IEOR 4004 Final Project

EcoBake: Strategic Optimization for Sustainable Bakery Production

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Abstract

This report presents a comprehensive optimization model for the bakery industry, aimed at balancing environmental sustainability with profitability. In light of the industry's significant energy costs and environmental impact, this study proposes a strategic approach to bakery production that emphasizes reducing carbon emissions while ensuring economic viability. Utilizing a detailed dataset comprising transactional, financial, qualitative, and environmental data, the report examines the carbon footprint of various bakery items. Through integer programming and rigorous mathematical formulations, the study develops a daily production plan that minimizes ecological impact and adheres to profit margins. The results showcase how bakeries can achieve a greener operational model without compromising on customer satisfaction or financial performance, offering a blueprint for sustainable practices in the food production sector.

1 Introduction

In an era where environmental sustainability intertwines with economic imperatives, the bakery industry stands at the forefront of a transformative shift. Our study emerges as a beacon of this evolution, encapsulating a comprehensive optimization model tailored for the bakery sector. This project is anchored in the dual objectives of environmental stewardship and profitability, addressing the significant energy costs and environmental impacts intrinsic to bakery operations. Bakeries, responding to environmental and economic pressures, are increasingly adopting sustainable practices, such as reducing energy consumption and innovating with eco-friendly packaging [1]. This shift is pivotal in the current landscape, where consumer preferences are rapidly evolving, especially among digitally-native generations who demand products aligning with sustainable and healthy lifestyles.

Our study leverages a detailed dataset that spans transactional, financial, qualitative, and environmental dimensions, examining the carbon footprint of various bakery items. Through sophisticated integer programming and rigorous mathematical formulations, "EcoBake" crafts a daily production plan that adeptly balances ecological considerations with profit margins. This approach is not only a response to the changing market dynamics but also a strategic alignment with global sustainable development goals. By optimizing bakery production for sustainability and profitability, "EcoBake" aims to foster environmental conservation within the baking sector, offering a blueprint for sustainable practices in the broader food production industry. Our results culminate in a comprehensive production plan for each daypart, culminating in a meticulously curated menu for the entire day, reflective of our commitment to sustainability, customer satisfaction, and financial performance.

2 Background and Motivation

The bakery industry stands at a pivotal juncture, confronting a myriad of challenges driven by a volatile mix of economic, epidemiological, and environmental factors. A significant driver reshaping this industry is the pronounced demographic shift towards digital generations, notably millennials and zoomers, who bring with them a new paradigm of demand [2]. This demographic, profoundly influenced by digital technologies and heightened environmental awareness, increasingly seeks bakery products that align with sustainable and healthy lifestyles [2]. These evolving demands, coupled with the global emphasis on sustainable development as framed by international agreements like the Paris Agreement, compel the bakery sector to reevaluate and transform its operational and strategic approaches [2].

In tandem with these market-driven changes, the bakery industry is grappling with the critical issue of environmental sustainability, particularly in energy consumption and efficiency. Commercial bakeries in the United States alone are responsible for over $870 million in annual energy consumption, with processes such as fermenting, baking, and freezing being the most energy-intensive [3]. The adoption of energy efficiency measures, therefore, becomes not only a strategic imperative to mitigate the impact of volatile energy markets but also a vital component in reducing greenhouse gas emissions. The sector's response to these challenges involves a multi-faceted approach, encompassing the adoption of sustainable development strategies, efficient energy management, and technological innovations aimed at minimizing the environmental footprint while enhancing economic viability [3]. This backdrop sets the stage for our project "EcoBake," an initiative that seeks to harmonize environmental stewardship with economic sustainability, driving the bakery industry towards a more resilient and eco-conscious future.

3 Literature Review

In synthesizing the recent literature on bakery manufacturing optimization and the broader context of food supply chain management, we find intersecting themes of operational efficiency, energy use, and environmental impact. Babor and Hitzmann (2022) and Babor et al. (2023) both emphasize the optimization of bakery production schedules, focusing on reducing makespan and oven idle time through algorithms like NSGA-II and MOPSO, revealing trade-offs between operational efficiency and energy consumption [4][5].

