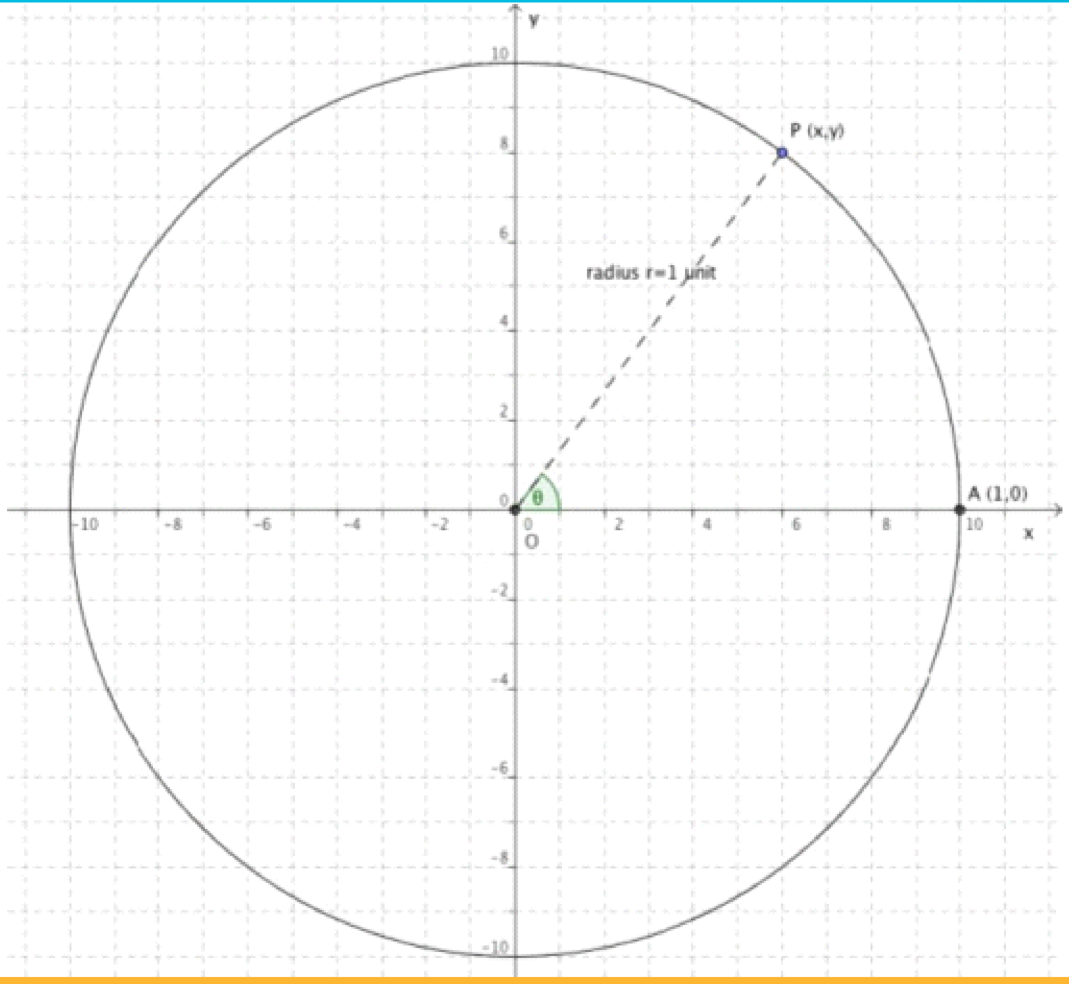
South Sudan

Secondary Mathematics 2

Teacher's Guide



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South Sudan

SECONDARY

2

Mathematics

Teacher's Guide 2

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Introduction

Teaching Mathematics is taking place in rapidly changing conditions. It is necessary to look for optimal didactic and educational solutions encompassing goals and contents as well as forms and teaching methods allowing for preparing students to face the challenges of the contemporary world.

The most significant role of educational system in terms of teaching Mathematics is developing and promoting subject competences as an important factor fostering student's personal development and the development of society. Well organised mathematical education facilitates logical thinking and expressing ideas, organizing own work, planning and organizing the learning process, collaboration and responsibility; it prepares for life in a modern world and enables to perform many jobs.

The teacher is required to pay more attention to students' awareness of developing learning skills and study habits, recognizing and analysing problems and predicting solutions to them. Undeniably, the implementation of modern teaching methods and techniques enhances students' curiosity about Mathematics and increases their understanding of the basis of mathematical and scientific knowledge. In accordance with the trends teaching Mathematics is supposed to help students understand and solve everyday problems.

The aim of teaching Secondary Mathematics is to encourage contemporary students to work in class, acquire knowledge and skills that are necessary in life. Moreover, research shows that teachers applying active methods assess the effectiveness of their work and how students respond to this way of teaching.

About this guide

The purpose of this guide is to offer suggestions that are helpful to Secondary 2 Mathematics teachers on planning, organizing, executing and evaluating the learning and teaching of mathematics. The suggestions will serve as useful starting points to the teachers who are expected to be dynamic innovative and creative to make the leaning process fit the learners.

The guide is to be used alongside Mathematics Students book for secondary 2. It consists of 4 units, in line with secondary 2 mathematics syllabus.

Each of the unit is structured to contain:

1. Introduction
2. Objectives
3. Teaching/Learning Activities
4. Answers to the exercises given in students secondary 2 book.

In each case, the introduction highlight the relevant work than learners are expected to have covered in their previous mathematics units and what they are expected to have covered previously. It also highlights what they are expected to cover in the unit. The teacher is expected to make a quick link up of previously learnt concepts. Learners should be able to make relevant references to their previous work. Where possible the mathematics teacher makes an entry behavior evaluation as a revision on previously learnt units related to the unit under study.

The unit objectives specify the skills (cognitive, affective, and psychomotor) that teachers will use to enable learners understand each unit. The objectives are likely to serve a useful purpose if they when stated to reflect the local conditions of the learner. For example, the type of students and the available learning resources. The teacher may break down the unit objectives to various objectives that enhance the learners understanding of the process involved and to suit different situations in the lesson, schools, society and the world at large.

Teaching/learning activities highlight the most noticeable and important. Points encountered in the learning process and suitable techniques to be used in handling each objective(s).

Answers to each exercise in the students' book are provided in these teachers guide. It is contemplated that the most conducive and favorable outcome from the guide will be realized if other sources of learning mathematics are properly organized and used.

