**Examining the Impact of Nonnormality on Parameter Estimation of Bifactor Graded Response Model**

**Purpose of the Study**

In psychology and psychiatric research areas, it is common to encounter a latent construct that is positively skewed. For example, most people are in the normal end of a psychiatric disorder spectrum, while a smaller number of individuals spread out along the continuum of the disorder end. However, many latent variable approaches, such as item response theory (IRT) and factor analytic methods assume the normality of the latent trait of interest. The impact of the nonnormality on parameter estimation of latent variable approaches has been attracting researchers’ attention (e.g., Wang et al., 2018). Previous research has primarily focused on exploring the effects of nonnormality on structural equation modeling (SEM) (Finch et al., 1997; Lai, 2018; Lei & Lomax, 2005; Maydeu-Olivares, 2017; Olsson et al., 2000; Ory & Mokhtarian, 2010), linear regression models (Islam & Tiku, 2005; Mardia, 1971; White & MacDonald, 1980), confirmatory factor analysis (CFA) (Curran et al., 1996; Hutchinson & Olmos, 1998; Savalei, 2008), and ANOVA (Luh & Guo, 2004; Seo et al., 1995). There has been relatively less research in investigating nonnormality in the context of item response theory (IRT) models (Svetina et al., 2017; Woods, 2014), particularly using the bifactor IRT model. Bifactor model has been gaining popularity in psychological and other social sciences because of its flexibility to incorporate a general factor and some specific factors for the multidimentional latent factors. To the best of our knowledge, no previous study utilized bifactor models to examine the impact of nonnormality on its parameter estimation, especially bifactor graded response model (Bifactor-GRM). This study will focus on the impact of the violation of the assumption of normality in the bifactor model with the graded response. It is an extension of previous studies focused on unidimensional IRT models (DeMars, 2012; Sen et al., 2016) and multidimensional IRT models (Svetina et al., 2017, Wang et al., 2018; Woods, 2014).

Compared to previous research studies designed for normality violation in unidimensional or multidimensional models, the current study uses Bifactor-GRM to check how the skewness and kurtosis of the general factor and specific factors affect the recovery of parameters, including item parameters and person ability estimates. The design factors included the severity of skewness of the general factor and specification factors, sample size, the number of factors, and the number of items per factor.

In psychological and psychometric research, the non-normality of the distribution of latent traits (θ) is a prevalent phenomenon. Most commercial software and open-source package offer one or more estimation methods to estimate the parameters of models, but most of them are based on the normal distribution. Scale scores in Graded Response Model (GRM) can be estimated by multiple methods, including Maximum Likelihood (ML), Maximum A Posteriori (MAP), Markov chain Monte Carlo (MCMC), Marginal Maximum Likelihood (MML), and Joint Maximum Likelihood (JML) (Baker & Kim, 2004). Maximum a posteriori (MAP) estimation is efficient in multidimensional model, and MAP approaches have been shown to achieve precise estimates with fewer items than the ML method; however, they are known to shrink the latent trait distribution towards the population mean (Brown, 2015). In this study, MAP and ML estimation are used to estimate the parameters of the bifactor IRT model, including item discrimination, threshold, and personal abilities on both general factor and specific factor.

The current study investigates the violation of normality assumptions in multi-dimensional models using Bifactor Graded Response Model (Bifactor-GRM). It examines how the skewness and kurtosis of the general factor and specific factors affect the recovery of parameters, including item parameters and person ability estimates. The research explores various design factors, including the severity of skewness in the general and specific factors, sample size, number of factors, and number of items per factor.

**Theoretical Framework**

**Grade Response Model**

The Graded Response Model (GRM), a component of the broader Item Response Theory (IRT) used in psychometrics, is a model specifically tailored for ordinal responses, such as Likert scales. GRM is effective for predicting the likelihood of a respondent selecting a specific response level or higher on a survey (Samejima, 1969; Baker & Kim, 2004).

**Bifactor Grade Response Model**

The Bifactor Graded Response Model (Bifactor-GRM) is an extension of the conventional Graded Response Model (GRM) and is a part of Item Response Theory (IRT). In a Bifactor-GRM, items are allowed to load onto a general factor (akin to a general ability or trait in the respondent) and one or more group factors (specific abilities or traits) (Reise et al., 2010). The probability that an examinee’s response falls at or above a particular ordered category given θ.

where P is the probability to provide a response equal to k or greater given a person's location on G and a specific trait S, category k's item-intercept as defined , and the conditional item slope parameter on G () and on S (). The person parameter represents person i's location on G, whereas represents person I’s location on S. For each person we have a number of specific trait scores equivalent to the number of specific traits defining the model (Toland et al., 2017).

