Continuations, mobile processes, all the things...

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Memo/garbage part

$M, N ::= x; \lambda x.M; MN$
$(\lambda x.M)N \to_\beta M[N/x]$ terms tend to get bigger

If $C[\ ]$ is a context, and $M \to_\beta N$
then $C[M] \to_\beta C[N]$

$P, Q ::= u(xy).P; \bar{u}xy.P; P|Q; (\nu x)P|!P$
$u(xy).P|\bar{u}ab.Q \to P[a/x, b/y]|Q$

If $P \to Q$ then $C[P] \to C[Q]$ (with necessary hypothesis on context C)
If $P \equiv P' \to Q \equiv Q'$ then $P \to Q$

Krivine Abstract Machine (KAM)
$M \star \Pi \star \mathcal{E}$

$$\begin{aligned} MN \star \Pi \star \mathcal{E} &\to M \star (N, \mathcal{E}).\Pi \star \mathcal{E} \\ \lambda x.M \star (N, \mathcal{E}).\Pi \star \mathcal{F} &\to M \star \Pi \star \mathcal{F}, s \mapsto (N, \mathcal{E}) \\ x \star \Pi \star \mathcal{E}, x \mapsto (M, \mathcal{F}) &\to M \star \Pi \star \mathcal{F} \end{aligned}$$

For exponentials :
  $!P \simeq !P|!P$
  $(\nu u)!u(x).P \simeq 0$
idea : $!P|Q \simeq !P|!P|Q \ \ \forall Q$

$$\begin{aligned} [\![(M, \mathcal{E}).\Pi]\!]_u &= (\nu m)(\nu v)(\bar{u}mv|!m(x)[\![M, \mathcal{E}]\!]_x|[\![\Pi]\!]_v) \\ [\![M, (x_i \mapsto (M_i, \mathcal{E}_i))_{i=1..k}]\!]_u &= (\nu x_1)\cdots(\nu x_k)([\![M]\!]_u|!x_1(u).[\![M_1, \mathcal{E}_1]\!]_u|\cdots) \\ [\![MN]\!]_u &= (\nu v)(\nu n)([\![M]\!]_v|\bar{v}nu|!n(x).[\![N]\!]_x) \\ [\![\lambda x.M]\!]_u &= u(xv).[\![M]\!]_v \\ [\![x]\!]_u &= \bar{x}u \end{aligned}$$

We want $M \star \Pi \star \mathcal{E} \to M' \star \Pi' \star \mathcal{E}'$ iff $[\![M, \mathcal{E}]\!]_u|[\![\Pi]\!]_u \to [\![M', \mathcal{E}']\!]_v|[\![\Pi']\!]_v$

— equiv $\simeq$ bisimulation
— the traduction goes well

Definition

A binary relation S is a reduction bisimulation if, forall $(P, Q) \in S$

(1) $P \xrightarrow{\tau} P'$ implies $Q \xrightarrow{\tau} Q'$ for some Q' with $(P', Q') \in S$
(2) $Q \xrightarrow{\tau} Q'$ implies $P \xrightarrow{\tau} P'$ for some P' with $(P', Q') \in S$

Definition (*Observability :*)

$P \downarrow_x$ if P can make an input action of subject x
$P \downarrow_{\bar{x}}$ if P can make an output action of subject x.

Definition (*Image-finite process :*)

P is image-finite if, for all derivative Q of P and any action $\alpha, \exists n \geq 0$ and $Q_1, \cdots Q_n$ such that $Q \stackrel{\alpha}{\Rightarrow} Q'$ implies $Q' = Q_i$ for some i.
where $\Rightarrow$ is the reflexive transitive closure of $\xrightarrow{\tau}$ and $\stackrel{\alpha}{\Rightarrow}$ is $\Rightarrow\xrightarrow{\alpha}\Rightarrow$ for some action α.

Rules for base-π

Value-typing TV-BASVAL TV-NAME

Process typing

T-PAR T-SUM
T-MAT T-NIL
T-REP T-RES T-TAU
T-INP T-OUT

Types : S, T ::=V value type
$|L$ link type
$| \diamond$ behaviour type

Value types : V ::= B basic type
Link types : L ::= $\sharp V$ connexion type
Environments : Γ ::= $\Gamma, x : L | \Gamma, x : V | \emptyset$

Transitions for base-π

OUT INP
TAU MAT
SUM-L PAR-L
COMM-L
RES REP-ACT
OPEN
REP-COMM
OUTERR INPERR MATERR

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simply-typed π-calculus : same but with value types V ::=B basic type
$|L$ link type