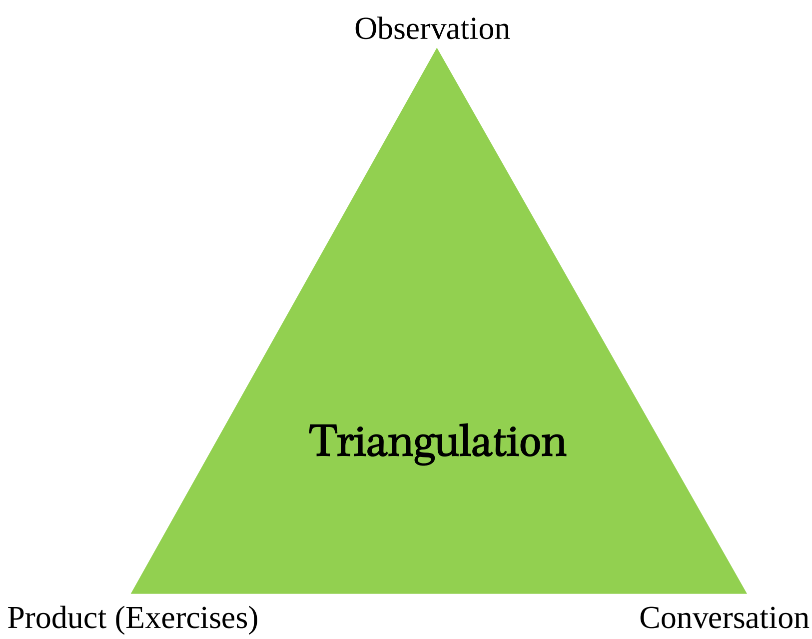
Among others, the following should be used alongside the guide:

1. The Schemes of work
2. The teacher's Lessons plans.

The Records of the work covered by the learners.

Making Classroom Assessment

- Observation – watching learners as they work to assess the skills learners are developing.
- Conversation – asking questions and talking to learners is good for assessing knowledge and understanding of the learner.
- Product – appraising the learner's work (writing report or finding, mathematics calculation, presentation, drawing diagram, etc.).



To find these opportunities, look at the “Learn About’ sections of the syllabus units. These describe the learning that is expected and in doing so they set out a range of opportunities for the three forms of opportunity.

UNIT 1: NUMERICAL CONCEPTS AND COMMERCIAL ARITHMETIC

Math: Secondary 2	Unit 1: Numerical Concepts & Commercial Arithmetic
Learn about	**Key inquiry questions**
Learners should investigate how an index or a power is used as short hand notation for easy calculations. They should know the rules of indices and evaluate fractional roots, zero indices, and negative indices as the reciprocal of positive power. They should understand and be able to calculate and apply surds which are basically an expression involving a root, squared or cubed, as part of the real number system, and their relationship with rational and irrational numbers. Learners should know how currencies in foreign exchange are calculated and they should derive and apply simple and compound interests formulae. They should know about how a value of an item depreciates/appreciates and how hire purchase and Income Tax are formulated and calculated, e.g. depreciation by 15% is calculated as 85/100 of the cost and appreciation by 12% is calculated as 112/100 of the cost.	- What are indices? - How does the use of indices save time and space when calculating large numbers? - What are rational and irrational numbers and how do they relate to surds? - What are foreign exchanges and what are simple and compound interests? - Why the market value of items depreciates/appreciates? - What is Income Tax and what is hire purchase
Learning outcomes	

Knowledge and understanding	**Skills**	**Attitudes**
- Know the rules of evaluating fractional, zero and negative indices - Understand and use surds - Understand rational and irrational numbers	- Use of spread sheet and computer for extensive calculations - Use of technology (calculators/computers) in opening accounts and calculating interest and balancing financial books - Calculate simple and compound interest, compound interest formula - Calculate depreciation and appreciation, hire purchase and Income Tax	- Appreciate the use of indices in easing extensive calculations - Appreciate the interdependence of technology and commercial arithmetic

Contribution to the competencies:

Critical thinking through analysis and investigation

Co-operation and communication through teamwork

Links to other subjects:

Science: where the natural logarithmic number e (e = 2.78) is widely used

Indices

Index or exponential notation is the use of repeated multiplication of prime numbers.

$2^2 = 4$ $3^1 = 3$

$2^3 = 8$ $3^2 = 9$

$2^4 = 16$ $3^3 = 9$

$2^5 = 32$ $3^4 = 81$

$2^6 = 64$ $3^5 = 243$ let the students also come up index notation for 5 and 7

$2^7 = 128$

Explain how indices can be used to perform operations on large numbers easily without using a calculator.

Activity 1

Group students in groups of at least 4. Supervise as they undertake the activity

1. Students to practice writing numbers using index notation. Check for understanding of how to write the numbers in index form.

2. $2^3 \times 2^2 = 8 \times 4 = 32 = 2^5$

$2^4 \times 2^3 = 16 \times 8 = 128 = 2^7$, let students fill in the missing numbers in this way until they can find a rule to finding the answer. $2^3 \times 2^2 = 2^5 = 32$. Therefore $y^a \times y^b = y^{a+b}$

The students should discover that the rule works even for large numbers as well as fractional numbers

3. a) Students should get the value of each individual exponent and divide. Then write the answer as an index. For example: $2^6 = 64$, $2^4 = 16$, therefore $64 \div 16 = 4 = 2^2$

b) Students should see the pattern that brings about the general rule. When numbers are being divided, the indexes are subtracted.

The general rule: $y^a \div y^b = y^{a-b}$

Students should test the rule by using their own examples.

Note that fractional indices also show roots of numbers. For example: $2^{\frac{1}{2}} = \sqrt{2}$ *and* $2^{\frac{1}{3}} = \sqrt[3]{2}$.

Laws of indices

Exercise 1

Students should work in pairs and show steps clearly.

Students should be able to use the rules of indices. Check for understanding.

1. i) 3 ii) 4 iii) 16 iv) $\frac{1}{8}$ v) $\frac{25}{16}$ vi) $\frac{1}{3}$

2. i) $8a^3$ ii) $2x^{-2}$ *or* $\frac{2}{x^2}$ iii) $p^{\frac{-4}{3}}$ *or* $\frac{1}{p^{\frac{4}{3}}}$ iv) $\frac{xy}{5x^2}$

Surds

Irrational numbers are numbers that cannot be written in the form $\frac{p}{q}$

For example: $0.75 = \frac{3}{4}, \sqrt{4} = 2 = \frac{2}{1}$ but $\sqrt{2} = 1.414213562\ldots$. It cannot be written as a fraction.

Link radicals with indices $2^{\frac{1}{2}} = \sqrt{2}$

Operations with radicals

a) Surds can only be added and subtracted if the number under the root sign is the same.

For example:

$\sqrt{2} + \sqrt{2} = 2\sqrt{2}$ treat them like algebraic expressions.

$\sqrt{3} + \sqrt{2}$ cannot be added.

b) Surds can be multiplied even if the number under the root symbol is different.

$\sqrt{2} \times \sqrt{2} = \sqrt{4} = 2$ and $\sqrt{2} \times \sqrt{3} = \sqrt{6}$

Check that students understand these principals.

Surds are simplified by finding the square factors of a number. Students should be aware of the square numbers: 1, 4, 9, 16, 25……

Check for understanding of the example on simplifying surds. Give other examples like $\sqrt{24}, \sqrt{48}$ etc.

Simplifying Surds

Exercise 3

1.a) $2\sqrt{3}$ b) $2\sqrt{6}$ c) $4\sqrt{3}$ d) $6\sqrt{3}$

Multiplication and division of surds

Exercise 4

1. a) Students should remember that $\sqrt{2} \times \sqrt{2} = 2$

b) $\frac{6}{\sqrt{2}} = \frac{3\times\sqrt{2}\times\sqrt{2}}{\sqrt{2}} = 3\sqrt{2}$

2. Yes. $\frac{3}{2}\sqrt{2}$

3. a) This is the introduction of rationalizing the denominator. Students should get the same answer in Q3 as in Q 2

5. a) $\frac{6}{5}\sqrt{5}$ b) $\frac{7}{2}\sqrt{6}$ c) $\frac{4}{3}\sqrt{15}$ d) $2\sqrt{13}$

Exercise 5

Review of “difference of two squares” concept may be necessary. Encourage discussion so that everyone understands how to identify the conjugate.

