

Homework 5.

(Due Oct. 6)

5.1.6 Suppose that $X \sim N(\mu, \sigma^2)$ and that

$$P(X \leq 5) = 0.8 \quad \text{and} \quad P(X \geq 0) = 0.6. \quad (1)$$

What are the values of μ and σ^2 ?

5.1.8 What are the upper and lower quartiles of a $N(0, 1)$ distribution? What is the interquartile range? What is the interquartile range of a $N(\mu, \sigma^2)$ distribution?

5.1.16 Bricks have weights that are independently distributed with a normal distribution that has a mean 1320 and a standard deviation of 15. A set of ten bricks is chosen at random. What is the probability that exactly three bricks will weigh less than 1300, exactly four bricks will weigh between 1300 and 1330, and exactly three bricks will weigh more than 1330?

5.2.12 Five students are waiting to talk to the TA when office hours begin. The TA talks to the students one at a time, starting with the first student and ending with the fifth student, with no breaks between students. Suppose that the time taken by the TA to talk to a student has a normal distribution with a mean 8 minutes and a standard deviation of 2 minutes, and suppose that the times taken by the students are independent of each other.

- (a) What is the probability that the total time taken by the TA to talk to all five students is longer than 45 minutes?
- (b) Suppose that the time that elapses between when the TA starts talking to the first student, and when the TA starts to have a headache, has a normal distribution with a mean of 28 minutes and a standard deviation of 5 minutes, which is independent of the times taken to talk to the students. What is the probability that the TA's headache starts at a time after TA has finished talking to the third student?

5.2.14 Suppose that $X \sim N(-1.9, 2.2)$, $Y \sim N(3.3, 1.7)$, and $Z \sim N(0.8, 0.2)$ are independent random variables. Find the probability that

- (a) $X - Y \geq -3$
- (b) $2X + 3Y + 4Z \leq 10$
- (c) $3Y - Z \leq 8$
- (d) $2X - 2Y + 3Z \leq -6$
- (e) $|X + Y - Z| \geq 1.5$
- (f) $|4X - Y + 10| \leq 0.5$

5.3.4 A multiple-choice test consists of a series of questions, each with *four* possible answers.

- (a) If there are 60 questions, estimate the probability that a student who guesses blindly at each question will get at least 30 questions right.

- (b) How many questions are needed in order to be 99% confident that a student who guesses blindly at each question scores no more than 35% on the test?

5.3.8 Suppose that a fair die is rolled 1000 times.

- (a) Estimate the probability that the number of 6s is between 150 and 180.
- (b) What is the smallest value of n for which there is a probability of at least 99% of obtaining at least 50 6s in n rolls of a fair die?

5.3.16 Calculate the following probabilities both exactly and by using a normal approximation:

- (a) $P(X \geq 7)$ where $X \sim B(10, 0.3)$
- (b) $P(9 \leq X \leq 12)$ where $X \sim B(21, 0.5)$
- (c) $P(X \leq 3)$ where $X \sim B(7, 0.2)$
- (d) $P(9 \leq X \leq 11)$ where $X \sim B(12, 0.65)$