Complementing these findings, Parashar et al. (2020) expand the scope to the entire food supply chain, emphasizing the significant environmental impact of food waste and energy consumption in food processing and transportation. The study highlights that food supply chains contribute substantially to greenhouse gas emissions, with the production and consumption of food accounting for over twenty percent of anthropogenic emissions [6]. It stresses the need for effective process and product management across the supply chain to minimize carbon footprints, a consideration that is vital for our project as we aim to optimize bakery operations for both efficiency and sustainability.

Integrating these studies, we see a clear imperative for our project to focus on optimizing bakery production not just for operational efficiency, but also for minimizing energy use and environmental impact. This holistic approach aligns with the broader goals of sustainable supply chain management and addresses the critical issue of its environmental repercussions.

4 Data

Our analytic approach encompasses a detailed examination of the bakery dataset, stretching from October 20, 2016, to September 4, 2017. Comprising 9,684 transactions and featuring 86 unique items, our methodology revolves around meticulous categorization and methodical metric assessment. This targeted analysis aims to extract nuanced insights, with a focus not solely on financial dimensions but extending to the ecological intricacies inherent in the bakery's operational data.

Dataset Overview

The dataset encompasses three sheets:

1. Bakery Sheet

The Bakery dataset, spanning from October 20, 2016, to September 4, 2017, comprises 9,684 transactions involving 86 unique items, recorded across five columns: Transaction, Items, Datetime, Daypart, and Daytype.

2. Pricing and Rating Sheet

Seamlessly integrating with the Bakery sheet, the Pricing and Rating sheet provides insights into the cost, sale price, and customer ratings for each of the 86 items. This integration facilitates a comprehensive view, empowering the calculation of item profits through a thorough analysis of cost and sale price data.

3. Carbon Footprint Sheet

The Carbon Footprint sheet assesses the environmental impact of each item. Categorization is derived from authoritative sources, specifically referencing insights from environmental impact databases, allowing for a meticulous categorization of items based on their carbon footprints. Each item's carbon footprint is then recorded in the sheet.

Derived Metrics

Building upon the foundation laid by these sheets, we perform additional calculations to extract meaningful metrics:

1. Average Number of Sales Per Daypart

Computed within the Bakery sheet dataset, this metric involves sorting 'Items' 'Date' and 'Daypart' columns. The count of sales for each item in each daypart unveils patterns of item popularity throughout the day.

2. Average Number of Transactions Per Day

Calculated using the Bakery sheet dataset, this metric involves counting transaction numbers for each day and computing their average. It provides a snapshot of the bakery's daily transactional activity.

3. Profit Calculation

Derived from the Pricing and Rating Sheet, the Profit Calculation entails subtracting the cost of production from the sale price for each item. This direct calculation provides a clear snapshot of individual item profitability.

4. Average Daily Profit

The Average Daily Profit is obtained by merging the Bakery and Pricing and Rating datasets. Daily profits are calculated by subtracting the cost from the sale price for each item, aggregated daily, and then averaged. This process offers insights into the bakery's financial performance over the analyzed time span.

In the process of data refinement, we took the expansive information from our original bakery dataset and, following comprehensive preprocessing, distilled it into a more streamlined and informative dataset. Our new *bakery.csv* file is a testament to our meticulous approach, presenting a clear depiction of customer ratings, profit generation, average sales, and carbon footprint calculations, itemized by daypart. This transformation of raw figures into a coherent and accessible format underscores our commitment to data-driven decision-making, aligning every product we offer with our overarching goals for customer satisfaction, profitability, and environmental stewardship.

Carbon Footprint Assessment

We employed an approach to categorize items based on their carbon footprint:

- High Carbon Footprint (Weight: 1):

  Items like Alfajores and Bacon, involving meat and dairy, contribute significantly to the environmental impact.

- Medium Carbon Footprint (Weight: 0.5):

  Items like Bakewell and Cake exhibit a moderate environmental impact, considering factors such as dairy and wheat.

- Lower Carbon Footprint (Weight: 0.3):

  Items like Baguette and Crisps, primarily composed of wheat, vegetables, and oats, contribute to a more sustainable footprint.

