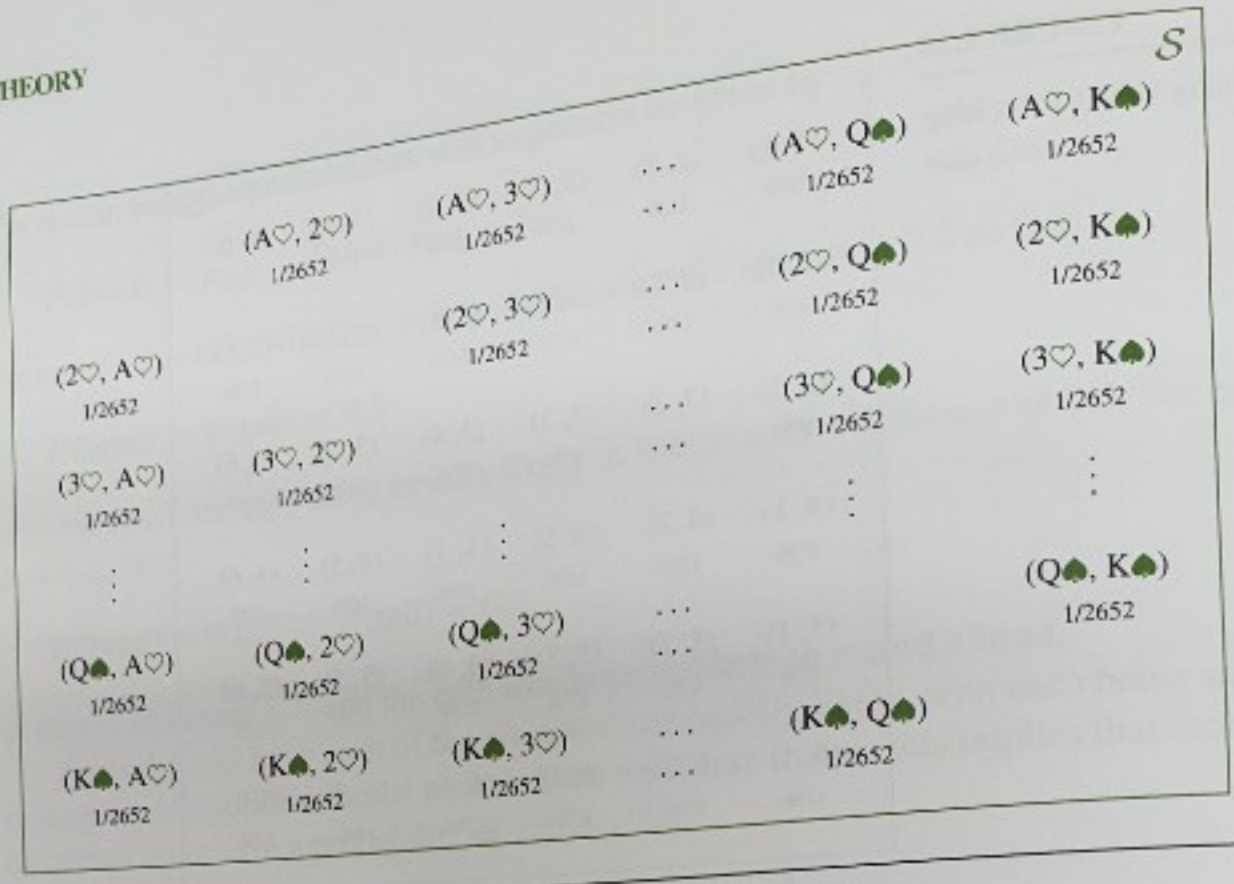


FIGURE 1.13
Probability values for choosing two cards without replacement

1.1.4 Problems

1.1.1 What is the sample space when a coin is tossed three times?

1.1.2 A bag contains balls that are either red or blue and either dull or shiny. What is the sample space when a ball is chosen from the bag?

1.1.3 What is the sample space for the number of aces in a hand of 13 playing cards?

1.1.4 What is the sample space for a person's birthday?

1.1.5 A car repair is performed either on time or late and either satisfactorily or unsatisfactorily. What is the sample space for a car repair?

1.1.6 An experiment has three outcomes, I, II, and III. If outcome I is twice as likely as outcome II, and outcome II is three times as likely as outcome III, what are the probability values of the three outcomes?

1.1.7 A probability value p is often reported as an *odds ratio*, which is $p/(1-p)$. This is the ratio of the probability that the event happens to the probability that the event does not happen.

(a) If the odds ratio is 1, what is p?

(b) If the odds ratio is 2, what is p?

(c) If $p = 0.25$, what is the odds ratio?

1.1.8 An experiment has five outcomes, I, II, III, IV, and V. If $P(\text{I}) = 0.13$, $P(\text{II}) = 0.24$, $P(\text{III}) = 0.07$, and $P(\text{IV}) = 0.38$, what is $P(\text{V})$?

