TD Automatique
continue n°1
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Laplace transform, transfer function and block diagram analysis of LTI dynamic systems

Exercise 1.1 - Solution of differential equations

Solve the following differential equations using the Laplace transform:

$$\dot{y}(t) + 2y(t) = \Gamma(t) \qquad \text{with } y(0) = 1$$
$$\ddot{y}(t) + 10\dot{y}(t) + 16y(t) = 10\delta(t) \qquad \text{with } \dot{y}(0) = y(0) = 0$$

Exercise 1.2 - Transfer function of a hydraulic system

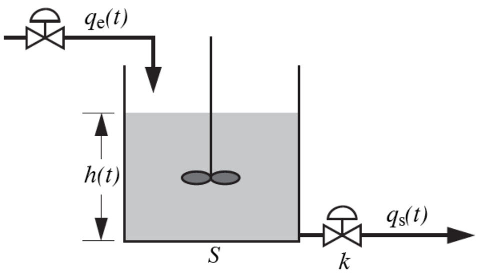

Figure 1.1: Hydraulic system.

Consider a tank represented in Figure 1.1. Using a mass balance and around a working point (H_0, Q_{e0}), the output flow $q_s(t)$ may be assumed to depend linearly on the height $h(t)$. The equation relating the variation of the liquid height $h(t)$ to the variation of the input flow $q_e(t)$ takes the form of the following ordinary differential equation with constant coefficients:

$$\frac{dh(t)}{dt} + ah(t) = bq_e(t)$$

1. Determine the transfer function $G(s) = \frac{H(s)}{Q_e(s)}$ and represent the system in the form of a block diagram.
2. Specify the system order, the steady-state gain, the pole(s) and zero(s) of the transfer function.

Exercise 1.3 - Transfer function of an electrical system

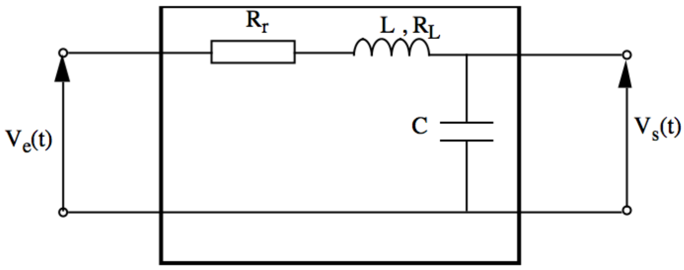

Figure 1.2: Electrical system.

Consider the RLC circuit shown in Figure 1.2. The differential equation relating the output voltage $V_s(t)$ to the input voltage $V_e(t)$ is:

$$LC\frac{d^2V_s(t)}{dt^2} + RC\frac{dV_s(t)}{dt} + V_s(t) = V_e(t) \qquad \text{with } R = R_r + R_L$$

1. Determine the transfer function $G(s) = \frac{V_s(s)}{V_e(s)}$ and represent the system in the form of a block diagram.
2. Specify the system order, the steady-state gain, the pole(s) and zero(s) of the transfer function.

Exercise 1.4 - Transfer function of a mechanical system
Consider a mechanical suspension system shown in Figure 1.3. of a damper and spring having the damping and stiffness coefficients of b and k respectively

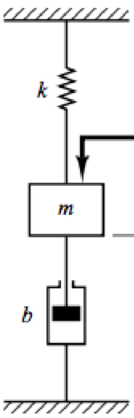

Figure 1.3: Mechanical system.

The differential equation of this mechanical system relating the vertical position $y(t)$ of the mass (output of the system) and the external force $u(t)$ applied to the system input is:

$$m\ddot{y}(t) + b\dot{y}(t) + ky(t) = u(t)$$

1. Determine the transfer function $G(s) = \frac{Y(s)}{U(s)}$ and represent the system in the form of a block diagram.
2. Specify the system order, the steady-state gain, the pole(s) and zero(s) of the transfer function.

Exercise 1.5 - Simplification of single input block diagrams
Consider the block diagrams displayed in Figures 1.4 and 1.5.
Derive their equivalent transfer function.

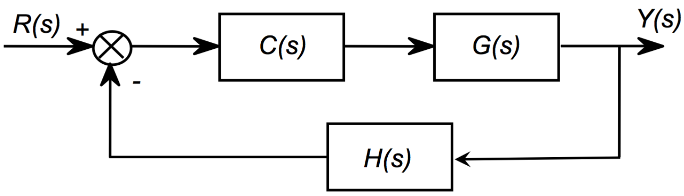

Figure 1.4:

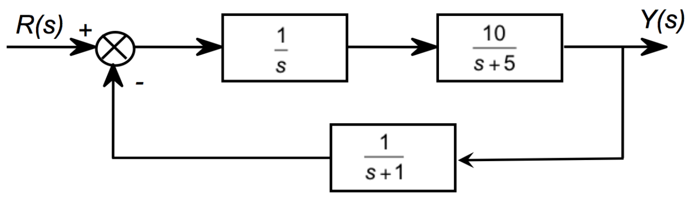

Figure 1.5:

Exercise 1.6 - Simplification of multiple input block diagrams
Consider the block diagrams shown in Figures 1.6 and 1.7.
Determine the transfer functions $T(s)$ and $S(s)$ in the following expression:

$$Y(s) = T(s)R(s) + S(s)D(s).$$

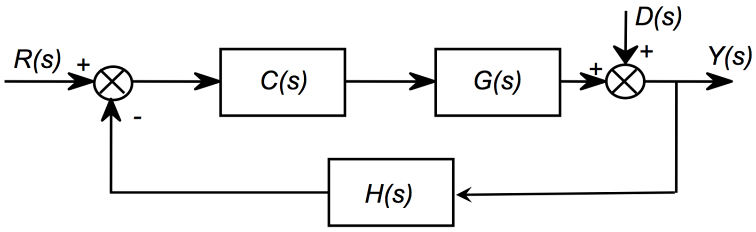

Figure 1.6:

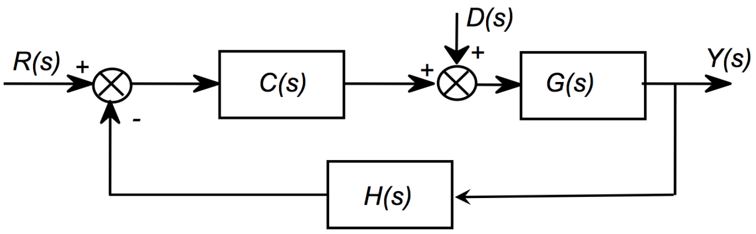

Figure 1.7: