HOMEWORK 3 - CC511

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3.1.12

(a) $P(B(10, 0.2) \geq 7) = P(X=7) + P(X=8) + P(X=9) + P(X=10)$

- $P(X=7) = \binom{10}{7}\left(\frac{1}{5}\right)^7\left(\frac{4}{5}\right)^{10-7} = 0.00079$
- $P(X=8) = \binom{10}{8}\left(\frac{1}{5}\right)^8\left(\frac{4}{5}\right)^{10-8} = 0.00007$
- $P(X=9) = \binom{10}{9}\left(\frac{1}{5}\right)^9\left(\frac{4}{5}\right)^{10-9} = 0.0000041$
- $P(X=10) = \binom{10}{10}\left(\frac{1}{5}\right)^{10}\left(\frac{4}{5}\right)^{10-10} = 0.0000001$

$P(B(10, 0.2) \geq 7) = 0.00079 + 0.00007 + 0.0000041 + 0.0000001 \approx \boxed{0.0009}$

(b) $P(B(10, 0.5) \geq 7) = \sum_{x_i=7}^{10} P(X=x_i)$

- $P(X=7) = \binom{10}{7}\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{10-7} = \binom{10}{7}\left(\frac{1}{2}\right)^{10}$
- $P(X=8) = \binom{10}{8}\left(\frac{1}{2}\right)^{10}$
- $P(X=9) = \binom{10}{9}\left(\frac{1}{2}\right)^{10}$
- $P(X=10) = \binom{10}{10}\left(\frac{1}{2}\right)^{10}$

$P(B(10, 0.5) \geq 7) = \left[\binom{10}{7} + \binom{10}{8} + \binom{10}{9} + \binom{10}{10}\right] \cdot \left(\frac{1}{2}\right)^{10} =$

$= 176 \times 9.7656 \cdot 10^{-4} \approx \boxed{0.1719}$

3.2.20

(a) $E(X) = \frac{1}{p} = \frac{1}{0.77} = \boxed{1.2987}$

(b) We consider a geometric distribution with $p = 0.23$,

Thus: $P(X=5) = (1-0.23)^{5-1} \times 0.23 = \boxed{0.0809}$

(c) Probability of $r=3$ when $x=6 \rightarrow$ Binomial distribution

$\binom{5}{2} \times (1-0.77)^3 \cdot 0.77^3 = \boxed{0.0555}$

5) 1(d) $P(B(8, 0.77) \geq 3) = 1 - P(B(8, 0.77) \leq 2)$

so,

$$\Sigma = \begin{cases} \cdot\ P(B(8, 0.77) = 0) = \binom{8}{0} (0.23)^8 = 7.83 \cdot 10^{-6} \\ \cdot\ P(B(8, 0.77) = 1) = \binom{8}{7} (0.23)^7 (0.77) = 2.40 \cdot 10^{-4} \\ \cdot\ P(B(8, 0.77) = 2) = \binom{8}{2} (0.23)^6 \cdot (0.77)^2 = \cancel{3} \ 2.46 \cdot 10^{-3} \end{cases}$$

$\Rightarrow P(B(0,77) \geq 3) = 1 - \Sigma \approx \boxed{0.9973}$

PAG 1/3 (bis)

$\boxed{3.38}$ $P(5 \le X \le 7) = P(X=5) + P(X=6) + P(X=7)$

$$= \frac{\binom{9}{4}\binom{6}{1}}{\binom{15}{5}} + \frac{\binom{9}{3}\binom{6}{2}}{\binom{15}{5}} + \frac{\binom{9}{2}\binom{6}{3}}{\binom{15}{5}} = [0.9111]$$

$\boxed{3.4.8}$ Poisson distribution with $\lambda = 9.2$

$$P(X) = \frac{e^{-9.2}\, 9.2^{x}}{x!}$$

(a) $P(6 \le X \le 10) = P(X=6) + P(X=7) + P(X=8) + P(X=9) + P(X=10)$

$$= e^{-9.2}\left(\frac{9.2^6}{6!} + \frac{9.2^7}{7!} + \frac{9.2^8}{8!} + \frac{9.2^9}{9!} + \frac{9.2^{10}}{10!}\right) \approx [0.5780]$$