At the end of the investigation, students should understand what the conjugate is. Give examples to test for understanding. E.g. conjugate of $\left(3-\sqrt{2}\right)$ is $(3+\sqrt{2})$

Still in groups, students will discuss the questions in Exercise 6 and individually write the answers in their books. Observe for understanding, organized work and critical thinking skills.

Exercise 6

1.a) $\frac{3-\sqrt{2}}{7}$ b) $\frac{6+2\sqrt{2}}{7}$ c) $\sqrt{2}+1$ d) $\frac{6-2\sqrt{3}}{3}$ e) $\frac{5\sqrt{35}+5\sqrt{14}}{3}$

f) $\frac{-12-5\sqrt{2}}{7}$ g) $\frac{-2+\sqrt{5}}{3}$ h) $\frac{3\sqrt{2}-2\sqrt{3}}{6}$

Commercial Arithmetic

Teacher should lead discussion on the different currencies around the world, the strongest, most trusted and used currency and give reasons for this.

Task

Using the table given, teacher should check for understanding of conversion

a) 1 SPP =KSh 0.8

$0.8 \times 10\ 000 = 8\ 000$

SSP 8 000

b) SSP 13 000 = $100

$ 1= SSP 130

$ 1000= **SSP 130,000**

c) SSP 17,500=£ 100

£ $1 = SSP\ 17.50$

£ $5000 = \boldsymbol{SSP\ 87,500}$

Simple interest

Exercise 8: To be done in groups

Divide the students in groups. Students should demonstrate understanding of the concepts, collaboration skills, critical thinking skills and organizational skills.

1. a) SSP 200

   b) $\frac{200}{1000} \times 100 = 20\%$

   c) SSP 2,400, 2,800

2. SSP 50

   b)16.7%

   c) SSP 1,166.70, SSP 1,334

3. a) SSP 50, SSP 550

   b) SSP 500, SSP 2,500

   c) Students should finally get the simple interest formula:

$$Simple\ interest = \frac{PRT}{100}$$

Income tax

Ensure students understand how income tax is calculated using the table given. Have a discussion with the class on why income is taxed and what the money is used for. Give real life examples of incomes and the tax paid. Use the task to help with this discussion.

Exercise 9

1. Find the general incomes of people working in different professions around or in your school. E.g. teachers, doctors, nurse, politician etc.
2. Using the rates given above, calculate the income tax they pay the government.

Compound Interest

Investigation: Still in groups, students should attempt the questions. Observe for constructive discussions.

1 a) $1000 \times \frac{10}{100} \times 1 = SSP\ 100$

$1000 + 100 = SSP\ 1100$

b) the new principal is the amount she owes at the end of year

$1100 \times \frac{10}{100} \times 1 = SSP\ 110$

$1100 + 110 = \boldsymbol{SSP\ 1210}$

2. $1000 \times \frac{10}{100} \times 1 = SSP\ 100$

$1000 + 100 = SSP\ 1{,}100$

$1100 \times \frac{10}{100} \times 1 = SSP\ 110$

$1100 + 110 = \boldsymbol{SSP\ 1210}$

$$\boldsymbol{Amount = P\left(1 + \frac{R}{100}\right)^{n}}$$

3. Students should calculate the interest paid in the first year and add to the principal. This amount is the principal for the second year.

They may also discover that amount at the end of the first year can be given by 1.08 (100+8)% . Use of the formula can be explained using a calculator.

SSP 2333.20

Appreciation/Depreciation

Investigation

Divide the students in groups. Each group should present their findings after their investigation.

Check that the students understand that appreciation is calculated like compound interest formula since each year an asset appreciates by a certain value.

$$appreciation = \boldsymbol{P\left(1+\frac{R}{100}\right)^{n}}$$

While depreciation is given by:

$$Depreciation = \boldsymbol{P\left(1-\frac{R}{100}\right)^{n}}$$

Depreciation

Exercise 11

a) A new car costs SSP 12 000. The car loses 10% of its value during the first year and 15% of its value during the second year.
How much is the car worth after 2 years?

b) A computer worth SSP 1500 has depreciated in value to SSP 900 in the past 3 years.
What is the percentage depreciation in the value of the computer?

Hire purchase

Teacher should guide students to understand what hire purchase is and how it is calculated. Give other examples of items that can be bought by hire purchase e.g. laptops.

Hire purchase items are more expensive than items paid in cash. Let students discuss this. Use the activity to build students understanding.

Exercise 12

1. a) Joyce: SSP 29,000

b) $29,000 = 25,000\left(1+\frac{r}{100}\right)^{12}$

$$1.16 = \left(1+\frac{r}{100}\right)^{12}$$

$$\sqrt[12]{1.16} = \left(1+\frac{r}{100}\right)$$

$$1.012 = \left(1+\frac{r}{100}\right)$$

$$0.012 = \frac{r}{100}$$

$r = 1.2\%$ use the example and this question to ensure students understanding of hire purchase

c) Students should present their discussion on this question to the whole class.

2. SSP 22,304

UNIT 2: GEOMETRIC FIGURES AND PYTHAGORAS THEOREM

<table>
<tr><td>Math:
Secondary 2</td><td colspan="2">Unit 2: Geometric figures and applying the Pythagoras Theorem</td></tr>
<tr><td colspan="2">Learn about</td><td>Key inquiry questions</td></tr>
<tr><td colspan="2">Learners should revisit prior learning on how to calculate surface area of 3D solids such as prism, pyramid and cone. They should apply this to the South Sudanese double-hut (double tukul) and frustum (cross-section) of a sphere or pyramid. Identify and investigate the regular polygons that make up the surface area of different solids. They should revisit nets of complex polyhedral and investigate plans, elevations, planes of symmetry and isometric drawing.

Learners should understand and use equations of straight lines, describe the straight line in a Cartesian coordinate system and explain how its equation is related to its slope.

Learners should investigate the proof of Pythagoras theorem and its application and explain trigonometric ratios from the unit circle and angle property of circle.</td><td>How do you calculate surface area of solids?
What is a straight line in a Cartesian coordinates system?
What is Pythagoras theorem and how do you apply it?
How do you find trigonometric ratios in a unit circle?</td></tr>
</table>

Learning outcomes		
Knowledge and understanding	**Skills**	**Attitudes**
Identify, name and calculate the areas of the shapes that constitute the surface area of solids (regular/irregular) Understand the use of equations of straight lines Understand trigonometric ratios from the unit circle, angle property of circle	Calculate the areas of geometric figures and the surface area of solids Draw straight lines on Cartesian coordinates and construction of squares on the sides of the right-angled triangles Investigate the proof of Pythagoras theorem	Appreciate the importance of Pythagoras' Theorem

Contribution to the competencies:

Critical thinking through analysis and investigation

Co-operation and communication through teamwork
Links to other subjects: Physics: measurements Geography: locating areas of the country using its map

Activity 1

1. Teacher should encourage discussion on the types of different shapes and their characteristics. As students find the surface area, let them draw the net of the solid and name the different shapes that make the solid.

a) Surface area = 592cm^2 b) 76cm^2 c) $Curved\ area\ of\ a\ cone =$ πrl =628.3 cm^2

2. Students should understand that the lampshade is a frustum. This is a shape cut out from an original shape, which is cone in this case.