1.1.9 An experiment has five outcomes, I, II, III, IV, and V. If $P(\text{I}) = 0.08$, $P(\text{II}) = 0.20$, and $P(\text{III}) = 0.33$, what are the possible values for the probability of outcome V? If outcomes IV and V are equally likely, what are their probability values?

1.1.10 What is the sample space for counting the number of females in a group of n people?

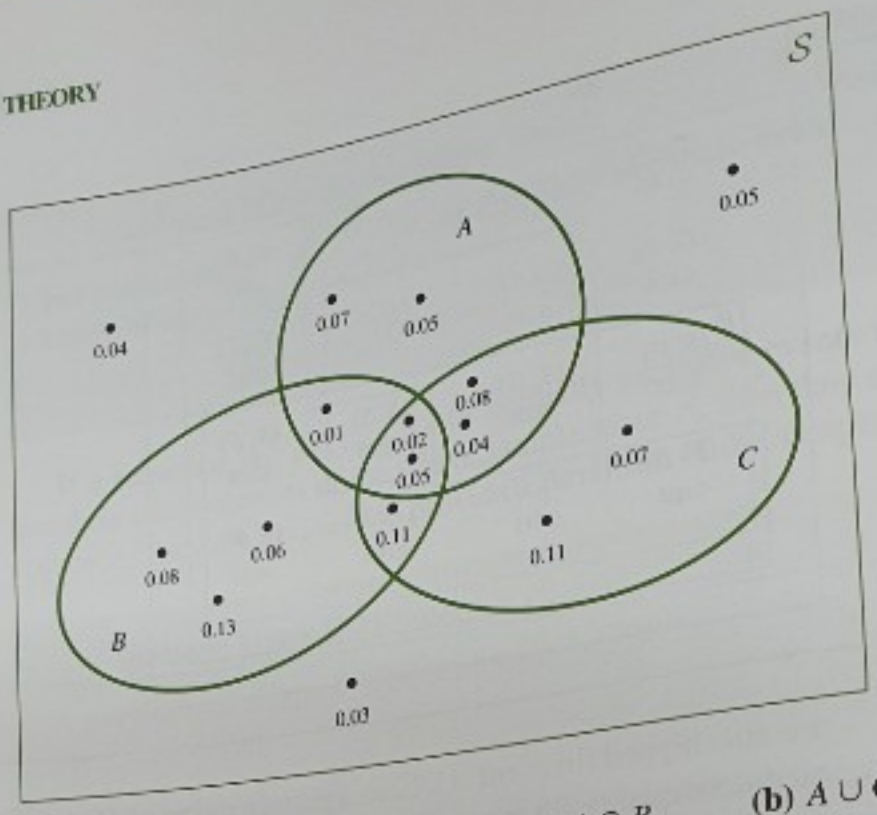
1.1.11 A company's advertising expenditure is either low with probability 0.28, average with probability 0.55, or high with probability p. What is p?

1.2 Events

1.2.1 Events and Complements

Interest is often centered not so much on the individual elements of a sample space, but rathe on collections of individual outcomes. These collections of

FIGURE 1.55



exclusive? What about the events that an ace is chosen and that a heart is chosen?

1.3.6 A bag contains 200 balls that are either red or blue and either dull or shiny. There are 55 shiny red balls, 91 shiny balls, and 79 red balls. If a ball is chosen at random, what is the probability that it is either a shiny ball or a red ball? What is the probability that it is a dull blue ball?

1.3.7 If $P(A) = 0.5$, $P(A \cap B) = 0.1$, and $P(A \cup B) = 0.8$, what is $P(B)$?

1.3.8 An evaluation of a small business by an accounting firm either reveals a problem with the accounts or it doesn't reveal a problem. Also, the evaluation is either done accurately or incorrectly. The probability that the evaluation is done accurately is 0.85. Furthermore, the probability that the evaluation is done incorrectly and that it reveals a problem is 0.10. If the probability that the evaluation is done accurately and it does not reveal a problem is 0.25, what is the probability that the evaluation does not reveal a problem?

A. 0.10 **B.** 0.20 **C.** 0.30 **D.** 0.40

1.3.9 A card is drawn at random from a pack of cards. A is the event that a heart is obtained, B is the event that a club is obtained, and C is the event that a diamond is obtained. Are these three events mutually exclusive? What is $P(A \cup B \cup C)$? Explain why $B \subset A'$.

1.3.10 A card is drawn from a pack of cards. A is the event that an ace is obtained, B is the event that a card from one of the two red suits is obtained, and C is the event that a picture card is obtained. What cards do the following events consist of?

