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Step responses of low-order systems and model identification from step response data

Exercise 2.1 - Step response of a first-order system
Consider the system described by the transfer function

$$G(s) = \frac{2}{1+10s}$$

1. Give the steady-state gain K, the time-constant T and the pole.
2. Recall the unit step response and calculate its slope at the origin.
3. Calculate the rise-times $T_m^{63\%}$ and $T_m^{95\%}$ as well as the settling-time $T_r^{5\%}$.
4. Plot precisely the step response and indicate on it its characteristic parameters calculated above.

Exercise 2.2 - Step response of a first-order plus time-delay system
Consider a first order system having a steady-state gain of 2, a time-constant of 10 seconds and a pure time-delay of 3 seconds

1. Give the system transfer function $G(s)$.
2. Without any calculation, plot precisely the unit step response and indicate on it its characteristic parameters computed above.

Exercise 2.3 - Step response of an underdamped second-order system
Consider the system whose dynamic behavior is governed by the following differential equation:

$$\ddot{y}(t) + 4\dot{y}(t) + 8y(t) = 2x(t) \qquad \text{with } \dot{y}(0) = 0, \quad y(0) = 0$$

1. Determine the transfer function $G(s)$ of the system.
2. Determine the order of the system, the steady-state gain K, the damping coefficient z, the undamped natural frequency ω_0, the poles and zeros.
3. Conclude about the type of step response: critical, overdamped or underdamped.
4. Calculate the values $D_{1\%}$ and $D_{2\%}$ of the first and second overshoot, the times of the first and second overshoot t_{D_1} and t_{D_2}.
5. Plot the step response and indicate on it the characteristic parameters computed above.

Exercise 2.4 - Identification of a heat exchanger model from step response data
The experimental step response of a heat exchanger is shown in 2.1.

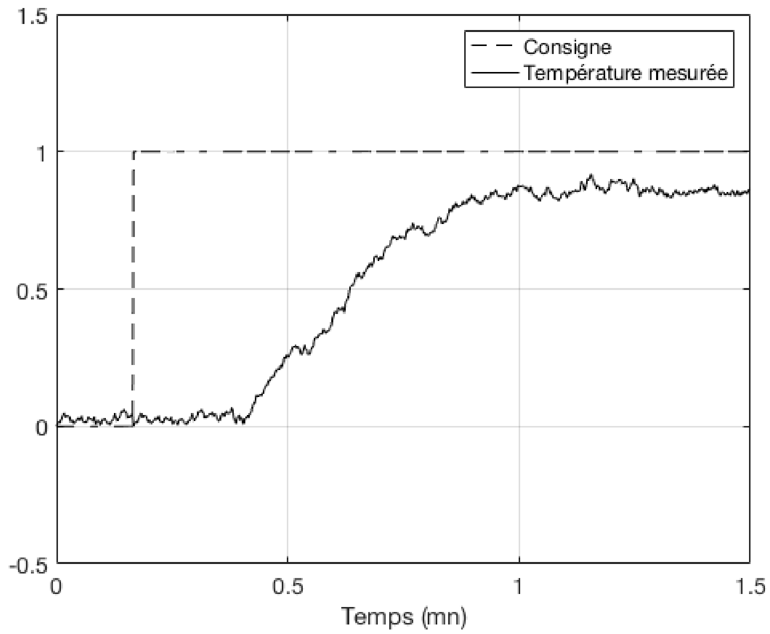

Figure 2.1: Step response of the heat exchanger (experimental data)

From the step response shown in Figure 2.1:

1. Propose a transfer function model form $G(s)$ for the heat exchanger. Justify your answer.
2. Determine the different parameters of your chosen transfer function model $G(s)$.
3. Deduce from your identified transfer function $G(s)$ the differential equation of the heat exchanger.

Exercise 2.5 - Model identification of a 2 inputs / 1 output system from step response data
Consider a LTI dynamic system with two inputs and one output described by:

$$Y(s) = G_1(s)X_1(s) + G_2(s)X_2(s)$$

In order to determine the transfer functions $G_1(s)$ and $G_2(s)$, the time response $y(t)$ of the system, shown in Figure 2.2, was observed experimentally for the following inputs:

$$x_1(t) = \Gamma(t) \quad \text{et} \quad x_2(t) = 2\Gamma(t-15)$$

1. Represent the system in the form of a block diagram.
2. Plot the time-evolution of the two inputs $x_1(t)$ and $x_2(t)$.
3. From the system response obtained for $x_1(t)$, propose a transfer function model $G_1(s) = \frac{Y(s)}{X_1(s)}$. Motivate your choice.
4. Determine the different parameters of your chosen model $G_1(s)$.
5. From the system response obtained at $x_2(t)$, propose a transfer function model $G_2(s) = \frac{Y(s)}{X_2(s)}$. Motivate your choice.
6. Determine the different parameters of your chosen model $G_2(s)$.