They should discuss ways of getting the surface area with this knowledge in mind.

Students need to find the slant length of the two cones, l and L

$$Surface\ area = Curved\ area\ of\ big\ cone - Curved\ area\ of\ small\ cone$$

$$\pi RL - \pi rl$$

$$= 220.5 - 54.95 = 165.6cm^2$$

3. Cone and hemisphere

$$Surface\ area = curved\ area\ of\ cone + curved\ area\ of\ sphere$$

$$\pi rl + \frac{4\pi r^2}{2}$$

$$= (\pi \times 3.5 \times 12.5) + (2 \times \pi \times 3.5^2)$$

$$13704 + 76.97 = \mathbf{214.3cm^2}$$

Coordinate geometry

Activity 2

Teacher should guide students on discussion on plotting points on the Cartesian plane. Let them understand that the x coordinate comes first every time and that you can plot points, shapes and lines on the Cartesian plane.

Exercise 1

1. Students can choose any point for x and find the corresponding y value. Guide them to see that if that for the equation $y = x$, each value of y will be equal to the corresponding value of x.

x is the independent variable while ***y*** is the dependent variable because it is dependent on ***x***.

4. $y = x + 1$

x	-1	0	1	2	3
y	0	1	2	3	4

5. $y = 2x + 2$

Students can pick any point for x and find the correct y value. A discussion on which line is steeper and how to tell from the graph should be held in class. Let students give other examples of steeper lines.

The equation of a line

Exercise 2

1. Line cuts the y axis at $y = -2$ **,** gradient =1

Students should try and make observations from the answers above and the equation of the line. The teacher may encourage students to draw the other lines in (f) so that the relationship can be noted.

Proving Pythagoras theorem

Task: In pairs, talk about various proofs of Pythagoras' theorem

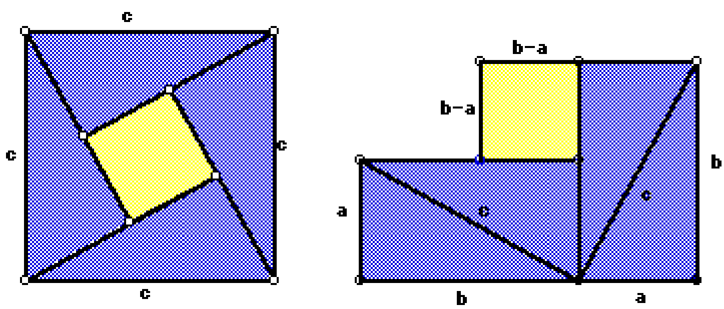
Teacher should divide the class in groups, guide the discussion so that students can solve:

$$a^2 + b^2 = c^2$$

Proof 2

Bhaskara's First Proof

Bhaskara's proof is also a dissection proof. It is similar to the proof provided by Pythagoras. Bhaskara was born in India. He was one of the most important Hindu mathematicians of the second century AD. He used the following diagrams in proving the Pythagorean Theorem.



In the above diagrams, the blue triangles are all congruent and the yellow squares are congruent. First we need to find the area of the big square two different ways. First let's find the area using the area formula for a square. Thus, $A=c^2$.
Now, lets find the area by finding the area of each of the components and then sum the areas.

Area of the blue triangles = $4(\frac{1}{2})ab$

Area of the yellow square = $(b-a^2$

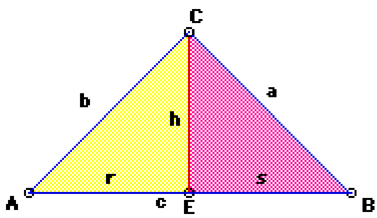
Area of the big square = $4(\frac{1}{2})ab + (b-a)^2$

$= 2ab + b^2 - 2ab + a^2$
$= b^2 + a^2$

Since, the square has the same area no matter how you find it
$A = c^2 = a^2 + b^2$,
concluding the proof.

Proof 3

Bhaskara's Second Proof of the Pythagorean Theorem
In this proof, Bhaskara began with a right triangle and then he drew an altitude on the hypotenuse. From here, he used the properties of similarity to prove the theorem.



Now prove that triangles ABC and CBE are similar.
It follows from the AA postulate that triangle ABC is similar to triangle CBE, since angle B is congruent to angle B and angle C is congruent to angle E. Thus, since internal ratios are equal $s/a = a/c$.
Multiplying both sides by ac we get $sc = a^2$.

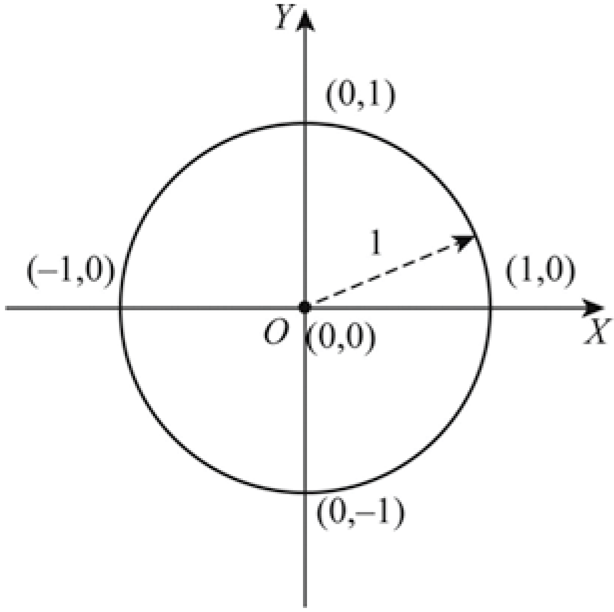
Now show that triangles ABC and ACE are similar.
As before, it follows from the AA postulate that these two triangles are similar. Angle A is congruent to angle A and angle C is congruent to angle E. Thus,

r/b=b/c. Multiplying both sides by bc we get $rc = b^2$.

Now when we add the two results we get
$sc + rc = a^2 + b^2$.
$c(s + r) = a^2 + b^2$
$c^2 = a^2 + b^2$, concluding the proof of the Pythagorean Theorem.

The unit circle

If a circle with centre (*O*) starting at the origin (0, 0) and a radius (*r*) is one unit, then the circle is said to be a unit circle.



In general, if (*x*, *y*) is a coordinate point on the circle with centre as origin and a radius as 1, then the equation of the circle would be, $x^2+y^2=1$. It is used to understand the trigonometric angle measures by forming the right triangle in the unit circle.