(a) $A \cap B$ **(b)** $A \cup C$
(c) $B \cap C'$ **(d)** $A \cup (B' \cap C)$

1.3.11 A car repair can be performed either on time or late and either satisfactorily or unsatisfactorily. The probability of a repair being on time and satisfactory is 0.26. The probability of a repair being on time is 0.74. The probability of a repair being satisfactory is 0.41. What is the probability of a repair being late and unsatisfactory?

1.3.12 If $P(A) = 0.4$ and $P(A \cap B) = 0.3$, what are the possible values for $P(B)$?

1.3.13 In a study of patients arriving at a hospital emergency room, the gender of the patients is considered, together with whether the patients are younger or older than 30 years of age, and whether or not the patients are admitted to the hospital. It is found that 45% of the patients are male, 30% of the patients are younger than 30 years of age, 15% of the patients are females older than 30 years of age who are admitted to the hospital, and 21% of the patients are females younger than 30 years of age. What proportion of the patients are females older than 30 years of age who are not admitted to the hospital?

1.3.14 Let A be the event that a person is *female*, let B be the event that a person has *black hair*, and let C be the event that a person has *brown eyes*. Describe the kinds of people in the following events:

(a) $A \cap B$ **(b)** $A \cup C'$
(c) $A' \cap B \cap C$ **(d)** $A \cap (B \cup C)$

1.3.15 Recall that an advertising campaign is canceled before launch with probability 0.10, is launched but canceled early with probability 0.18, is launched and runs its

What is the probability of the chosen ball being shiny conditional on it being red? What is the probability of the chosen ball being dull conditional on it being red?

1.4.6 When a company receives an order, there is a probability of 0.42 that its value is over \$1000. If an order is valued at over \$1000, then there is a probability of 0.63 that the customer will pay with a credit card.

(a) What is the probability that the next three independent orders will each be valued at over \$1000?

(b) What is the probability that the next order will be valued at over \$1000 but will not be paid with a credit card?

1.4.7 Assess whether the probabilities of the events (i) increase, decrease, or remain unchanged when they are conditioned on the events (ii).

(a) (i) It rains tomorrow, (ii) it is raining today.

(b) (i) A lottery winner has black hair, (ii) the lottery winner has brown eyes.

(c) (i) A lottery winner has black hair, (ii) the lottery winner owns a red car.

(d) (i) A lottery winner is more than 50 years old, (ii) the lottery winner is more than 30 years old.

1.4.8 Suppose that births are equally likely to be on any day. What is the probability that somebody chosen at random has a birthday on the *first day* of a month? How does this probability change conditional on the knowledge that the person's birthday is in March? In February?

1.4.9 Consider again Figure 1.24 and the battery lifetimes. Calculate the probabilities:

(a) A type I battery lasts longest conditional on it not failing first

(b) A type I battery lasts longest conditional on a type II battery failing first

(c) A type I battery lasts longest conditional on a type II battery lasting the longest

(d) A type I battery lasts longest conditional on a type II battery not failing first

1.4.10 Consider again Figure 1.25 and the two assembly lines. Calculate the probabilities:

(a) Both lines are at full capacity conditional on neither line being shut down

(b) At least one line is at full capacity conditional on neither line being shut down

(c) One line is at full capacity conditional on exactly one line being shut down

(d) Neither line is at full capacity conditional on at least one line operating at partial capacity

1.4.11 The length, width, and height of a manufactured part are classified as being either within or outside specified tolerance limits. In a quality inspection 86% of the parts are found to be within the specified tolerance limits for width, but only 80% of the parts are within the specified tolerance limits for all three dimensions. However, 2% of the parts are within the specified tolerance limits for width and length but not for height, and 3% of the parts are within the specified tolerance limits for width and height but not for length. Moreover, 92% of the parts are within the specified tolerance limits for either width or height, or both of these dimensions.

(a) If a part is within the specified tolerance limits for height, what is the probability that it will also be within the specified tolerance limits for width?

(b) If a part is within the specified tolerance limits for length and width, what is the probability that it will be within the specified tolerance limits for all three dimensions?

1.4.12 If $A \subset B$ and $B' \neq \emptyset$, is $P(A)$ larger or smaller than $P(A|B)$? Provide some intuitive reasoning for your answer.

1.4.13 A manufactured component has its quality graded on its performance, appearance, and cost. Each of these three characteristics is graded as either pass or fail. There is a probability of 0.40 that a component passes on both appearance and cost. There is a probability of 0.31 that a component passes on all three characteristics. There is a probability of 0.64 that a component passes on performance. There is a probability of 0.19 that a component fails on all three characteristics. There is a probability of 0.06 that a component passes on appearance but fails on both performance and cost.

(a) What is the probability that a component passes on cost but fails on both performance and appearance?

(b) If a component passes on both appearance and cost, what is the probability that it passes on all three characteristics?