- Very Low or Unclear Carbon Footprint (Weight: 0.1):

  Items like Mineral Water and Fairy Doors possess a minimal or uncertain environmental impact, potentially including non-food items.

Calculation Methodology:

Items were sorted by category, and a weighted approach was applied:

- High: 1, Medium: 0.5, Lower: 0.3, Very Low: 0.1.

In our assessment of the carbon footprint associated with various food products, we employed a weighted scoring methodology grounded in the quantifiable metric of greenhouse gas emissions (GHG) per kilogram of food product. Our source for the emissions data was Our World in Data, a reputable repository for global scientific data. According to this source, GHG emissions are presented in terms of kilograms of carbon dioxide-equivalents (kg COe/kg), which incorporate the global warming potential of non-CO gases over a centennial timescale.

Recognizing the inherent uncertainty in the absence of precise ingredient proportions and serving sizes, we estimated and provided a generalized weight score. This score was based on the main ingredients identified as the predominant contributors to the GHG emissions of each food product. While this approach does not account for the full complexity of the food items, it allows for a comparative analysis using the most impactful ingredients as proxies for the overall carbon footprint.

To categorize the carbon footprint of different food items, we classified them into four distinct groups, each assigned a relative weight based on their estimated environmental impact. The *High Carbon Footprint* category, with a weight of **1**, included items predominantly comprising meat and dairy—two sectors known for their substantial GHG emissions. For instance, beef from dairy herds was listed with a significant emission factor of 99.48 kg CO2-e/kg, while lamb and mutton registered at 39.72 kg CO2-e/kg. In the context of our report, items such as Alfajores and Bacon, which typically involve meat and/or dairy, were placed in this high-impact group.

The *Medium Carbon Footprint* category, assigned a weight of **0.5**, encompassed food products with moderate emissions, such as Bakewell and Cake, which contain dairy and wheat. Wheat Rye has a lower emission factor of 1.57 kg COe/kg, and milk stands at 3.15 kg CO2-e/kg. The *Lower Carbon Footprint* category, with a weight of **0.3**, was designated for items like Baguette and Crisps, where the primary ingredients are wheat (1.57 kg CO2-e/kg), vegetables, and oats, signaling a more sustainable environmental contribution.

Finally, the *Very Low or Unclear Carbon Footprint* category, with a weight of **0.1**, was applied to items such as Mineral Water and Fairy Doors, which either have a minimal carbon footprint or fall into a non-food category, making their environmental impact uncertain or negligible.

The weight scores were assigned by analyzing the primary ingredients of each food item and mapping them to the corresponding GHG emission factors provided by Our World in Data. This methodology, while acknowledging its estimative nature, was guided by the principle that the primary ingredient's carbon intensity serves as a proxy for the item's overall carbon footprint. Such an approach provides a comparative framework to evaluate the relative environmental impacts of food products, thereby enabling us to make more informed choices to further perform the optimization. The weight scores, therefore, reflect a considered approximation, drawing on the most significant contributors to GHG emissions within the composition of each food item.

5 Methods

In our project, we have chosen to utilize a mixed integer programming (MIP) model due to its robustness and precision in handling complex decision-making processes, as we have learned in our classes. MIP is particularly well-suited for our bakery optimization problem because it allows us to incorporate both continuous variables (like production quantities and carbon footprint) and discrete variables (such as the binary decision of whether to produce a specific item). Moreover, MIP's ability to enforce integer constraints ensures realistic and practical solutions, especially important in production planning where fractional outputs are not feasible. Our project also involves multiple objectives and constraints, such as minimizing carbon footprint while maximizing profit and adhering to production capacities. MIP excels in handling such multi-objective optimization problems, allowing us to define and balance these competing priorities effectively.

Problem Formulation

Decision Variables

- X_{ij}: Integer variable representing the quantity of item i produced in daypart j.
- $Produce_{ij}$: Binary variable indicating whether item i is produced in daypart j (1 if produced, 0 otherwise).

Parameters

- $Profit_{ij}$: Profit for producing item i in daypart j.