Unit circle in the complex plane:

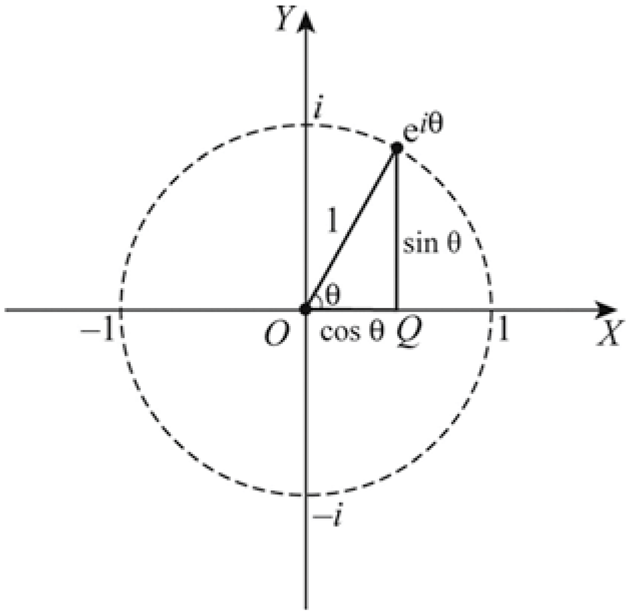
The unit circle in the complex plane is the set of complex numbers with magnitude one in which the extreme values are taken as follows:

• Right extreme value is 1.

• Left extreme value is − 1.

• Top extreme value is *i*.

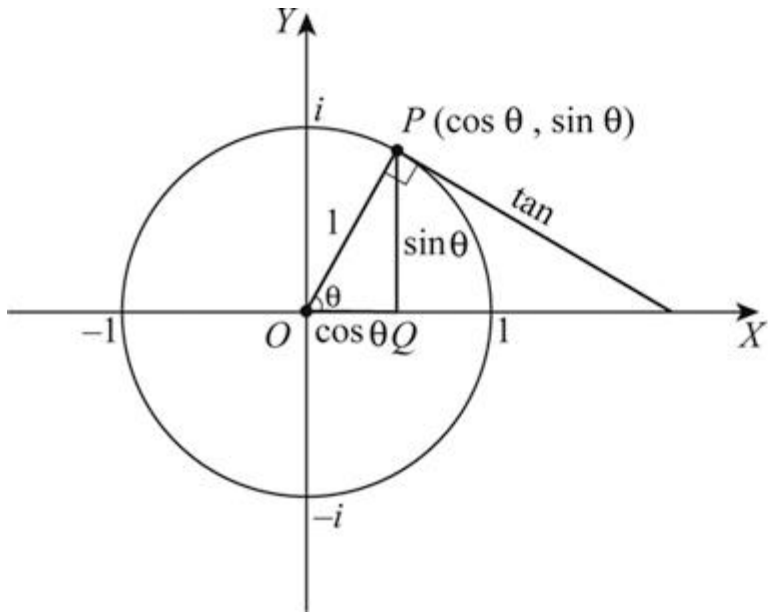
• Bottom extreme value is − *i* .



That is, the set of complex numbers *z* is of the form,

$$z = e^{i\theta}$$
$$= \cos(\theta) + i\sin(\theta)$$

Consider the coordinate point of *P* as $(\cos\theta, \sin\theta)$.



Thus, the radius is obtained below:

$$r = \sqrt{x^2 + y^2}$$
$$= \sqrt{\cos^2\theta + \sin^2\theta}$$
$$= \sqrt{1}$$
$$= 1$$

Thus, it forms a unit circle.

To be done in groups. Each group to present their findings to the class.

Students should notice that sin θ = the y co-ordinate of the point and cos θ= the x co-ordinate of the point P

They should also make a summary of which quadrant sin θ *and* cos θ *is* ***positive or negative***

Students should also notice that $\boldsymbol{\tan\theta = \frac{\sin\theta}{\cos\theta}}$

Trigonometric ratios

Exercise 3

Answers:

1) $\frac{5}{13}$ 2) $\frac{12}{13}$ 3) $\frac{5}{12}$ 4) $\frac{13}{5}$ 5) $\frac{13}{12}$ 6) $\frac{12}{5}$ 7) $\frac{12}{13}$ 8) $\frac{5}{13}$ 9) $\frac{12}{5}$ 10) $\frac{13}{12}$ 11) $\frac{13}{5}$ 12) $\frac{5}{12}$

UNIT 3: ALGEBRA

Math: Secondary 2	**Unit 3: Algebra**
Learn about	**Key inquiry questions**
Learners should revisit the expansion of algebraic expressions, with special attention to how expansion changes signs (+ and -) of terms in the bracket as they are multiplied or divided by the term(s) outside the bracket. They should know how quadratic expressions and equations are factorized, expanded and identified. They should investigate the difference between an expression and an equation. Learners should investigate and distinguish between vectors and scalar quantities, column, position and equivalent vectors. They should learn operations on vectors and vector translation, and represent equivalent and parallel vectors in column and row positions. Learners should investigate sets, set notation, how Venn diagram is used to represent sets when solving problems involving not more than three sets. They should learn about matrices to understand determinant, inverse, transposition of matrices, similarities and enlargement are determined and performed, and how the matrix method is used to solve simultaneous equations. Learners learn about functions: functional notation, inverse of simple functions, composite functions and their inverses.	What is a quadratic expression (equation)? How do you factorize and expand quadratic expressions? What is a vector and how do vectors differ from scalar quantities? How do you use Venn diagram in representing sets and solving problems involving sets? What is a function and how elements of two different sets, e.g. F(x) = y = 2x, are related in a function. What are the inverses of simple and composite functions?

Learning outcomes		
Knowledge and understanding	**Skills**	**Attitudes**
Understand quadratic expressions and equations (1): Expansion, Identification, Factorization Understand vectors (I): vectors and scalar quantities, column, position and equivalent vectors, operation on vectors, vector translation Understand sets: set notation, Venn diagrams and solving problems up to three sets Understand matrices (I): Understand determinant, inverse, transposition of matrices, similarities and enlargement Understand functions**:** functional notation, inverse of simple functions, composite functions and their inverses	Use Venn diagram in solving set problems Sketch graphs Calculate matrices' determinants	Appreciate and value Venn diagram and matrix method as tools for solving set and simultaneous equations

Contribution to the competencies:

Critical thinking through analysis and investigation

Co-operation and communication through teamwork

Links to other subjects:

Physics: Vectors

Social Studies: Venn diagram to represent demographic sectors

Algebraic expressions & equations

Exercise 1

The teacher should guide the students to understand expansion of expressions. Check that students connect expansion to finding the various areas of the rectangles

1c) $a(b+c)= ab +bc$

2e) $(a+b)(c+d) = ac + ad + bc + bd$ – area of big rectangle is equal to the sums of the areas of the smaller rectangles. This introduces the distributive law: $a(c+d) + b(c+d) = ac + ad + bc + bd$

3. Check for understanding of the expansion principle. Students should simplify like terms after expanding.

a) $(x+2)(x+3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$

b) $(2x+2)(x+3) = 2x^2 + 6x + 2x + 6 = 2x^2 + 8x + 6$

c) $(x-2)(x-3) = x^2 - 3x - 2x + 6 = x^2 - 5x + 6$

d) $(2x-2)(x+3) = 2x^2 + 6x - 2x - 6 = 2x^2 + 4x - 6$

4. $Area\ of\ big\ square - area\ of\ small\ unshaded\ square$

$(a^2 - b^2)$

c) $(a^2 - b^2) = (a+b)(a-b)$

i) $(x^2 - 4)$ ii) $(4x^2 - 4)$ iii) $(25 - y^2)$ iv) $(25x^2 - 4)$

5. $e)$ $(a+b)^2 = a^2 + 2ab + b^2$

Teacher should ensure students understanding of the summary at the end of this task by going through the questions in the task with the rules in mind.