1.4.14 Recall that an advertising campaign is canceled before launch with probability 0.10, is launched but canceled early with probability 0.18, is launched and runs its targeted length with probability 0.43, and is launched and is extended beyond its targeted length with probability 0.29. If the advertising campaign is launched, what is the probability that it runs at least as long as targeted?

1.4.15 There is a 4% probability that the plane used for a commercial flight has technical problems, and this causes a delay in the flight. If there are no technical problems

with the plane, then there is still a 33% probability that the flight is delayed due to all other reasons. What is the probability that the flight is delayed?

1.4.16 In a reliability test there is a 42% probability that a computer chip survives more than 500 temperature cycles. If a computer chip does not survive more than 500 temperature cycles, then there is a 73% probability that it was manufactured by company A. What is the probability that a computer chip is not manufactured by company A and does not survive more than 500 temperature cycles?

1.4.17 Recall that a company's revenue is considerably below expectation with probability 0.08, is slightly below expectation with probability 0.19, exactly meets expectation with probability 0.26, is slightly above expectation with probability 0.36, and is considerably above expectation with probability 0.11. If revenue is not below expectation, what is the probability that it exactly meets expectation?

1.4.18 A car repair is either on time or late and either satisfactory or unsatisfactory. If a repair is made on time, then there is a probability of 0.85 that it is satisfactory. There is a probability of 0.77 that a repair will be made on time. What is the probability that a repair is made on time and is satisfactory?

1.5 Probabilities of Event Intersections

1.5.1 General Multiplication Law

It follows from the definition of the conditional probability $P(A|B)$ that the probability of the intersection of two events $A \cap B$ can be calculated as

$$P(A \cap B) = P(B)\,P(A|B)$$

That is, the probability of events A and B both occurring can be obtained by multiplying the probability of event B by the probability of event A conditional on event B. It also follows from the definition of the conditional probability $P(B|A)$ that

$$P(A \cap B) = P(A)\,P(B|A)$$

so that the probability of events A and B both occurring can also be obtained by multiplying the probability of event A by the probability of event B conditional on event A. Therefore, it does not matter which of the two events A or B is conditioned upon.

More generally, since

$$P(C|A \cap B) = \frac{P(A \cap B \cap C)}{P(A \cap B)}$$

the probability of the intersection of three events can be calculated as

$$P(A \cap B \cap C) = P(A \cap B)\,P(C|A \cap B) = P(A)\,P(B|A)\,P(C|A \cap B)$$

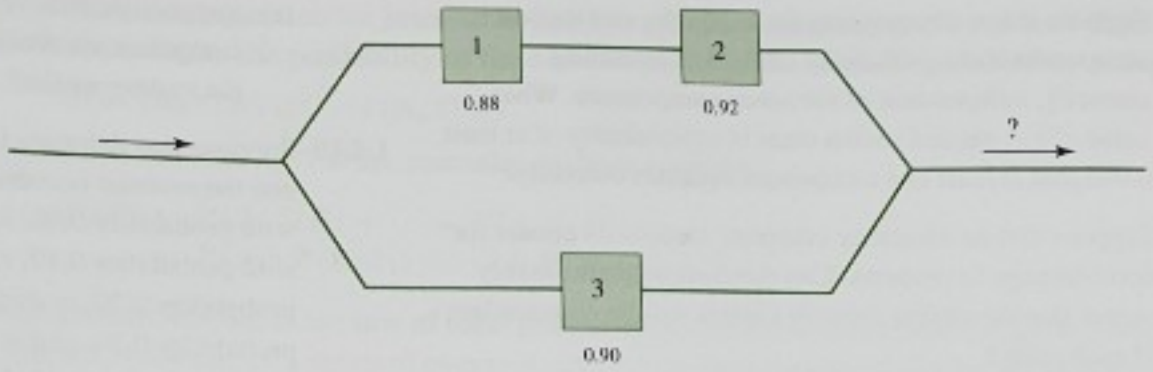
Thus, the probability of all three events occurring can be obtained by multiplying together the probability of one event, the probability of a second event conditioned on the first event, and the probability of the third event conditioned on the intersection of the first and second events. This formula can be extended in an obvious way to the following **multiplication law** for the intersection of a series of events.

Probabilities of Event Intersections

The probability of the **intersection of a series of events** $A_1, \ldots, A_n$ can be calculated from the expression

$$P(A_1 \cap \cdots \cap A_n) = P(A_1) \times P(A_2|A_1) \times P(A_3|A_1 \cap A_2) \times \cdots \times P(A_n|A_1 \cap \cdots \cap A_{n-1})$$

FIGURE 1.66
Switch diagram



have different birthdays is

$$\frac{364}{365} \times \frac{363}{365}$$

and extend this pattern to show that the probability that n people chosen at random all have different birthdays is

$$\frac{364}{365} \times \cdots \times \frac{366 - n}{365}$$

What then is the probability that in a group of n people, at least two people will share the same birthday? Evaluate this probability for $n = 10$, $n = 15$, $n = 20$, $n = 25$, $n = 30$, and $n = 35$. What is the smallest value of n for which the probability is larger than a half? Do you think that birthdays are equally likely to be on any day of the year?