- $CarbonFootprint_{ij}$: Carbon footprint weight for producing item i in daypart j.
- $AverageSale_{ij}$: Average sale for item i in daypart j.
- $CustomerRating_{ij}$: Customer rating for item i in daypart j.
- $AverageDailyProfit$: The average daily profit, given as 500.
- $MinItemsPerDaypart$: Minimum number of different items to be produced in each daypart.

Objective Function

Minimize the total daily carbon footprint:

$$\text{minimize} \quad \sum_{i,j} CarbonFootprint_{ij} \times X_{ij} \tag{1}$$

Explanation: This objective seeks to minimize the environmental impact of the bakery by reducing the total carbon footprint weight associated with producing the bakery items.

Constraints

In constructing our mathematical model for bakery production optimization, we intricately wove a series of constraints to align with strategic and operational imperatives, ensuring the bakery's activities resonate with business goals.

1. **Total Profit Constraint:**

$$\sum_{i,j} Profit_{ij} \times X_{ij} \geq AverageDailyProfit \tag{2}$$

Explanation: This mandates that the total profit from producing bakery items must meet or exceed a predetermined average daily profit target, set conservatively at $500 based on our data insights. This decision stems from preprocessed data indicating an average daily profit of $410.228, thus ensuring the bakery's financial robustness. This constraint ensures the bakery's production remains financially viable and profitable.

2. **Least Total Production Constraint:**

$$\sum_{i,j} X_{ij} \geq 130 \tag{3}$$

Explanation: This constraint sets a minimum level of total production to ensure sufficient bakery output. Drawing from our data preprocessing insights, which revealed an average of 127.969 transactions per day, we established the Production Constraint. This ensures that daily production meets or exceeds 130 items, thereby addressing customer demand while preventing the potential for insufficient production due to the model's carbon footprint minimization directive.

3. **Minimum Different Items in Each Daypart:** For each daypart d:

$$\sum_{i \in d} Produce_{ij} \geq MinItemsPerDaypart_d \tag{4}$$

Explanation: This constraint ensures a diverse range of bakery items in each daypart, fostering variety in the product offerings. This was critically formulated to foster a diverse product offering, enhancing customer choice. Our observations dictated setting a minimum threshold for the variety of items produced in each daypart—5 items for each morning and afternoon and 1 item for each evening and night respectively, reflecting the transactional diversity observed.

4. **Maximum Production per Item and Daypart:** For each item i and daypart j:

$$X_{ij} \leq \min(15, AverageSale_{ij} + 1) \tag{5}$$

Explanation: This constraint limits the production quantity of each item in a daypart to prevent overproduction and to prevent excessive inventory and potential wastage. This cap of 15 aligns the production quantities with both inventory management strategies and the bakery's operational rationality, ensuring no item is overproduced.

5. **No Production for Low Sales Items:** For each item i and daypart j with $AverageSale_{ij} \leq 2$:

$$X_{ij} = 0 \tag{6}$$

Explanation: This constraint avoids the production of items that historically have low sales, optimizing resource allocation. This is a strategic measure to calibrate the production plan with empirical sales data. Recognizing that the average of average sales across all items and dayparts is calculated at 1.72, the model imposes a non-production directive on items whose average sales do not exceed the threshold of two units in any daypart. This constraint refines inventory management to be congruent with established sales patterns, ensuring an optimized stock level that reflects genuine customer demand.

6. **Produce More Than Average if Produced:** For each item i and daypart j:

$$X_{ij} \geq Produce_{ij} \times (AverageSale_{ij} - 1) \tag{7}$$

Explanation: This constraint ensures that if an item is produced, its quantity should be greater than its historical average sale minus one. This balances the need to minimize carbon footprint with meeting customer demand

7. **Minimum Average Customer Rating:**

$$\frac{\sum_{i,j} CustomerRating_{ij} \times X_{ij}}{\sum_{i,j} X_{ij}} \geq 4.0 \tag{8}$$

Explanation: This constraint maintains a high level of customer satisfaction, a key tenet of the bakery's ethos, by ensuring the average rating of the produced items is above a specified threshold. We set the threshold to 4.0 to reflect the bakery's commitment to quality and customer experience:

6 Results

The optimization model's application to our bakery's production scheduling has yielded results that are both strategically insightful and operationally transformative. With the model's optimal objective value at a minimized carbon footprint of approximately 42.9 units, we are affirming our commitment to sustainability—a cornerstone of our operational philosophy. The resultant daily production plan is meticulously itemized across different dayparts, meticulously calculated to align with our environmental objectives while ensuring economic viability.