Based on Equation (1), the category response functions, which indicate the probability of responding to a particular category given θ, can be calculated by subtraction of adjacent boundary functions,

**Method**

**Design Factors**

This study is a Monte Carlo simulation study of the bifactor model with one general factor and two, three or four specific factors (Fs = 2, 3, 4), using the manipulated factors that have been implemented in previous research (Auné,2020; Mao,2022; Rijmen,2011; Svetina et al., 2017; Wang et al., 2018), including sample size (three levels: N= 250, 500, 1000 ), number of item per factor (two levels: I = 5, 10), and the degree of nonnormality on population’s latent traits (three levels at general factor and three levels at specific/group factor; see Table 1).

In bifactor models, each subject (participant or patient) has one latent trait on general factor (θg) and several latent traits on specific factors (θsk), in which k is the number of specific factors. We simulate three levels of nonnormality on general factor and specific factors: normality (with skewness and kurtosis values of 0), moderate nonnormality (skewness: 2, kurtosis: 7), and severe nonnormality (skewness: 3, kurtosis: 21). There are nine different combinations of nonnormality for the general factor and specific factors. All the nonnormalities of the latent traits pertaining to the specific factors (θsk) are set to the same values in both 2, 3, and 4 specific factors settings.

**Table 1 Simulation Design**

|  |  |  |
| --- | --- | --- |
| Design factors | Number of levels | Values of levels |
| **Data Structure** |  |  |
| Sample size (N) | 3 | N = 250, 500, 1000 |
| Number of Item per Factor (I) | 2 | I = 5, 10 |
| Number of Specific Factor (Fs) | 3 | Fs = 2, 3, 4 |
|  |  |  |
| **Nonnormality of Latent Traits (Theta)** |  |  |
| Nonnormality on general factor (θg) | 3 | Normal: Skewness = 0, Kurtosis = 0  Moderate: Skewness = 2, Kurtosis = 7  Severe: Skewness = 3, Kurtosis = 21 |
| Nonnormality on general factor (θsk) | 3 | Normal: Skewness = 0, Kurtosis = 0  Moderate: Skewness = 2, Kurtosis = 7  Severe: Skewness = 3, Kurtosis = 21 |

3 × 2 × 3 × 3 × 3 = Each of these conditions was replicated 500 times using the R package "SimMultiCorrData" in R (R Core Team, 2021).

**Item parameter**

In psychological and psychiatric research, the general factor discrimination is usually positive and falls within the range of 1.1 to 2.8(Atkinson, 2018; Auné, 2020; Berkeljon, 2012; Raines, 2015). Previous studies have consistently shown that specific factor discriminations are typically smaller than the general factor, ranging from 0 to 1.5 (Wang et al., 2018), used as specific factor in this study. In the bifactor model, the general factor and specific factor are considered independent, with no correlation between them. In this study, the discrimination values for the general factor are set to range from 1.1 to 2.8, while the discrimination values for the specific factors are established within the range of 0 to 1.5.

Item difficulty values can theoretically range from negative infinity to positive infinity, but in practice, they typically vary from -2 to +2 (Hambleton, 1993; Hambleton & Swaminathan, 1985). Psychological and psychiatric tests often use a four-point Likert scale to measure latent traits or personalities (Auné et al., 2020; Rijmen,2011). According Wang (2018), this study generated normally distributed thresholds, b1[−2, −0.67], b2[−0.67, 0.67], and b3[0.67, 2], for three thresholds (locations) to distinguish the possibilities of choosing each item.

**Person ability parameter**

Blanca (2013) showed the measures of cognitive ability and of other psychological variables were included. The results showed that skewness ranged between −2.49 and 2.33. The values of kurtosis ranged between −1.92 and 7.41. The values for asymmetry and kurtosis between -2 and +2 are considered acceptable in order to prove normal univariate distribution (George & Mallery, 2010). Hair et al. (2010) and Bryne (2010) argued that data is normal if skewness is between ‐2 to +2 and kurtosis is between ‐7 to +7. Since, we simulate three level of nonnormality, normality (skewness: 0, kurtosis: 0), moderate nonnormality (skewness: 2, kurtosis: 7), and severe nonnormality (skewness: 3, kurtosis: 21).There were nine combinations of non-skewed, moderately skewed, and severely skewed. All latent traits on specific factors (θs) are set equally.