1.5.5 Repeat Problem 1.5.4, except that the drawings are made *with replacement*. Compare your answers with those from Problem 1.5.4.

1.5.6 Show that if the events A and B are independent events, then so are the events
(a) A and B' **(b)** A' and B **(c)** A' and B'

1.5.7 Consider the network given in Figure 1.66 with three switches. Suppose that the switches operate independently of each other and that switch 1 allows a message through with probability 0.88, switch 2 allows a message through with probability 0.92, and switch 3 allows a message through with probability 0.90. What is the probability that a message will find a route through the network?

1.5.8 If a fair die is rolled six times, what is the probability that each score is obtained exactly once? If a fair die is rolled seven times, what is the probability that a 6 is not obtained at all?

1.5.9 Suppose that 17 lightbulbs in a box of 100 lightbulbs are broken and that 3 are selected at random without replacement. Construct a probability tree for this problem. What is the probability that there will be no broken lightbulbs in the sample? What is the probability that there will be no more than 1 broken lightbulb in the sample? (This problem is continued in Problem 1.7.8.)

1.5.10 Repeat Problem 1.5.9, except that the drawings are made *with replacement*. Compare your answers with those from Problem 1.5.9.

1.5.11 Suppose that a bag contains 43 red balls, 54 blue balls, and 72 green balls, and that 2 balls are chosen at random without replacement. Construct a probability tree for this problem. What is the probability that 2 green balls will be chosen? What is the probability that the 2 balls chosen will have different colors?

1.5.12 Repeat Problem 1.5.11, except that the drawings are made *with replacement*. Compare your answers with those from Problem 1.5.11.

1.5.13 A biased coin has a probability p of resulting in a head. If the coin is tossed twice, what value of p minimizes the probability that the same result is obtained on both throws?

1.5.14 Consider four advertising campaigns where for each one it is canceled before launch with probability 0.10, it is launched but canceled early with probability 0.18, it is launched and runs its targeted length with probability 0.43, and it is launched and is extended beyond its targeted length with probability 0.29. If the advertising campaigns are independent, what is the probability that all four campaigns will run at least as long as they are targeted?

1.5.15 **(a)** If a fair die is rolled five times, what is the probability that the numbers obtained are all even numbers?
(b) If a fair die is rolled three times, what is the probability that the three numbers obtained are all different?
(c) If three cards are taken at random from a pack of cards with replacement, what is the probability that there are two black cards and one red card?
(d) If three cards are taken at random from a pack of cards without replacement, what is the probability that there are two black cards and one red card?

In conclusion, the chemical company should realize that it is wasteful to disregard off-hand batches that are indicated to have high impurity levels. Further investigation of these batches should be undertaken to identify the large proportion of them that are in fact satisfactory products.

■ 1.6.4 Problems

1.6.1 Suppose it is known that 1% of the population suffers from a particular disease. A blood test has a 97% chance of identifying the disease for diseased individuals, but also has a 6% chance of falsely indicating that a healthy person has the disease.

(a) What is the probability that a person will have a positive blood test?

(b) If your blood test is positive, what is the chance that you have the disease?

(c) If your blood test is negative, what is the chance that you do not have the disease?

1.6.2 An advertising campaign is canceled before launch with probability 0.10, in which case the marketing company is fired with probability 0.74; is launched but canceled early with probability 0.18, in which case the marketing company is fired with probability 0.43; is launched and runs its targeted length with probability 0.43, in which case the marketing company is fired with probability 0.16; and is launched and is extended beyond its targeted length with probability 0.29, in which case the marketing company is fired with probability 0.01. What is the probability that the marketing company is fired? If the marketing company is fired, what is the probability that the advertising campaign was not canceled before launch?

1.6.3 A class had two sections. Section I had 55 students of whom 10 received A grades. Section II had 45 students of whom 11 received A grades. Now 1 of the 100 students is chosen at random, with each being equally likely to be chosen.

(a) What is the probability that the student was in section I?

(b) What is the probability that the student received an A grade?

(c) What is the probability that the student received an A grade if the student is known to have been in section I?

(d) What is the probability that the student was in section I if the student is known to have received an A grade?

1.6.4 An island has three species of bird. Species 1 accounts for 45% of the birds, of which 10% have been tagged. Species 2 accounts for 38% of the birds, of which 15% have been tagged. Species 3 accounts for 17% of the birds, of which 50% have been tagged. If a tagged bird is observed, what are the probabilities that it is of species 1, of species 2, and of species 3?