Delving into the daypart-specific details elucidated by the model, we observe a production strategy that is environmentally conscious and customer-oriented. The model prescribes a morning production plan (Table 1) consisting of a total of 37 items, featuring an assortment of bakery products including bread, brownie, and hot chocolate, each with a production quantity set at 3 units, and a higher production of 5 units for pastries and Scandinavian sandwiches, ensuring a varied and ample selection for the start of the day.

daypart	items	quantity
Morning	Bread	3
	brownie	3
	cake	3
	cookies	3
	Hot chocolate	3
	medialuna	4
	muffin	3
	pastry	5
	scandinavian	3
	tea	4
	toast	3
	total	**37**

Table 1: Production Plan for Morning

Transitioning to the afternoon (Table 2), the diversity of offerings expands with 73 items scheduled for production, spanning an eclectic mix from traditional bread and cakes to international flavors such as tacos/fajitas and Spanish brunch, both set at a higher production quantity of 6 units. This variety caters to a broader customer palette, anticipating the dynamic and varied afternoon demand.

daypart	items	quantity
Afternoon	bakewell	3
	bread	3
	brownie	3
	cake	6
	cookies	3
	frittata	3
	hearty&seasonal	3
	Hot chocolate	3
	juice	3
	Kids biscuit	4
	medialuna	3
	muffin	3
	pastry	5
	sandwich	3
	scandinavian	3
	soup	4
	Spanish brunch	3
	tacos/fajita	6
	tea	6
	tiffin	3
	total	**73**

Table 2: Production Plan for Afternoon

As evening approaches, the model recommends a focused production of 7 units of pintxos (Table 3), a specialty item likely chosen for its popularity and alignment with the evening clientele's preferences. This targeted approach signifies a strategic anticipation of customer demand during this daypart.

For the night (Table 4), the model strategically scales back, proposing the production of 8 units of the Vegan feast, a singular item that likely resonates with the night-time customer base and reflects the bakery's adaptive strategy to different daypart dynamics.

daypart	items	quantity
Evening	pintxos	7
	total	**7**

Table 3: Production Plan for Evening

daypart	items	quantity
Night	Vegan feast	8
	total	**8**

Table 4: Production Plan for Night

The comprehensive production plan delineated by our optimization model presents not just a mere schedule of bakery items to be prepared but a fluid orchestration of the daily menu. The specified quantities—meticulously calculated for each daypart—do not restrict the gamut of offerings available at any given moment but rather serve to augment the array of goods with freshly produced fare. The numbers detailed in the tables represent a proactive blueprint for production, laying out the precise amount of each item that should be freshly available for sale during the designated daypart.

However, this does not imply a rigid constraint on the daypart's offerings. The dynamic nature of customer demand and the inherent variability of daily sales means that these quantities are but a part of a larger, adaptive menu strategy. For instance, while the evening daypart specifically calls for the preparation of 7 pintxos, this does not preclude the continued availability of items like bread and cookies from earlier in the day. Any surplus from the morning or afternoon becomes part of the evening's inventory, ready to be offered to customers alongside the freshly prepared pintxos.

Figure 1: Ecobake Menu

This approach ensures a robust selection that maximizes customer choice and minimizes food waste, a reflection of our bakery's commitment to sustainability and responsiveness to consumer needs. As such, the items produced throughout the day, including any carryover from earlier dayparts, collectively constitute the menu for the day, shown in Figure 1.

The implementation of this optimization model underscores a viable pathway for bakeries to balance sustainability with profitability. By producing a selective assortment of items, the bakery not only maintains a daily profit of approximately $514.89 but also achieves this within the environmental confines of a reduced carbon footprint. This strategic approach not only solidifies our bakery's dedication to sustainable operations but also exemplifies how businesses can contribute positively to environmental conservation without compromising customer service or financial viability.

