1.

a)

Not one-to-one: $f(-2) = 4$ and $f(2) = 4$
Not onto: if x is negative such as, -1, x^2 cannot reach a negative

b)

Both one-to-one and onto

c)

Is One-to-one
Not onto: the number 3 cannot be reached

2.

a)

Is one-to-one
Not onto: $\{0000, 0100, 0110, 0010, 1111, 1011, 1001, 1101\}$

b)

Not one-to-one: $g(010) = 010$ and $g(011) = 010$
Is onto

c)

Is a bijection: $f^{-1}(x) =$ the reverse of x
Example: $f^{-1}(100) = 001$

d)

Is a bijection: $f^{-1}(x) =$ copy the first 3 bits, reverse those bits, then add then to the original bits, x
Example: $f^{-1}(1101) = 1101011$, we copy the fist 3 bits (110), reverse them, (011), then add them to the original 1101011

3.

No they are not a bijection. For them to be a bijection the cardinality of both functions must be the same, by definition of a bijection.
Set for first function: $\{00, 10, 11, 01\}$
Set for second function: $\{000, 010, 001, 100, 101, 110, 011, 111\}$
As stated above the cardinalities are not the same, therefore it cannot be a bijection.

4.

a)

Assume $|N| = |F|$
Using the given $f : N \to \{0, 1\}$ we can represent any Natural Number using 0's or 1's
With that we can represent Natural Numbers in the following manner

x is a natural number; a can either be 0 or 1; i is some number

$$\begin{aligned}
x_1 &= a_{11}a_{12}a_{13}a_{14}...a_{1i}\\
x_2 &= a_{21}a_{22}a_{23}a_{24}...a_{2i}\\
x_3 &= a_{31}a_{32}a_{33}a_{34}...a_{3i}\\
x_4 &= a_{41}a_{42}a_{43}a_{44}...a_{4i}\\
... &= ...\\
x_i &= a_{i1}a_{i2}a_{i3}a_{i4}...a_{ii}
\end{aligned}$$

let y = $a_{11}a_{22}a_{33}a_{44}...a_{ii}$, which will be represented as y = $e_1e_2e_3e_4...e_i$
so by definition $e_i = a_{ii}$
y is a natural number, therefore $y \epsilon N \Rightarrow y = x_i$ meaning y is a natural number, x_i
this is a contradiction, as y cannot equal both $e_1e_2e_3e_4...e_i$ and x_i
Therefore $|N| = |F|$ must be true

b)

$|\mathcal{P}(N)| > |N|$
This is proven by the answer in part a of question 4, we proved that $|N| \neq |F|$. With this we showed that the every natural number cannot represent itself which means that, by definition of a Powerset: all subsets of a set or in this case all sets of natural numbers, has to be greater than the cardinality of natural numbers.