Expansion

Exercise 2

Students should be encouraged to see that expansion reverses factorization and vice versa. Let students discuss question 1 and have them explain to whole group their observations.

Students the observations to factorise.

3. $i)$ $(x + 9)(x + 2)$

ii) $(x + 8)(x + 3)$

iii) $(x + 9)(x + 4)$

iv) $(x + 4)(x + 3)$

v) $(x + 9)(x + 6)$

4. i) $(x - 3)(x + 2)$

ii) $(x + 9)(x - 5)$

iii) $(x - 4)(x - 3)$

iv) $(x - 25)(x + 4)$

5. i) $3(x^2 + 2x - 24) = 3(x + 6)(x - 4)$

ii) $2(x^2 + 9x + 14) = 2(x + 7)(x + 2)$

iii) $5(x^2 + 4x + 3) = 5(x + 1)(x + 3)$

Solving quadratic equations

Exercise 3

1. Students need to understand that if the product of some numbers is zero at least one of the numbers must be zero. Teacher should check for understanding from the discussions.

i) $x = 0, x = -2$ ii) $x = 2, x = -2$ iii) $x = -5, x = -2$

iv) $x = 0, x = 4$

v) $x = 8, x = -8$ vi) $x = 1, x = 4$ vii) $x = 7, x = -4$

viii) $x = 1, x = 2$

Vectors

Exercise 4

Teacher should ensure that students understand position vectors, column vectors, addition and subtraction of vectors. Students should notice that $3\boldsymbol{a} = \boldsymbol{b}$ means that **a** and **b** are parallel vectors and that **b** is 3 times longer than **a**.

1. $\boldsymbol{a} + \boldsymbol{b} = \begin{pmatrix}7\\1\end{pmatrix}$ ii) $\boldsymbol{a} + \boldsymbol{c} = \begin{pmatrix}2\\3\end{pmatrix}$ iii) $\boldsymbol{b} + \boldsymbol{c} = \begin{pmatrix}1\\0\end{pmatrix}$

Task 2: Translation vectors

Teacher should ensure students show understanding of what translation is and how the vector moves an object to get the image.

Exercise 5: To be done in pairs

1 Given that $\bar{a} = \begin{pmatrix}4\\2\end{pmatrix}$, $\bar{b} = \begin{pmatrix}3\\-1\end{pmatrix}$, $\bar{c} = \begin{pmatrix}-2\\1\end{pmatrix}$.

a) Draw the vectors

i) $\bar{a}, \bar{b}, \bar{a}+\bar{b}$

ii) $\bar{a}, \bar{c}, \bar{a}+\bar{c}$

iii*)* ***b***, ***c***, ***b*** **+** ***c***

2. Consider the points A(2, 3) and B(1, 5).

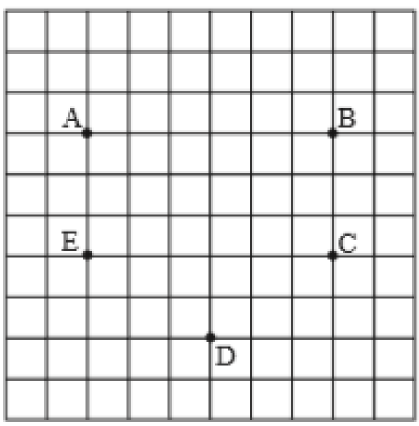
a) Draw a Cartesian plane on squared paper or graph paper and mark the points A and B
b) Join Point A to the origin and put an arrow on the line segment OA going upwards. This represents the vector OA. Notice that the vector is moving upwards because we are starting our movement from A and ending at B.
c) Join point B to the origin and mark the vector OB. Note that this vector will also move upwards since we are starting from O and moving to B.

d) What is the column vector OA and OB?

3. Use the points in the grid below to write the vectors given in column vector form

a) AB b) AC c) DE d) BE e) EB f) AD g) CD h) DC

What is the relationship between AC and CA



4. Consider the vectors $\boldsymbol{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$

a) Draw vectors **a** and **b** on a grid.
b) What relationship do they have? Express it in equation form?
c) Without drawing vector $\boldsymbol{c} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$ describe the relationship that exists between **a** and **c.**

Sets & Venn diagrams

Activity 1: Work in pairs.

Teacher to guide learners in completing the activity in pairs. Both learners should participate in asking and giving answers to the questions. The teacher should ensure that the learners are articulate in providing reasons to back up their answers.

Are the following sets well-defined?

1. The set of all groups of size three that can be selected from the members of this class.
2. The set of all books written by John Grisham.
3. The set of great rap artists.
4. The best fruits.
5. The 10 top-selling recording artists of 2017.

Solution

1. You can determine if a group has three people and whether or not those people are members of this class so this is well-defined.
2. You can determine whether a book was written by John Grisham or not so this is also a well-defined set.
3. A rap artist being great is a matter of opinion so there is no way to tell if a particular rap artist is in this collection, this is not well-defined.
4. Similar to the previous set, best is an opinion, so this set is not well-defined.
5. This is well-defined, the top selling recording artists of any particular year are a matter of record.

Equality, the Universal Set and the Empty Set

Activity 2: Work in pairs.

$$A = \{0, 1, 2, 3, 4, 5\}$$
$$B = \{1, 3, 4\}$$
$$C = \{6, 4, 3, 1\}$$
$$D = \{0, 1, 2, 5, 3, 4\}$$
$$E = \{\}$$

Which of the sets B, C, D, E are subsets of A?

The teacher to point out the following:

$B \subseteq A$ since it's elements 1, 3, and 4 are all also in A. C is NOT a subset of A (C 6 $\subseteq A$) since there is a 6 in C and there is no 6 in A. D is a subset of A since everything that is in D is also in A; in fact $D = A$. Finally, E is a subset of A; this is true since any element that is in E is also in A.

Notice that every set is a subset of itself and the empty set is a subset of every set. If $A \subseteq B$ and A 6 $= B$, then we say that A is a proper subset of B. The notation is only a bit different: $A \subset B$. Note the lack of the "equal" part of the wymbol.

Complement

Activity 3: To be done in groups

The teacher to guide the group work.