1.6.5 After production, an electrical circuit is given a quality score of A, B, C, or D. Over a certain period of time, 77% of the circuits were given a quality score A, 11% were given a quality score B, 7% were given a quality score C, and 5% were given a quality score D. Furthermore, it was found that 2% of the circuits given a quality score A eventually failed, and the failure rate was 10% for circuits given a quality score B, 14% for circuits given a quality score C, and 25% for circuits given a quality score D.

(a) If a circuit failed, what is the probability that it had received a quality score either C or D?

(b) If a circuit did not fail, what is the probability that it had received a quality score A?

1.6.6 Bag A contains 3 red balls and 7 blue balls. Bag B contains 8 red balls and 4 blue balls. Bag C contains 5 red balls and 11 blue balls. A bag is chosen at random, with each bag being equally likely to be chosen, and then a ball is chosen at random from that bag. Calculate the probabilities:

(a) A red ball is chosen.

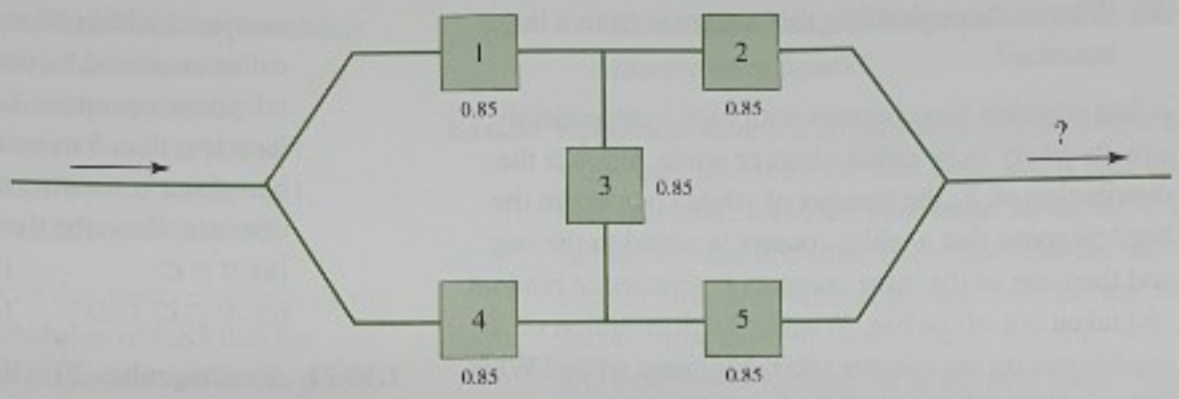
(b) A blue ball is chosen.

(c) A red ball from bag B is chosen.

If it is known that a red ball is chosen, what is the probability that it comes from bag A? If it is known that a blue ball is chosen, what is the probability that it comes from bag B?

1.6.7 A valve can be used at four temperature levels. If the valve is used at a cold temperature, then there is a probability of 0.003 that it will leak. If the valve is used at a medium temperature, then there is a probability of 0.009 that it will leak. If the valve is used at a warm temperature, then there is a probability of 0.014 that it will leak. If the valve is used at a hot temperature, then there is a probability of 0.018 that it will leak. Under standard operating conditions, the valve is used at a cold temperature 12% of the time, at a medium

FIGURE 1.75
Switch diagram



1.10.5 If a card is chosen at random from a pack of cards, what is the probability of choosing a diamond picture card?

1.10.6 A hand of 10 cards is chosen at random without replacement from a deck of 52 cards. What is the probability that the hand contains exactly two aces, two kings, three queens, and three jacks?

1.10.7 Two fair dice are thrown. A is the event that the sum of the scores is no larger than four, and B is the event that the two scores are identical. Calculate the probabilities:
(a) $A \cap B$ **(b)** $A \cup B$ **(c)** $A' \cup B$

1.10.8 Two fair dice are thrown, one red and one blue. Calculate:
(a) P(red die is 5|sum of scores is 8)
(b) P(either die is 5|sum of scores is 8)
(c) P(sum of scores is 8|either die is 5)

1.10.9 Consider the network shown in Figure 1.75 with five switches. Suppose that the switches operate independently and that each switch allows a message through with a probability of 0.85. What is the probability that a message will find a route through the network?

1.10.10 Which is more likely: obtaining at least one head in two tosses of a fair coin, or at least two heads in four tosses of a fair coin?

1.10.11 Bag 1 contains six red balls, seven blue balls, and three green balls. Bag 2 contains eight red balls, eight blue balls, and two green balls. Bag 3 contains two red balls, nine blue balls, and eight green balls. Bag 4 contains four red balls, seven blue balls, and no green balls. Bag 1 is chosen with a probability of 0.15, bag 2 with a probability of 0.20, bag 3 with a probability of 0.35, and bag 4 with a probability of 0.30, and then a ball is chosen at random from the bag. Calculate the probabilities:
(a) A blue ball is chosen.
(b) Bag 4 was chosen if the ball is green.
(c) Bag 1 was chosen if the ball is blue.