The outcomes of this model are a testament to the synergy between algorithmic precision and operational strategy. By leveraging data-driven insights and computational rigor, our bakery can now navigate the complex interplay of customer satisfaction, environmental responsibility, and economic performance. These results not only validate our innovative spirit but also mark a progressive step towards a more sustainable and prosperous future, setting a new standard for the industry at large.

7 Conclusion

Our project "EcoBake" has culminated in a substantial contribution to the bakery industry's approach to sustainability and operational efficiency. The optimization model we developed has not only successfully minimized the carbon footprint to approximately 42.9 units, while maintaining a daily profit of $514.89, but also crafted a holistic production plan that strategically aligns with each daypart, ensuring the availability of a diverse and appealing menu throughout the day. This model, with its core focus on environmental stewardship and profitability, has set a precedent for sustainable practices that do not sacrifice customer satisfaction or financial performance.

The broader implications of our findings extend beyond the operational confines of a single bakery. They underscore a pressing need for the food production sector to adapt to and mitigate the environmental challenges that are increasingly shaping consumer behavior and business strategies. By demonstrating that a bakery can significantly reduce its carbon footprint while enhancing profitability, we provide a scalable blueprint for the industry at large. This innovative approach resonates with the growing consumer demand for environmentally responsible and health-conscious products, particularly among the younger, digitally-native generations.

Our study also sheds light on the potential for data-driven models to inform and transform industry practices, offering insights that could catalyze a shift towards more sustainable food systems globally. As climate change and resource depletion continue to pose significant threats, the necessity for industries to pivot towards more eco-conscious operations becomes ever more critical. "EcoBake" exemplifies how businesses can leverage algorithmic precision and computational rigor to navigate the intricate balance between sustainability, customer satisfaction, and economic viability.

In conclusion, the "EcoBake" project not only reflects our bakery's innovative spirit but also serves as an impetus for broader systemic change. It is a step towards a more sustainable and prosperous future, embodying a commitment to the well-being of our planet and its inhabitants while setting a new standard for the industry. We hope our work inspires and facilitates a transition towards more responsible production practices, fostering a more sustainable world for current and future generations.

References

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* The GitHub Repository link for this project is:
`https://github.com/jingwen67/IEOR4004Project-Team16.git`.