**Estimation**

The item parameters in this study were estimated using the "bfactor()" function from the R package "mirt", limited in 2000 iterations. For estimating the person ability parameters, two estimation methods, namely, maximum a posteriori (MAP) and maximum likelihood (ML), were utilized. Within the R package "mirt," the estimation of person ability parameters involved utilizing the "fscores()" function. In this package, the thresholds or locations are calculated as cjk, as described in Equation (1).

***Evaluation criteria***

The accuracy of parameter recovery in this study is assessed through the calculation of bias, root mean squared error (RMSE), and Pearson correlations. These measures are calculated for both the two discrimination parameters, the three boundary parameters, and two personal parameters, for each replication.

**Bias.** The relative bias is estimated for all the parameters of model, including item parameter (ag, as, c1, c2, c3) and personal parameter (θg, θs) as,

where is the estimated parameters () across valid replications and is the true parameters (agj, asj, c1j, c2j, c3j , θgj, θsj,). In the , j represents the item number, ranging from 1 to J. The total number of items J is computed by multiplying the number of items in each specific factor by the number of specific factors. For each condition, a total of 500 replications are carried out, denoted as R in equation (3).

**RMSE**. The RMSE is estimated for all the parameters of model, including item parameter (ag, as, c1, c2, c3) and personal parameter (θg, θs) as,

where is the estimated parameters () across valid replications and is the true parameters (agj, asj, c1j, c2j, c3j , θgj, θsj,).

**Correlation**. Correlation measures the strength and direction of a linear relationship between two variables. It is a statistical measure that ranges from -1 to 1. In this study, a positive correlation (closer to +1) indicates that when true personal traits (θ) increases, the other tends to increase as well. A correlation close to 0 indicates a weak or no linear relationship between the variables.

To aid in comparing the effects and interactions among the manipulated variables, effect size (η2) was computed for each recovered item parameter.

**Preliminary Results**

**Item Parameter Estimation**

When analyzing item parameters, none of the interaction terms had an effect size larger than 0.05. We focused on ag (discrimination on the general factor), as (discrimination on the specific factor), and three locations c1, c2, and c3. In the bias test, we discovered that as the skewness and kurtosis of the population's general factor increased, the bias in estimating ag grew significantly. However, the bias in as estimation was not impacted. When the skewness and kurtosis of the population's specific factor increased, there was a slight increase in the bias of estimating as, but it did not affect the estimation of ag. For estimating the location parameter c, we took an average of c1, c2, and c3 instead of treating them separately. The results showed that the skewness and kurtosis of the population's general factor positively influenced the estimation of c, while the non-normality of the population's specific factor, sample size, and item number per factor had limited impact.

Regarding RMSE estimation, as the skewness and kurtosis of the population's general factor increased, the RMSE of estimating ag became noticeably higher. However, the increase in skewness and kurtosis of the population's specific factor had an imperceptible effect on the RMSE of estimating as. Item number per factor and sample size effectively impacted as. Sample size emerged as a major factor influencing all item parameters, including ag, as, and c.

**Personal Parameter Estimation**

The choice of algorithm used for estimating theta plays a significant role in personal parameter measurement. This is especially true for theta related to general factors, as it can introduce bias, increase the root mean square error (RMSE), and impact the correlation between estimated theta and the true theta. However, the algorithm mainly affects the RMSE of theta related to specific factors, with no significant impact on bias and only a slight influence on correlation.

When there is a greater deviation from normality in both general and specific factors, the bias and RMSE in estimating theta for these factors separately become more pronounced. Additionally, the correlation between the population's theta for general factors and the true theta decreases.

The number of specific factors can affect the bias, RMSE, and correlation in estimating theta for general factors, as well as the bias and RMSE in estimating theta for specific factors.

Another factor that increases the RMSE in estimating theta for both general and specific factors is the sample size. At the same time, it decreases the correlation between the estimated theta and the true theta for specific factors.

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