(b) $P(X \le 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$$= e^{-9.2}\left(\frac{9.2^0}{0!} + \frac{9.2^1}{1!} + \frac{9.2^2}{2!} + \frac{9.2^3}{3!} + \frac{9.2^4}{4!}\right) \approx [0.0486]$$

$\boxed{3.5.4}$

(a) $P(X_6 = 3, X_5 = 3) = \frac{15!}{3!\,3!\,9!}\left(\frac{1}{6}\right)^3\left(\frac{1}{6}\right)^3\left(\frac{2}{3}\right)^9 = [0.0558]$

(b) $P(X_6 = 3, X_5 = 3, X_4 = 4) = \frac{15!}{3!\,3!\,4!\,5!}\left(\frac{1}{6}\right)^3\left(\frac{1}{6}\right)^3\left(\frac{1}{6}\right)^4\left(\frac{4}{6}\right)^5 = [0.0065]$

(c) $P(X_6 = 2) = \frac{15!}{2!\,13!}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{13} = [0.2726]$

→ Expected number of 6s: $np_i = 15 \cdot \frac{1}{6} = [2.5]$

$\boxed{3.8.4}$ Poisson distribution: we want $E(X) = \lambda = \frac{40}{60} = \frac{2}{3}$

cousin minute

so $P(X) = \frac{e^{-\frac{2}{3}}\left(\frac{2}{3}\right)^x}{x!}$

(a) $P(X=0) = \frac{e^{-\frac{2}{3}}\left(\frac{2}{3}\right)^0}{0!} = [0.5134]$

(b) $P(X=1) = \frac{e^{-\frac{2}{3}}\left(\frac{2}{3}\right)^1}{1!} = [0.3423]$

(c) $P(X \ge 3) = 1 - P(X \le 2) = 1 - [P(X=0) + P(X=1) + P(X=2)]$

so, $P(X=2) = \frac{e^{-\frac{2}{3}}\left(\frac{2}{3}\right)^2}{2!} = 0.1141$

$P(X \ge 3) = 1 - [0.5134 + 0.3423 + 0.1141] = [0.0302]$

$\boxed{3.8.10}$ (a) We can use a negative binomial distribution with $p = 0.55$ for Team A winning and $r = 4$ to denote the number of successes needed for winning the series.

(b) $P(X=7) = \binom{6}{3} \cdot (1-0.55)^3 \cdot 0.55^4 = [0.1668]$

(c) $P(X=6) = \binom{5}{3} (1-0.55)^2 \cdot 0.55^4 = [0.1853]$

(d) Probability game is over for $X=5$ = $P(A \text{ winning}) + P(B \text{ winning})$

$\to P(A \text{ winning}, X=5) = \binom{4}{3} 0.45^1 0.55^4 = [0.1647]$

$\to P(B \text{ winning}, X=5) = \binom{4}{3} (1-0.45)^1 0.45^4 = [0.0902]$

$\to P(X=5, \text{ winning one Team}) = 0.1647 + 0.0902 = [0.2549]$

(e) $P(X=4) = \binom{3}{3} 0.45^0 \cdot 0.55^4 = 0.55^4 = 0.0915$

↳ A winning at game 4

$\to$ So, we have $P(A \text{ winning}) = P(X=4) + P(X=5) + P(X=6) + P(X=7)$

$= 0.0915 + 0.1647 + 0.1853 + 0.1668 = [0.6083]$

$\boxed{3.8.14}$ (a) If we want to find the probability to have at least 16 cells, then at least 3 have to be created.

$\to P(X \geq 16) \to P(X_{new} \geq 3) = 1 - [P(X_{new}=0) + P(X_{new}=1) + P(X_{new}=2)]$

$= 1 - \left[\left(\frac{13!}{13!}\right)(0.6)^{13} + \left(\frac{13!}{1!\,12!}\right)(0.4)\cdot(0.6)^{12} + \left(\frac{13!}{2!\,11!}\right)(0.4)^2\cdot(0.6)^{11}\right] =$

$= 1 - (0.0013 + 0.0113 + 0.0453) = 0.9421$

(b) $E(X) = np_i = 13 \cdot 0.4 = 5.2$ new cells

$\to 13 + 5.2 = [18.2]$

old cells   new cells