HOMEWORK 1 - PROBABILITY AND STATISTICS

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Ex 1.8

$P(I) + P(II) + P(III) + P(IV) + P(V) = 1$, so

$P(V) = 1 - P(I) - P(II) - P(III) - P(IV)$

$= 1 - 0.13 - 0.24 - 0.09 - 0.38 = \boxed{0.18}$

Ex 3.12

we know that $P(A \cup B) = P(A) + P(B) - P(A \cap B) \leq 1$

thus, $P(B) \leq 1 - 0.4 + 0.3 = 0.9$ (first constraint)

Moreover, the second condition is that $P(B) \geq P(A \cap B) = 0.3$

$\Rightarrow$ The condition becomes: $\boxed{0.3 \leq P(B) \leq 0.9}$

Ex 4.10

(a) $A \rightarrow$ both lines at full capacity, with $\rightarrow \{(F,F)\}$ outcome

$B \rightarrow$ neither line is shut down $\rightarrow \{(P,P),(P,F),(F,P),(F,F)\}$

$A \cap B = \{(F,F)\}$

So, $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.19}{0.14 + 0.2 + 0.21 + 0.19} = \boxed{0.257}$

(b) $C \rightarrow$ just one line is at full capacity $\rightarrow$ outcomes:

$\{(F,P),(F,S), \cancel{(P,F)}, \cancel{(S,F)}\}$

$(F,F),(S,F),(P,F)$

So, $P(C|B) = \dfrac{P(C \cap B)}{P(B)} = \dfrac{0.19}{0.14 + 0.2 + 0.21 + 0.19} = \boxed{0.817}$

(c) $D \rightarrow$ one line at full capacity $\rightarrow \{(F,P),(F,S),(P,F),(S,F)\}$

$E \rightarrow$ one line is shut down $\rightarrow \{(S,P),(S,F),(P,S),(F,S)\}$

$D \cap E = \{(S,F),(F,S)\}$

So, $P(D|E) = \dfrac{P(D \cap E)}{P(E)} = \dfrac{0.06 + 0.05}{0.06 + 0.05 + 0.07 + 0.06} = \boxed{0.458}$

(d) $G \rightarrow$ neither is at full capacity $\rightarrow \{(S,S),(P,S),(S,P),(P,P)\}$

$H \rightarrow$ at least one line at partial capacity $\{(S,P),(P,S),(P,P),(P,F),(F,P)\}$

thus, $F \cap G = \{(S,P),(P,S),(P,P)\}$

So, $P(F|G) = \dfrac{P(F \cap G)}{P(G)} = \dfrac{0.06 + 0.07 + 0.14}{0.06 + 0.07 + 0.14 + 0.2 + 0.21} = \boxed{0.397}$

Ex 4.18 Event T: On time repair
Event S: Satisfactory repair

- $P(S|T) = 0.85$ $P(T) = 0.77$

~~[illegible]~~ Given the conditional probability:

$$P(S|T) = \frac{P(T \cap S)}{P(T)}$$ we conversely obtain

The probability of on time and satisfactory repairs, which is:

$$P(T \cap S) = P(S|T) \times P(T) = 0.85 \times 0.77 = 0.6545$$

Ex 5.14 Probability of Targeted campaign =

$= P($ (launched and Targeted ~~[illegible]~~ $\cup$ extended and Targeted $)$

$= 0.43 + 0.29 = 0.72$

Since C_1, C_2, C_3, C_4 are independent, Then we have to multiply:

$0.72 \times 0.72 \times 0.72 \times 0.72 = \boxed{0.2687}$

Ex 6.6 R = Red Ball B = Blue Ball

(a) $P(R) = (P(R \mid \text{Bag } A) \times P(\text{Bag } A)) + (P(R \mid \text{Bag } B) \times P(\text{Bag } B)) +$
$+ (P(R \mid \text{Bag } C) \times P(\text{Bag } C)) = \left(\frac{1}{3} \times \frac{3}{10}\right) + \left(\frac{1}{3} \times \frac{8}{12}\right) + \left(\frac{1}{3} \times \frac{5}{10}\right) =$
$= \boxed{0.426}$

(b) $P(B) = 1 - P(R) = 1 - 0.426 = 0.574$

(c) $P(R \text{ from Bag } B) = P(\text{Bag } B) \times P(R \mid \text{Bag } B) = \frac{1}{3} \times \frac{8}{12} = \boxed{\frac{2}{9}}$

- If a Red ball is chosen, probability it comes from Bag A?

$$P(\text{Bag } A \mid R) = \frac{P(\text{Bag } A \cap R)}{P(R)} = \frac{P(\text{Bag } A) \times P(R \mid \text{Bag } A)}{P(R)} =$$

$$= \frac{\frac{1}{3} \times \frac{3}{10}}{0.426} = \boxed{0.235}$$

- If a Blue ball is chosen, probability it comes from Bag B?

$$P(\text{Bag } B \mid B) = \frac{P(\text{Bag } B \cap B)}{P(B)} = \frac{P(\text{Bag } B) \times P(B \mid \text{Bag } B)}{P(B)} =$$

$$= \frac{\frac{1}{3} \times \frac{4}{12}}{0.574} = \boxed{0.194}$$

Ex 10.8 Coordinates: (a, b) where a = red die result
b = blue die result

(a) Outcomes for the "sum of the two scores is eighth"

$\Rightarrow$ $\{(2,6), (3,5), (4,4), (5,3), (6,2)\}$

So, looking at the result table, we have: → total of 36 results

$P(\text{red die is } 5 \mid \text{sum of scores is } 8) = \dfrac{P(\text{red die is } 5 \cap \text{sum of scores is } 8)}{P(\text{sum of scores is } 8)}$

$= \dfrac{\left(\frac{1}{36}\right)}{\frac{5}{36}} = \left[\frac{1}{5}\right]$

independent events

(b) $P(\text{either score is } 5 \mid \text{sum of scores is } 8) = 2 \times \frac{1}{5} = \left[\frac{2}{5}\right]$

(c) The score on either die is 5 has these outcomes:

$\{(1,5), (2,5), (3,5), (4,5), (5,5), (6,5), (5,6), (5,4), (5,3), (5,2), (5,1)\}$

So we can calculate:

$\dfrac{P(\text{sum of scores is } 8 \cap \text{either score is } 5)}{P(\text{either score is } 5)} = \dfrac{\left(\frac{2}{36}\right)}{\left(\frac{11}{36}\right)} = \dfrac{2}{11}$

EX 10.20

(a) $B \cap C'$: calls answered by an experienced operator last over 5 minutes

(b) $(A \cup B') \cap D$: complaint handled successfully and answered either within 10 seconds or by an inexperienced operator, or both

(c) $A' \cap C' \cap D'$: calls answered after ten seconds that lasted more than five minutes and not handled successfully.

(d) $(A \cap C) \cup (B \cap D)$: calls either answered within ten seconds and lasted less than five minutes, or handled by an experienced operator successfully.

EX 10.32 we need to find the probability to have at least one uncorrupted file, so:

$P(\text{at least one uncorrupted file}) = 1 - P(\text{both files are corrupted})$

$= 1 - (0.0005 \times 0.001) = \left[\begin{array}{l} 0.9999995 \\ \text{or } 99.99995\ \% \end{array}\right]$

EX 10.36

four contestants are: A, B, C, D

winner is x, runner up is y → (x, y)

The sample space is:

$S = \{(A,B), (A,C), (A,D), (B,A), (B,C), (B,D), (C,A), (C,B), (C,D), (D,A), (D,B), (D,C)\}$