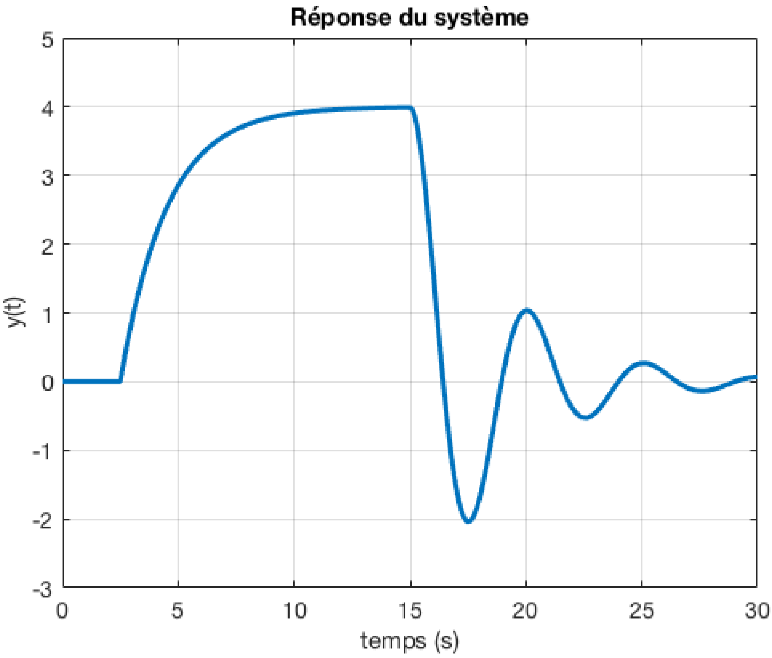

Figure 2.2: Step response of a 2 inputs - 1 output system

Exercise 2.6 - Step response analysis
Figure 2.3 shows the unit step responses of five different linear time-invariant systems. Pair each of the 5 step responses to one of the 7 transfer functions below. Justify your answers.

$$G_1(s) = \frac{0.1}{s+0.1}; \qquad G_2(s) = \frac{4}{s^2+2s+4}; \qquad G_3(s) = \frac{0.5}{s^2-0.1s+2}$$
$$G_4(s) = \frac{-0.5}{s^2+0.1s+2}; \qquad G_5(s) = \frac{1}{s+1}; \qquad G_6(s) = \frac{4}{s^2+0.8s+4}$$
$$G_7(s) = \frac{2}{s^2+s+3}$$

You can use Matlab to verify your solutions.

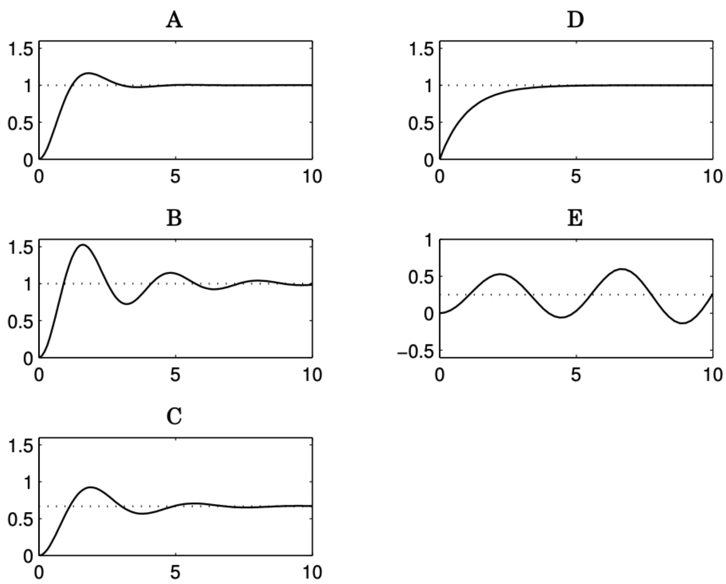

Figure 2.3: Step responses of different LTI systems