumption
$\neg P((x)^2+3(x)+1$	Universal instantiation
$\neg P((2x+1)(x+1)$	Distribution
$\neg P((2x+1)(x+1)$	because the equation can be a factor of 2 numbers it is not prime
$\therefore \exists P(2n^2+3n+1)$	Invalid (Proof by contradiction)

Because the negation is proved to be true the hypothesis is invalid.

b)

$x = 0$	assumption
$P(2(0)^2+3(0)+2)$	
$P(2)$	
2 is prime	
$\therefore \exists P(2n^2+3n+1)$	Valid (Definition of $\exists$)

2.

$\frac{a}{b}$ =some rational number where a and b are integers	assumption/definition of a rational number
$\frac{n}{m}$ =some rational number where a and b are integers	assumption/definition of a rational number
$x =$ some irrational number	assumption
$\frac{a}{b} - x \quad = \quad \frac{n}{m}$	
$-x \quad = \quad \frac{n}{m} - \frac{a}{b}$	
$x \quad = \quad \frac{a}{b} - \frac{n}{m}$	
$x \quad = \quad \frac{am-nb}{bm}$	
$\frac{am-nb}{bm}$	by definition is a rational number
$\therefore \frac{a}{b} - x$	cannot be irrational via contradiction above

3.

$\lvert x\rvert + \lvert y\rvert \geq \lvert x+y\rvert$	assumption
$\lvert n\rvert \quad = \quad \binom{n\geq 0\to n}{n<0\to(-n)}$	

Case 1.

$x, y \geq 0 \Rightarrow \lvert x\rvert + \lvert y\rvert : x + y$	definition of absolute value
$x, y \geq 0 \Rightarrow x + y \geq 0$	
$x + y = \lvert x+y\rvert$	definition of absolute value
$\therefore \lvert x\rvert + \lvert y\rvert = \lvert x+y\rvert$	when x and $y \geq 0$

Case 2.

$x, y < 0 \Rightarrow \lvert x\rvert + \lvert y\rvert : (-x) + (-y)$	definition of absolute value
$(-x) + (-y) = -(x+y)$	
$x, y < 0 \Rightarrow -(x+y) < 0$	
$-(x+y) = \lvert x+y\rvert$	definition of absolute value
$\therefore \lvert x\rvert + \lvert y\rvert = \lvert x+y\rvert$	when x and $y < 0$

Case 3.

$x \geq 0, y < 0 \Rightarrow \|x\| + \|y\| : x + (-y)$	definition of absolute value
$y < 0 \Rightarrow x + (-y) > x$	because we are subtracting a negative number it makes x bigger
$\|x + y\|$ can either be $x + y \geq 0$ or $-(x + y) < 0$	can do this via definition of absolute value
$x \geq 0, y < 0 \Rightarrow x + y < x$	because $y < 0, x$+a negative number will be less than x
$x + (-y) > x \Rightarrow x + (-y) > x + y$	because $x + y < x$ we can say $x + (-y) > x + y$
$-(x + y) = -x - y$	distribution
$x \geq 0 \Rightarrow x > -x$	
$x + (-y) = x - y \Rightarrow x - y > -x - y$	
$\therefore \|x\| + \|y\| > \|x + y\|$	when $x \geq 0, y < 0$

Case 4.

$x < 0, y \geq 0 \Rightarrow \|x\| + \|y\| : (-x) + y$	definition of absolute value
$x < 0 \Rightarrow (-x) + y > y$	because we are adding a negative, negative number it makes y bigger
$\|x + y\|$ can either be $x + y \geq 0$ or $-(x + y) < 0$	can do this via definition of absolute value
$x < 0, y \geq 0 \Rightarrow x + y < y$	because x $< 0, y$+a negative number will be less than x
$(-x) + y > y \Rightarrow (-x) + y > x + y$	because $x + y < y$ we can say $(-x) + y > x + y$
$-(x + y) = -x - y$	distribution
$y \geq 0, \Rightarrow y > -y$	
$(-x) + y > -x - y$	because $y > -y$ adding -x to y will always be greater than (-x) - y, as
$\therefore \|x\| + \|y\| > \|x + y\|$	when $x \geq 0, y < 0$

Because all the cases of $|x| + |y|$ are either $\geq$ or $=$, $|x| + |y| \geq |x + y|$

4.

$E(x) = $	x is even
$O(x) = $	x is odd
$E(x) = $	$\neg O(x)$
$O(x) = $	$\neg E(x)$
$E(x^2 + x + 1) \to O(x)$	
$\neg O(x) \to \neg E(x^2 + x + 1)$	start proof by contrapositive
$E(x) \to O(x^2 + x + 1)$	definition
$k =$ some arbitrary int	assumption
$2k =$ even $2k + 1 =$ odd	definition of odd and even
$2k^2 + 2k + 1 = (2k^2 + 2k) + 1$	
$2k^2 + 2k =$ some integer $+1 = 2k + 1$	definition of odd
$\therefore E(x^2 + x + 1) \to O(x)$	proof by contrapositive

5.

$3a + 5b = 7m$, where m is some integer	$3c + 5d = 7n$, where n is some integer
$3(a + c) + 5(b + d) = 7z$	$z =$some integer
$3a + 3c + 5b + 5d = 7z$	distribute
$3a + 3b + 3c + 5d = 7z$	
$7m + 7n = 7z$	definition
$7(m + n) = 7z$	distribution
$m + n =$some integer	integer + integer = some integer
7(some integer) = 7(some integer)	
$\therefore 3(a + c) + 5(b + d) = 7z$	valid