Homework 10.

(Due Nov. 24)

12.1.6 Suppose that the purity of a chemical solution y is related to the amount of a catalyst x through a linear regression model with $\beta_0 = 123.0$, $\beta_1 = -2.16$, and with an error standard deviation $\sigma = 4.1$.

(a) What is the expected value of the purity when the catalyst level is 20?

(b) How much does the expected value of the purity change when the catalyst level increases by 10?

(c) What is the probability that the purity is less than 60.0 when the catalyst level is 25?

(d) What is the probability that the purity is between 30 and 40 when the catalyst level is 40?

(e) What is the probability that the purity of a solution with a catalyst level of 30 is smaller than the purity of a solution with a catalyst level of 27.5?

12.2.4 **Oil Well Drilling Costs**
Estimating the costs of drilling oil wells is an important consideration for the oil industry. DS 12.2.1 contains the total costs and the depths of 16 offshore oil wells located in the Philippines (taken from "Identifying the major determinants of exploration drilling costs: A first approximation using the Philippine case" by Gary S. Makasiar, Energy Exploration and Exploitation, 1985).

(a) Fit a linear regression model with costs as the dependent variable and depth as the explanatory variable.

(b) What does your model predict as the cost increase for an additional depth of 1000 feet?

(c) What cost would you predict for an oil well of 10,000 feet depth?

(d) What is the estimate of the error variance?

(e) What could you say about the cost of an oil well of depth 20,000 feet?

12.2.10 A data set has $n = 20$, $\sum_{i=1}^{20} x_i = 8.552$, $\sum_{i=1}^{20} y_i = 398.2$, $\sum_{i=1}^{20} x_i^2 = 5.196$, $\sum_{i=1}^{20} y_i^2 = 9356$, and $\sum_{i=1}^{20} x_i y_i = 216.6$. Calculate $\hat{\beta}_0, \hat{\beta}_1$, and $\hat{\sigma}^2$. What is the fitted value when $x = 0.5$?

12.3.2 In a simple linear regression analysis with $n = 22$ data points, an estimate $\hat{\beta}_1 = 56.33$ is obtained with s.e.($\hat{\beta}_1$)= 3.78.

(a) Calculate a two-sided 95% confidence interval for the slope parameter β_1.

(b) Test the null hypothesis $H_0 : \beta_1 = 50.0$ against a two-sided alternative hypothesis.

12.3.8 **Vacuum Transducer Bobbin Resistances**
Consider the data set of vacuum transducer bobbin resistance given in DS 12.2.6.

(a) What is the standard error of $\hat{\beta}_1$?

(b) Construct a two-sided 99% confidence interval for the slope parameter β_1.

(c) Test the null hypothesis $H_0 : \beta_1 = 0$. Is it clear that resistance increases with temperature?

12.4.2 In a simple linear regression analysis with $n = 17$ data points, the estimates $\hat{\beta}_0 = 12.08$, $\hat{\beta}_1 = 34.60$, and $\hat{\sigma}^2 = 17.65$ are obtained. If $S_{XX} = 1096$ and $\bar{x} = 53.2$, construct a two-sided 95% confidence interval for the expected response at $x* = 40.0$.

12.4.8 **Vacuum Transducer Bobbin Resistances**
Consider the data set of vacuum transducer bobbin resistance given in DS 12.2.6. Construct a two-sided 99% confidence interval for the average resistance of a vacuum transducer bobbin at a temperature of 70°F.

12.5.4 **VO2-max Aerobic Fitness Measurements**
Consider the data set of aerobic fitness measurements given in DS 12.2.3. Construct and interpret a two-sided 95% prediction interval for the VO2-max measurement of a new experimental subject who is a 50-year-old male.

12.5.10 The amount of catalyst (x) and the yield (y) of a chemical experiment are analyzed using a simple linear regression model. There are 30 observations (x_i, y_i), and it is found that the fitted model is

$$y = 51.98 + 3.44x$$

Suppose that the sum of squared residual is $\sum_{i=1}^{30} e_i^2 = 329.77$, and that $\sum_{i=1}^{30} x_i = 603.36$, $\sum_{i=1}^{30} x_i^2 = 12578.22$. If an additional experiment is planned with a catalyst level 22, give a range of values with 95% confidence for the yield that you think will be obtained for that experiment.

12.6.6 **Truck Unloading Times**
Consider the data set of the times taken to unload a truck at a warehouse given in DS 12.2.2. Compute the analysis of variance table and calculate the coefficient of determination R^2. Check that the F-statistic is the square of the t-statistic for testing $H_0 : \beta_1 = 0$, calculated earlier. What is the implication of the p-value in the analysis of variance table?