Appendix A Gurobi Code

```python
import pandas as pd
import numpy as np
from gurobipy import Model, GRB, quicksum

# Load the data
bakery_data = pd.read_csv('bakery.csv')

# Correcting the 'Daypart' to have consistent capitalization
bakery_data['Daypart'] = bakery_data['Daypart'].str.capitalize()

# Create a multi-index for the items and dayparts
item_daypart = pd.MultiIndex.from_frame(bakery_data[['Item', 'Daypart']])

# Initialize the model
model = Model('BakeryOptimization')

# Decision variables:
X = model.addVars(item_daypart, vtype=GRB.INTEGER, name="X")
produce_item = model.addVars(item_daypart, vtype=GRB.BINARY, name="Produce")

# Parameters
profits = bakery_data.set_index(['Item', 'Daypart'])['Profit'].to_dict()
carbon_footprints = bakery_data.set_index(['Item', 'Daypart'])['Carbon Footprint'].to_dict()
average_sales = bakery_data.set_index(['Item', 'Daypart'])['Average Sale'].to_dict()
customer_ratings = bakery_data.set_index(['Item', 'Daypart'])['Customer Rating'].to_dict()
average_daily_profit = 500

# Objective: Minimize the total daily carbon footprint
model.setObjective(quicksum(carbon_footprints[i] * X[i] for i in item_daypart), GRB.MINIMIZE)

# Constraints

# Constraint 1: The total profit must be at least the average daily profit
model.addConstr(quicksum(profits[i] * X[i] for i in item_daypart) >= average_daily_profit, 'MinProfit')

# Constraint 2: Cap the total production to x items per day
model.addConstr(quicksum(X[i] for i in item_daypart) >= 130, 'TotalProductionCap')

# Constraint 3: Ensure at least 5 different items are produced in each Morning and Afternoon, and at least 1 item for each Evening and Night
min_items_per_daypart = {'Morning': 5, 'Afternoon': 5, 'Evening': 1, 'Night': 1}  # Dictionary to hold minimum items per daypart

for daypart in ['Morning', 'Afternoon', 'Evening', 'Night']:
    items_in_daypart = bakery_data[bakery_data['Daypart'] == daypart]['Item'].unique()
    min_items = min_items_per_daypart[daypart]  # Get the minimum number of items for the current daypart
    # Add a constraint that the sum of the binary variables for each daypart should be at least the minimum number
```

```python
    model.addConstr(quicksum(produce_item[item, daypart] for item in items_in_daypart) >= min_items, f'MinDiffItems_{daypart}')

# Constraint 4: If an item is produced in a specific daypart, its production quantity should be at most 15
for item, daypart in item_daypart:
    max_prod = min(15, average_sales[item, daypart] + 1)  # The maximum of 15 or average sales + 1
    model.addConstr(X[item, daypart] <= max_prod, name=f"max_production_{item}_{daypart}")

# Constraint 5: Do not produce items with average sales of 2 or less
for item, daypart in item_daypart:
    if average_sales[item, daypart] <= 2:
        model.addConstr(X[item, daypart] == 0, name=f"no_production_low_sales_{item}_{daypart}")

# Constraint 6: If an item is produced, produce greater than its average sale - 1
for item, daypart in item_daypart:
    model.addConstr(X[item, daypart] >= (produce_item[item, daypart] * (average_sales[item, daypart] - 1)), name=f"produce_more_than_average_{item}_{daypart}")

# Constraint 7: Produce items with average customer rating for the entire day greater than 4
total_rating = quicksum(customer_ratings[i] * X[i] for i in item_daypart)
total_production = quicksum(X[i] for i in item_daypart)
model.addConstr(total_rating >= 4.0 * total_production, 'MinAverageRating')

# Optimize the model
model.optimize()

# Prepare to output the results
if model.status == GRB.OPTIMAL or model.status == GRB.SUBOPTIMAL:
    # Retrieve and print the production plan
    production_plan = {v.varName: v.x for v in model.getVars() if 'X' in v.varName and v.x > 0}
    for item_daypart, quantity in production_plan.items():
        print(f"{item_daypart}: {quantity}")
    objective_value = model.ObjVal
    print(f"Objective Value (Minimized Carbon Footprint): {objective_value}")
```

Listing 1: Python Code for Bakery Optimization

Appendix B Data Processing Code

```
# Read data
xls = pd.ExcelFile('Bakery.xlsx')
df1 = pd.read_excel(xls, 'Bakery')
df2 = pd.read_excel(xls, 'Pricing and Rating')
df3 = pd.read_excel(xls, 'Carbon Footprint')

# Process data
df2.rename(columns={'Item': 'Items'}, inplace=True)
df1 = pd.merge(df1, df2, on='Items')
df1['Date'] = df1['DateTime'].dt.date
# Calculate Profit of each Item
df1['Profit'] = df1['Sale Price'] - df1['Cost to Produce']

# Calculate Average Daily Profit
number_profit = df1.groupby(['Date'])['Profit'].sum().reset_index(name="
    number_profit")
average_daily_profit = number_profit['number_profit'].sum() / len(
    number_profit)

# Calculate Average Number of Transactions Per Day
per_day_transaction_num = df1.groupby(['Date'])['TransactionNo'].count().
    reset_index(name="number_perday")
average_num_transac_per_day = per_day_transaction_num['number_perday'].sum
    () / len(per_day_transaction_num)

# Calculate Average Number of Sales Per Daypart
number_sale = df1.groupby(['Items','Date', 'Daypart'])['TransactionNo'].
    count().reset_index(name="Sale")
average_sale= number_sale.groupby(['Items', 'Daypart'])['Sale'].mean().
    reset_index(name='Average Sale')
df = pd.merge(average_sale, df2, on='Items').sort_values(by=['Items','
    Daypart'])
df3.rename(columns={'Item': 'Items'}, inplace=True)
df = pd.merge(df, df3, on='Items')
```

Listing 2: Python Code for Data Pre-Processing