1.10.12 A fair die is rolled. If an even number is obtained, then that is the recorded score. However, if an odd number is obtained, then a fair coin is tossed. If a head is obtained, then the recorded score is the number on the die, but if a tail is obtained, then the recorded score is *twice* the number on the die.
(a) Give the possible values of the recorded score.
(b) What is the probability that a score of ten is recorded?
(c) What is the probability that a score of three is recorded?
(d) What is the probability that a score of six is recorded?
(e) What is the probability that a score of four is recorded if it is known that the coin is tossed?
(f) If a score of six is recorded, what is the probability that an odd number was obtained on the die?

1.10.13 How many sequences of length 4 can be made when each component of the sequence can take five different values? How many sequences of length 5 can be made when each component of the sequence can take four different values? In general, if $3 \leq n_1 < n_2$, are there more sequences of length n_1 with n_2 possible values for each component, or more sequences of length n_2 with n_1 possible values for each component?

1.10.14 A business tax form is either filed on time or late, is either from a small or a large business, and is either accurate or inaccurate. There is an 11% probability that a form is from a small business and is accurate and on time. There is a 13% probability that a form is from a small business and is accurate but is late. There is a 15% probability that a form is from a small business and is on time. There is a 21% probability that a form is from a small business and is inaccurate and is late.
(a) If a form is from a small business and is accurate, what is the probability that it was filed on time?

(b) What is the probability that a form is from a large business?

1.10.15 A bag contains two counters with each independently equally likely to be either black or white. What is the distribution of X, the number of white counters in the bag? Suppose that a white counter is added to the bag and then one of the three counters is selected at random and taken out of the bag. What is the distribution of X conditional on the counter taken out being white? What if the counter taken out of the bag is black?

1.10.16 A random sample of 10 fibers is taken from a collection of 92 fibers that consists of 43 fibers of polymer A, 17 fibers of polymer B, and 32 fibers of polymer C.

(a) What is the probability that the sample does not contain any fibers of polymer B?

(b) What is the probability that the sample contains exactly one fiber of polymer B?

(c) What is the probability that the sample contains three fibers of polymer A, three fibers of polymer B, and four fibers of polymer C?

1.10.17 When asked to select their favorite opera work, 26% of the respondents selected a piece by Puccini, and 22% of the respondents selected a piece by Verdi. Moreover, 59% of the respondents who selected a piece by Puccini were female, and 45% of the respondents who selected a piece by Verdi were female. Altogether, 62% of the respondents were female.

(a) If a respondent selected a piece that is by neither Puccini nor Verdi, what is the probability that the respondent is female?

(b) What proportion of males selected a piece by Puccini?

1.10.18 A warehouse contains 500 machines. Each machine is either new or used, and each machine has either good quality or bad quality. There are 120 new machines that have bad quality. There are 230 used machines. Suppose that a machine is chosen at random, with each machine being equally likely to be chosen.

(a) What is the probability that the chosen machine is a new machine with good quality?

(b) If the chosen machine is new, what is the probability that it has good quality?

1.10.19 A fair coin is tossed five times. What is the probability that there is not a sequence of three outcomes of the same kind?

1.10.20 Consider telephone calls made to a company's complaint line. Let A be the event that the call is answered within 10 seconds. Let B be the event that the call is answered by one of the company's experienced telephone operators. Let C be the event that the call lasts less than 5 minutes. Let D be the event that the complaint is handled successfully by the telephone operator. Describe the following events.

(a) $B \cap C'$ (b) $(A \cup B') \cap D$

(c) $A' \cap C' \cap D'$ (d) $(A \cap C) \cup (B \cap D)$

1.10.21 A manager has 20 different job orders, of which 7 must be assigned to production line I, 7 must be assigned to production line II, and 6 must be assigned to production line III.

(a) In how many ways can the assignments be made?

(b) If the first job and the second job must be assigned to the same production line, in how many ways can the assignments be made?

(c) If the first job and the second job cannot be assigned to the same production line, in how many ways can the assignments be made?

1.10.22 A hand of 3 cards (without replacement) is chosen at random from an ordinary deck of 52 playing cards.

(a) What is the probability that the hand contains only diamonds?

(b) What is the probability that the hand contains one ace, one king, and one queen?

1.10.23 A hand of 4 cards (without replacement) is chosen at random from an ordinary deck of 52 playing cards.

(a) What is the probability that the hand does not have any aces?

(b) What is the probability that the hand has exactly on ace?

Suppose now that the 4 cards are taken with replacement.

(c) What is the probability that the same card is obtained four times?

1.10.24 Are the following statements true or false?

(a) If a fair coin is tossed three times, the probability obtaining two heads and one tail is the same as th probability of obtaining one head and two tails.