Let A be the set of all numbers from 1 to 20, then A= {1, 2, 3, 4, 5, 6 ….20}

a) Write down the number of elements of A
b) Let B be the set of all odd numbers from 1 to 20, write down the set B. How many elements does B have?
c) Are the following statements true?

i) B I A

ii) 5I B

i) n(A) = 9

Answers

1. a) 20

b) {1,3,5,7,9,11,13,15,17,19},10 members

c)i) Yes ii) Yes iii) No

Activity 4: To be discussed in groups

1. a) $A \cap B = \{6\}$

b) $A \cup B = \{2,3,4,6,8,9\}$

c) $B' = \{1,2,4,5,7,8\}$

2. a(i) $A' = \{-3, -1,1\}$

ii) $B' = \{-1,0,2\}$

iii) $A \cup B = \{-3, -2,0,1,3\}$

iv) $A \cap B = \{-2,3\}$

v) $A \cap B' = \{0,2\}$

vi) $A' \cup B' = \{-3, -1,0,1,2\}$

b) 5

3. $B' = \{3,4,6\}$

4. a) $F = \{1,2,3,4,6,8,12\}$ $M = \{4,8,12,16,20\}$ $M' = \{0,1,2,3,5,6,7,9,10,11,13,14,15,17,18,19\}$

$F \cap M = \{4,8,12\}$ $F \cap M' = \{1,2,3,6\}$ $F \cup M = \{1,2,3,4,6,8,12,16,20\}$ $n(F \cap M') = 4$

Venn diagrams

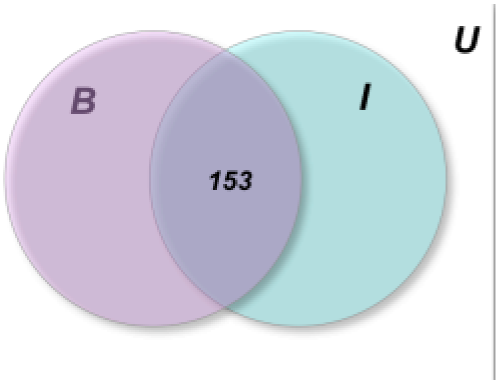
Teacher should check for correct understanding of putting values in the Venn diagrams.

Activity 5: Work in groups

Two programs were broadcast on television at the same time; one was the Big Game and the other was Ice Stars. The Nelson Ratings Company uses boxes attached to television sets to determine what shows are actually being watched. In its survey of 1000 homes at the midpoint of the broadcasts, their equipment showed that 153 households were watching both shows, 736 were watching the Big Game and 55 households were not watching either. How many households were watching only Ice Stars? What percentage of the households were not watching either broadcast?

After group presentations, the teacher to guide whole class discussion.

We begin by constructing a Venn diagram, we will use B for the Big Game and I for Ice Stars. Rather than entering the name of every household involved, we will put the cardinality of each set in its place within the diagram. So, since they told us that 153 households were watching both broadcasts, we know that $n(B \cap I) = 153$, this number goes in the dark purple "football" area.



We are told that 736 were watching the Big Game, $n(B) = 736$, since we already have 153 in that part of B that is in common with I, the remaining part of B will have 736−153 = 583. This tells us that 583 households were watching only the Big Game.

We are also told that 55 households were watching neither program, $n(\overline{(A \cup B)}) = 55$, so that number goes outside of both circles.

Finally, we know that the total of everything should be 1000, $n(U) = 1000$. Since only one area does not yet contain a number it must be the missing amount to add up to 1000. We add the three numbers that we have, 583 + 153 + 55 = 791, and subtract that total from 1000, 1000 − 791 = 209, to get the number that were watching only Ice Stars. Filling in this number, we have a complete Venn diagram representing the survey.

Now, we have the information needed to answer any questions about the survey results. In particular, we were asked how many households were watching only Ice Stars, we found this number to be 209. We were also asked what percentage of the households were watching only the Big Game. The number watching only the game was found to be 583, so we compute the percentage, (583/1000) $*$ 100% = 58.3%.

Applications.

In this section, we will illustrate the use of Venn diagrams in some examples.

Task: Work in groups

Two programs were broadcast on television at the same time; one was the Big Game and the other was Ice Stars. The Nelson Ratings Company uses boxes attached to television sets to determine what shows are actually being watched. In its survey of 1000 homes at the midpoint of the broadcasts, their equipment showed that 153 households were watching both shows, 736 were watching the Big Game and 55 households were not watching either.

a) Draw a Venn diagram.

b) How many households were watching only Ice Stars?
c) What percentage of the households were not watching either broadcast?

Task

In a recent survey people were asked if they took a vacation in the summer, winter, or spring in the past year. The results were 73 took a vacation in the summer, 51 took a vacation in the winter, 27 took a vacation in the spring, and 2 had taken no vacation. Also, 10 had taken vacations at all three times, 33 had taken both a summer and a winter vacation, 18 had taken only a winter vacation, and 5 had taken both a summer and spring but not a winter vacation.

1. Draw a Venn diagram.
2. How many people were surveyed?
3. How many people took vacations at exactly two times of the year?
4. How many people took vacations during at most one time of the year?
5. What percentage took vacations during both summer and winter but not spring?

Exercise 6: To be discussed in groups and answered individually

1. Consider the universal set U={0,1,2,3,4,5,6,7,8,9}. Illustrate on a Venn diagram the sets:

   a) A = {2,3,5,7} and B = {1,2,4,6,7,8}

   b) A = {2,3,5,7} and B = {4,6,8,9}

   c) A = {3,4,5,6,7,8} and B = {4,6,8}

   d) A = {0,1,3,7} and B = {0,1,2,3,6,7,9}

2. a) Come up with your own universal set

   b) From this set create 2 others set A and B. List the elements of A and B. 9 Make sure some values are shared by A and B)

c) Draw Venn diagrams and shade regions represented by i) A E B
ii) A Ç B

d) List i) A E B ii) A Ç B

Exercise 7: Discuss in groups

1. In a class of 30 students, 7 have black hair and 24 are right handed. If 2 students neither have black hair nor are they right handed, how many students:
    a) have both black hair and are right handed.
    b) have black hair but are not right handed?
2. 46% of people in a town ride a bicycle and 45% ride a motor bike. 16% ride neither a bicycle nor a motor bike.
    a) Illustrate this information on a Venn diagram
    b) How many people ride:

        i) both a bicycle and a motor bike

        ii) either a bicycle or a motor bike or both

ii) a bicycle only

Matrices

Discussion

1. Why do we state the rows first, then columns when we specify the matrix size?
2. What is the usefulness of organizing data in matrix form?
3. Collect data that can be arranged in matrix form. Arrange it and state the size of the matrix formed.

Lead learners in holding a discussion about matrices. They should be able to point out the notation of matrices. They should also talk about the usefulness of organizing matrices a certain way.

Allow learners to go out of class if necessary to collect data that can be arranged in matrix form. Findings can then be presented to the whole class.

Exercise 8

A restaurant served 72 men, 84 women and 49 children on Friday night. On Saturday night they served 86 men, 72 women and 46 children.

a. Express this information in two column matrices.
b. Use matrices to find the totals of men, women and children served over the two day period.

Scalar Multiplication

Exercise 9

a) 1. Consider the matrix $A = \begin{pmatrix} 2 & 4 \\ 3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ -5 & 2 \end{pmatrix}$ the identity matrix $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

b) a) Evaluate AB, AI, BI ABI, BA

c) What do you observe? Discuss

d) 2. Given that

e) **i)** $A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$,

f) ii) $A = \begin{pmatrix} 7 & 4 \\ 5 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -4 \\ -5 & 7 \end{pmatrix}$

g) Evaluate AB. What do you notice?

h) What observation can you make about the elements in the diagonals of A and B?

i) When two matrices A and B are such that $AB = I$ then A is said to the inverse of B and vice versa.

