CS 321 - Algorithms - Fall 2018 | *Your name(s) go here*

Assignment 2

Due BEFORE 8:00AM on Monday 9/17/2018

On time / 20% off / no credit

Total points: 45

You are allowed to work with a partner on this assignment. If you decide to form a pair, make sure to include both names above, but submit only one file to D2L.

This assignment will test your knowledge of asymptotic notation. You must write up your solutions to this assignment IN THIS FILE using LaTeX by filling in all of the boxes below. If your submitted `.tex` file does not compile, then you will receive 0 points.

All of the proofs that you write for this and subsequent assignments should be **as detailed as possible**. You must write EACH step on its own line so that it is very easy to see how each step follows directly from the previous step. Furthermore, be sure to explain/justify each step in your proof. For example, state that you added 5 to each side, that you simplified a fraction, etc. Do NOT skip a bunch of algebraic steps in your proofs. For example, do not simply say $\frac{k(k+1)}{2} + (k+1) = \frac{(k+1)(k+2)}{2}$. It is a true statement, but it is not trivial. It is not obvious that the left hand side equals the right hand side. If you are unsure whether a step is needed or not, ask me first before omitting it. Use the symbol $\square$ (i.e., `\qed` in your `.tex` file) to denote the end of each proof.

Here is an example of proper justification of algebraic steps and formatting of a sequence of equations:

$$\begin{aligned} \frac{k(k+1)}{2} + (k+1) &= \frac{k(k+1)}{2} + \frac{2(k+1)}{2} && \text{unified denominators} \\ &= \frac{k(k+1) + 2(k+1)}{2} && \text{added fractions} \\ &= \frac{(k+1)(k+2)}{2} && \text{factored out common factor} \end{aligned}$$

Note how this list of equations is aligned vertically into three columns around the equal signs and with the justifications on the right. This is the REQUIRED format for your submissions whenever applicable.

You should NOT add any LaTeX packages to your `.tex` file.

Submission procedure:

1. Complete this file, called `a2.tex`, with your full name and answers typed up below.
2. Compile this file to produce a file called `a2.pdf`. Make sure that this file compiles properly and that its contents and appearance meet the requirements described in this handout.
3. Create a directory called `a2` and copy exactly two files into this directory, namely:
   - `a2.tex` (this file with all of your answers and name(s) added)
   - `a2.pdf` (the compiled version of the file above)
4. Zip up this directory to yield a file called `a2.zip`

5. Submit this zip file to the D2L dropbox for A2 before the deadline above.
6. Submit a single-sided, hard copy of your `a2.pdf` file BEFORE the beginning of class on the due date above.

Problem statements

1. **(5 points) Prove $10 \cdot \log_2 N^{100} = O(\log_{16} N)$. Your proof MUST use the definition on slide 2-4. For full credit, your proof must use the smallest possible value for N_0.**

   Proof:

   Your proof goes here

2. **(10 points) Prove that $12N^2 - 36N + 24 = \Theta(N^2)$. Your proof MUST use:**
   (a) **the definition of the $\Theta(.)$ notation on slide 2-8 (and thus also those on slides 2-4 and 2-6), and**
   (b) **the constants defined at the bottom of page 46 in our text.**

   Proof:

   Your proof goes here

3. **(10 points) Prove or disprove $3^N = \Theta(N!)$. For full credit, your proof MUST use the definition on slide 2-8 or its negation. In other words, you must specify the required constant(s).**

   Proof:

   Your proof goes here

4. **(10 points) Prove or disprove $15^{\log_2 N} = o(N^4)$. For full credit, your proof MUST use the formal definition on slide 3-5 or its negation. In other words, you must specify the required constant(s).**

   Proof:

   Your proof goes here

5. **(10 points) Given a list of functions, your goal is to order them according to their rate of growth, from smallest to largest. For example, if given the following functions:**
N^2 $10N$ $N^2 - 5N + 12$ N $N \log N$ 2^N
the correct answer would be the following table:

N	$10N$
$N \log N$	
N^2	$N^2 - 5N + 12$
2^N	

in which all functions in a given row are big-theta of each other and little-o of all functions in the following row (if any). The left-to-right order within each row is not significant.

For this problem, logs **are base 2 unless otherwise specified. You must build an appropriately sized table that shows the correct ordering of the following 12 functions:**

$N\sqrt{N}$ $\log \log N$ 3^N $N^{0.01} \log_3 N$ $N \log N$ 3^{2N}
$N^{\frac{N}{\log_3 N}}$ N^3 $\sqrt{N^2 2^{\log \log^2 N}}$ $3^{N \log N^3}$ $N^{\frac{3}{\log_3 N}}$ 3^{N+2}

Your table goes here