H. Garnier
F. Collin

Automatic cruise control

Automatic cruise control is an excellent example of a feedback control system found nowadays in many vehicles. The purpose of the cruise control system is to maintain a constant vehicle speed despite external disturbances, such as changes in wind or road grade. This is usually accomplished by measuring the vehicle speed, comparing it to the desired or reference speed, and automatically adjusting the throttle according to a control law.

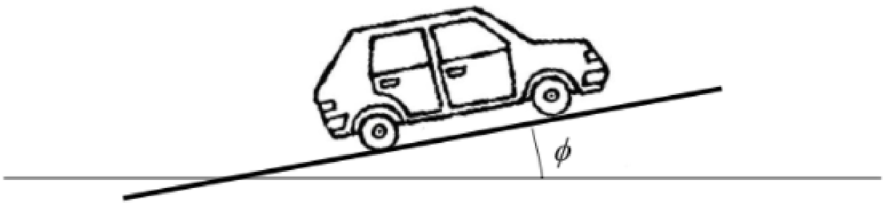

Figure 4.1: Simplified model of a car

We consider here a simple model of the vehicle represented in Figure 4.1. The vehicle, of mass m, is acted on by a control force, $f(t)$ (in N). The force $f(t)$ represents the force generated at the road/tire interface and $\phi(t)$ (in rad) denotes the angle of the road with the horizontal axis. For this simplified model we will assume that we can control the force $f(t)$ directly and will neglect the dynamics of the powertrain, tires, etc., that go into generating the force. The resistive forces due to rolling resistance and wind drag, are assumed to vary linearly with the vehicle velocity, $v(t)$ (in m/s) through the damping coefficient b, and act in the direction opposite the car motion.
Summing forces in the horizontal direction and applying Newton's second law, we arrive at the following differential equation:

$$m\dot{v}(t) + bv(t) = f(t) - mg\sin(\phi(t)) \tag{1}$$

where g is the acceleration due to gravity.
For this example, we assume that the physical parameters of the system are:

- $m = 1000$ kg
- $b = 200$ Ns/m
- $g \approx 10$ m/s^2.

1. **Modelling**

   1.a. Define the output $y(t)$, the input $u(t)$ and the disturbance variable $d(t)$ and their unit.

   1.b. Is the model describing the vehicle dynamics linear? Justify.

   1.c. Assuming that the road angle with respect to the horizontal remains small, linearise the model.

   1.d Let $Y(s)$, $U(s)$ and $D(s)$ denote the Laplace transforms of $y(t)$, $u(t)$ and $d(t)$ respectively. Show that equation (1) can be written in the Laplace domain as:

   $$Y(s) = G(s)U(s) + G_D(s)D(s)$$

   where $G(s) = \dfrac{K}{1+Ts}$ and $G_D(s) = \dfrac{K_D}{1+Ts}$.
   Express the value of the two steady-state gains K and K_D along with the time-constant T in terms of m, b and g.

1.e Compute the poles of each model and conclude about the stability.

1.f Represent the system in the form of a block diagram.

2. **Proportional feedback control (P)** Let $Y_r(s)$ denote the Laplace transform of the speed reference (or setpoint) $y_r(t)$. We want to drive at a constant speed of $y_r(t) = 25\Gamma(t)$ (25 m/s = 90 km/h). We initially assume that the road is flat $\phi(t) = 0$

3.a. We choose now to implement a simple feedback proportional (P) controller where the throttle is automatically adjusted according to the following control law:

$$u(t) = k_p \varepsilon(t) \qquad \text{with } \varepsilon(t) = y_r(t) - y(t)$$

where $k_p > 0$. Determine the controller transfer function $C(s)$ and recall its common name.

3.b. Represent the closed-loop block diagram of the vehicle automatic cruise control.

3.c. Determine the open-loop $F_{OL}(s)$ and closed-loop transfer function $F_{CL}(s)$.

3.d. Determine the range of values for k_p that ensures the stability of the closed-loop P control.

3.e. By using the final value theorem, determine the steady-state error

$$\lim_{t \to +\infty} \varepsilon(t) = \lim_{t \to +\infty} (y_r(t) - y(t))$$

in terms of k_p for the following setpoint $y_r(t) = 25\Gamma(t)$ (25 m/s = 90 km/h). Conclude about the accuracy of this feedback P control.

3.f Compute the steady-state error when k_p=100 and k_p=900. What are the practical limits to the latter large value of proportional gain?

3. **Proportional and integral (PI) feedback control**

4.a. We choose now to implement a proportional integral (PI) controller, with the following transfer function:

$$C(s) = k_p + \frac{k_i}{s} = \frac{k_p s + k_i}{s}$$

with $k_p = 500$ and $k_i = 300$.

4.b. Determine the new open-loop transfer function $F_{OL}(s)$.

4.c. From this, determine the transfer functions $F_{CL}(s)$ and $F_D(s)$ in the following relation:

$$Y(s) = F_{CL}(s) Y_r(s) + F_D(s) D(s)$$

4.d. Study the stability $F_{CL}(s)$ and $F_D(s)$.

4.e. By using the final value theorem, determine the steady-state error

$$\lim_{t \to +\infty} \varepsilon(t) = \lim_{t \to +\infty} (y_r(t) - y(t))$$

for the following setpoint $y_r(t) = 25\Gamma(t)$ (25 m/s = 90 km/h). Conclude about the accuracy of this feedback PI control when the road is flat.

4.f. The PI controller settings are retained. The road at $t = t_0$ suddenly starts to climb with a low constant slope ϕ_0, such as $\phi(t) = \phi_0 \Gamma(t - t_0)$. After the transient phase, is the steady-state speed of the car maintained at its initial value? Justify.

4.g. Now the slope of the road increases constantly such as $\phi(t) = \phi_0 \, r(t - t_0)$. Is the steady-state speed of the car maintained at its initial value? Justify.