(b) If a card is drawn at random from a deck of cards the probability that it is a heart increases if it is conditioned on the knowledge that it is an ace.

(c) The number of ways of choosing five different letters from the alphabet is more than the numbe seconds in a year.

(d) If two events are independent, then the probability that they both occur can be calculated by multiplying their individual probabilities.
(e) It is always true that $P(A|B) + P(A'|B) = 1$.
(f) It is always true that $P(A|B) + P(A|B') = 1$.
(g) It is always true that $P(A|B) \leq P(A)$.

1.10.25 There is a probability of 0.55 that a soccer team will win a game. There is also a probability of 0.85 that the soccer team will not have a player sent off in the game. However, if the soccer team does not have a player sent off, then there is a probability of 0.60 that the team will win the game. What is the probability that the team has a player sent off but still wins the game?

1.10.26 It is found that 28% of orders received by a company are from first-time customers, with the other 72% coming from repeat customers. In addition, 75% of the orders from first-time customers are dispatched within one day, and overall 30% of the company's orders are from repeat customers whose orders are not dispatched within one day. If an order is dispatched within one day, what is the probability that it was for a first-time customer?

1.10.27 A class has 250 students, 113 of whom are male, and 167 of whom are mechanical engineers. There are 52 female students who are not mechanical engineers. There are 19 female mechanical engineers who are seniors.
(a) If a randomly chosen student is not a mechanical engineer, what is the probability that the student is a male?
(b) If a randomly chosen student is a female mechanical engineer, what is the probability that the student is a senior?

1.10.28 Twenty copying jobs need to be done. If there are four copy machines, in how many ways can five jobs be assigned to each of the four machines? If an additional copier is used, in how many ways can four jobs be assigned to each of the five machines?

1.10.29 (a) If four cards are taken at random from a pack of cards without replacement, what is the probability of having exactly two hearts?
(b) If four cards are taken at random from a pack of cards without replacement, what is the probability of having exactly two hearts and exactly two clubs?
(c) If four cards are taken at random from a pack of cards without replacement and it is known that there are no clubs, what is the probability that there are exactly three hearts?

1.10.30 Applicants have a 0.26 probability of passing a test when they take it for the first time, and if they pass it they can move on to the next stage. However, if they fail the test the first time, they must take the test a second time, and when applicants take the test for the second time there is a 0.43 chance that they will pass and be allowed to move on to the next stage. Applicants are rejected if the test is failed on the second attempt.
(a) What is the probability that an applicant moves on to the next stage but needs two attempts at the test?
(b) What is the probability that an applicant moves on to the next stage?
(c) If an applicant moves on to the next stage, what is the probability that he or she passed the test on the first attempt?

1.10.31 A fair die is rolled five times. What is the probability that the first score is strictly larger than the second score, which is strictly larger than the third score, which is strictly larger than the fourth score, which is strictly larger than the fifth score (i.e., the five scores are strictly decreasing).

1.10.32 A software engineer makes two backup copies of his file, one on a CD and another on a flash drive. Suppose that there is a probability of 0.05% that the file is corrupted when it is backed-up onto the CD, and a probability of 0.1% that the file is corrupted when it is backed-up onto the flash drive, and that these events are independent of each other. What is the probability that the engineer will have at least one uncorrupted copy of the file?

1.10.33 A warning light in the cockpit of a plane is supposed to indicate when a hydraulic pump is inoperative. If the pump is inoperative, then there is a probability of 0.992 that the warning light will come on. However, there is a probability of 0.003 that the warning light will come on even when the pump is operating correctly. Furthermore, there is a probability of 0.996 that the pump is operating correctly. If the warning light comes on, what is the probability that the pump really is inoperative?

1.10.34 Two cards are drawn from a pack of cards. Is it more likely that two hearts will be drawn when the drawing is with replacement or without replacement?

1.10.35 There are 11 items of a product on a shelf in a retail outlet, and unknown to the customers, 4 of the items are overage. Suppose that a customer takes 3 items at random.

(a) What is the probability that none of the overage products are selected by the customer?
(b) What is the probability that exactly 2 of the items taken by the customer are overage?

1.10.36 What is the sample space when a *winner* and a *runner-up* are chosen in a tournament with four contestants.

1.10.37 The marketing division of a company profiles its potential customers and grades them as either likely or unlikely purchasers. Overall, 16% of the potential customers are graded as likely purchasers. In reality, 81% of the potential customers graded as likely purchasers actually make a purchase, while only 9% of the potential customers graded as unlikely purchasers actually make a purchase. If somebody made a purchase, what is the probability that they had been graded as a likely purchaser?

"When solving mysteries like this one, it's always a question of prior probabilities and posterior probabilities." (From *Inspector Morimoto and the Two Umbrellas*, by Timothy Hemion)