Homework 4.

(Due Sep. 29)

4.1.4 When employees undergo an evaluation, their scores are independent and uniformly distributed between 60 and 100.

(a) If six employees take the evaluation, what is the probability that half of them score more than 85 and half less?

(b) If six employees take the evaluation, what is the probability that two of them score less than 80, two of them score between 80 and 90, and the remaining two score more than 90?

4.2.6 Imperfections in an optical fiber are distributed according to a Poisson process such that the distance between imperfections in meters has an exponential distribution with parameter $\lambda = 2\text{m}^{-1}$.

(a) What is the expected distance between imperfections?

(b) What is the probability that the distance between two imperfections is longer than 1 meter?

(c) What is the distribution of the number of imperfections in a 3-meter stretch of fiber?

(d) What is the probability that a 3-meter stretch of fiber has no more than four imperfections?

4.2.12 A researcher plants 12 seeds whose germination times in days are independent exponential distributions with $\lambda = 0.31$.

(a) What is the probability that a given seed germinates within five days?

(b) What are the expectation and variance of the number of seeds germinating within five days?

(c) What is the probability that no more than nine seeds have germinated within five days?

4.3.6 Recall problem 4.2.7 concerning the arrivals at a factory first-aid room.

(a) What is the distribution of the time between the first arrival of the day and the fourth arrival?

(b) What is the expectation of this time?

(c) What is the variance of this time?

(d) By using (i) the gamma distribution and (ii) the Poisson distribution, show how to calculate the probability that this time is longer than 3 hours.

4.2.7 [not a homework problem] The arrival times of workers at a factory first-aid room satisfy a Poisson process with an average of 1.8 per hour.

4.4.8 The time to failure in hours of an electrical circuit subjected to a high temperature has a Weibull distribution with parameter $a = 3$ and $\lambda = 0.5$.

(a) What is the median failure time of a circuit?

(b) [not homework] The circuit engineers can be 99% confident that a circuit will last as long as what time?

(c) What are the expectation and variance of the circuit failure times?

(d) If a circuit has three equivalent backup circuits that have independent failure times, what is the probability that at least one circuit is working after 3 hours?

4.5.6 Consider the beta probability density function

$$f(x) = Ax^9(1-x)^3 \tag{1}$$

for $0 \le x \le 1$ and $f(x) = 0$ elsewhere.

(a) What are the values of the parameters a and b?

(b) Use the answer to part (a) to calculate the value of A.

(c) What is the expectation of this distribution?

4.8.5 Figure 4.25 shows the probability density function of a *triangle distribution* $T(a, b)$ with endpoints a and b.

(a) What is the height of the probability density function at $(a + b)/2$?

(b) If the random variable X has a $T(a, b)$ distribution, what is $P(X \le a/4 + 3b/4)$?

(c) What is the variance of a $T(a, b)$ distribution?

(d) Calculate the cumulative distribution function of a $T(a, b)$ distribution.

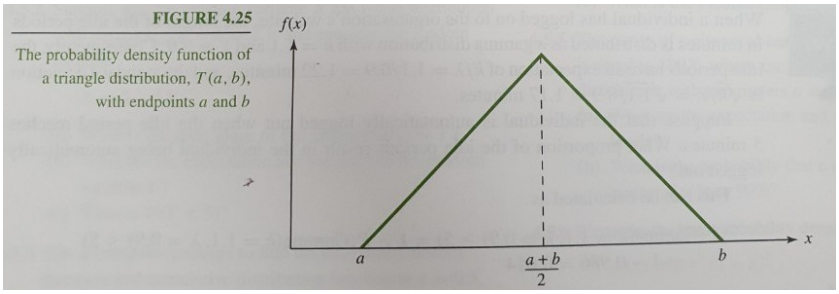

FIGURE 4.25
The probability density function of a triangle distribution, $T(a, b)$, with endpoints a and b

4.8.8 Suppose that customer waiting times are independent and can be modeled by a Weibull distribution with $a = 2.3$ and $\lambda = 0.09$ per minute. What is the probability that out of ten customers, exactly three wait less than 8 minutes, exactly four wait between 8 and 12 minutes, and exactly three wait more than 12 minutes?