Determinant Matrix

Exercise 10

Find the inverse of the following matrices:

a) $B = \begin{pmatrix} 7 & 9 \\ 3 & 4 \end{pmatrix}$ b) $A = \begin{pmatrix} 6 & 13 \\ -2 & -4 \end{pmatrix}$ c) $C = \begin{pmatrix} 5 & 10 \\ 5 & 9 \end{pmatrix}$

Solving simultaneous equations using matrices

Investigation: To be done in groups

1. Consider the following equations

$$3x + y = 7$$
$$5x + 2y = 12$$

   a. Using methods you have learnt before, either, elimination, substitution or graphical method solve these equations. What is the value of x and y?
   b. The above equations can be written as $\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 12 \end{pmatrix}$.This is a matrix equation. Discuss how this is done.
   c. The first matrix $\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$ is called the **coefficient**s matrix while $\begin{pmatrix} 7 \\ 12 \end{pmatrix}$ is called the **constant**s matrix. Find the inverse of the coefficients matrix.
   d. Pre-multiply the inverse on both sides of the matrix equation. What happens?
   e. What are the values of x and y?

2. Use the method above to solve the following equations:

a) $3x + y = 8$
$2x - y = -3$

b) $\begin{array}{l} x + 3y = 5 \\ 2x + 6y = 7 \end{array}$

c) $\begin{array}{l} 3y - 2x = 3 \\ 3y + x = 4 \end{array}$

Use elimination or substitution to confirm your answers.

Task

Task notes

The general straight line graph has equation $y = mx + c$. The gradient is *m* and the *y*-intercept is *c*.

Provided *c* is the same, graphs will have the same *y*-intercept.

Provided *m* is the same, the graphs will be parallel.

If the graphs are perpendicular, the product of the gradients is -1 e.g. $y = 2x - 4$ and $y = 2x + 6$ are both perpendicular to $y = -\frac{1}{2}x + 1$ since $2 \times -\frac{1}{2} = 1$

Exercise 11

Solutions

1. $y = 3x + k, k\epsilon\mathbb{R}\backslash -4$ (any real number excluding -4)
2. $y = -\frac{1}{3}x + k, k\epsilon\mathbb{R}$, since the product of the gradients is -1
3. $y = -3x - 4$
4. $y = -3x + 4$

Functions

Exercise 12

a) Given that $h(x) = z^2 - 4z + 9$, find i) h(-3) ii) h(2)

b) Given that $g(x) = x^2 - 2$, find g(4)

c) Given that $f(x) = 4x - 6$ $\qquad g(x) = 2x^2 - x$

$$h(x) = \frac{(x-6)^2}{2}$$

Find

i) $5f(-4)$ ii) $h(10)$ iii) $h(3)$ iv) $g(-3)$ iv) $2h(7)$

Composite functions

Activity 1: Work in groups

Explore why it is important to pay attention to the order in which the composition of a function is written.

In many cases (f o g)(x) is not the same as (go f)(x). Let's look at why the order is so important:

(f o g)(x) = f(g(x)), the g function is inside of the f function

(go f)(x) = g(f(x)), the f function is inside of the g function

(f o g)(x) and (go f)(x) are often different because in the composite(f o g)(x), f(x) is the outside function and g(x) is the inside function. Whereas in the composite (go f)(x), g(x) is the outside function and f(x) is the inside function. This difference in order will often be the reason why we will get different answers for

(f o g)(x) and (go f)(x). This means we need to make sure that we pay close attention to the way the problem is written when we are trying to find the composition of two functions.

Activity 2: Work in groups.

How Do You Find the Composition of Two Functions?

After investigation and research, learners should be able to outline the following steps. The teacher to provide guidance where necessary.

Here are the steps we can use to find the composition of two functions:

Step 1: Rewrite the composition in a different form.
For example, the composition (f o g)(x) needs to rewritten as $f(g(x))$.

Step 2: Replace each occurrence of x found in the outside function with the inside function. For example, in the composition of (f o g)(x) = f(g(x)), we need to replace each x found in $f(x)$, the outside function, with $g(x)$, the inside function.

Step 3: Simplify the answer.

Inverse Functions

Activity 3: Work in groups.

The teacher to guide learners in exploring the following problems given what they have learned so far. A discussion can be held after group presentations by the heads of different groups to justify the answers arrived at. The teacher to provide guidance where necessary.

1. Explain the difference in meaning of the notation $f(2) = 5$ versus the notation $f^{-1}(5) = 2$
2. Suppose the point (10, 5) lies on a graph of a function f, what point lies on the graph f^{-1}.
3. The number of people in thousands in a city is given by the function $f(t) = 20 + 0.4t$ where t is the number of years since 1970.
    a) In the context of this problem, explain what $f(25)$ and $f^{-1}(25)$ mean (no calculation required).
    What is the unit measure (number of people or number of years) for f(25) and $f^{-1}(25)$
    b) Now calculate $f^{-1}(25)$

4. The total cost, C, in South Sudanese pounds (SSP) for a clothing factory to make $'j'$ jackets is given by the function $C = f(j)$. Interpret the meaning of the following notation within the context of the story given.
   a) $f(30) = 678$
   b) $f^{-1}(30) = 678$

Activity 4: Work in groups.

Explore how you can determine if a function is a one-to-one function or not.

In looking at the graph of the function we can determine if a function is a one-to-one function or not by applying the Horizontal Line Test, or HLT. If the graph of the function passes the Horizontal Line Test, then the function is a one-to-one function. If the graph of the function fails the Horizontal Line Test, then the function is not a one-to-one function.

***Horizontal Line Test** – The HLT says that a function is a one-to-one function if there is no horizontal line that intersects the graph of the function at more than one point.*

By applying the Horizontal Line Test not only can we determine if a function is a one-to-one function, but more importantly we can determine if a function has an inverse or not.

Exercise 13

Use the steps above to answer the following questions:

1. $f(x) = \frac{2x+3}{5}$ find $f^{-1}(x)$

2. $f(x) = \frac{2x+3}{2}$ find $f^{-1}(x)$

UNIT 4: STATISTICS

Math Secondary 2	**Unit 4: Statistics**
Learn about	**Key inquiry questions**
Learners should work in pairs or groups to investigate the use of assumed mean and to interpret cumulative frequency tables in a range of situations. They should gather appropriate date to create cumulative frequency tables of their own and to explain them to the class. The should investigate a range of cumulative frequency tables from different sources and comment on their effectiveness. They should find out how to use median, quartiles, ogives (cumulative histograms) and dispersion, and investigate problems using these tools, and perform calculations involving probability.	- What is assumed by the word mean? - What are median, quartiles and dispersion? - How would you perform calculations involving probability?

Learning outcomes

Knowledge and understanding	**Skills**	**Attitudes**
- Understand 'assumed mean' - Understand ogives, median, quartiles, depression	- Create and interpret cumulative frequency tables - Identify and analyze statistical data - Perform and carry out calculations and solve problems of statistics and probability	- Appreciate and value the concept of statistics.

Contribution to the competencies: Critical thinking through analysis and investigation Co-operation and communication through teamwork
Links to other subjects: Social Studies: statistics and probability (study of demography) Physical Education: predicting the outcomes of matches Science

Measures of central tendency

Exercise 1

Teacher should review work on mean, median and mode. Students should organize their data into a frequency table and show understanding of what the three measures mean. Each group to present their findings before the class.

Assumed Mean

Exercise 2

A: mean – 50 B: mean – 62 C: mean – 42

Mean B is 12 more than A, each of the values of A are 12 more than B

Mean C is 8 less than A, each of the values of C are 8 less than A

When you add/ subtract a constant to a particular data set, the means also increases/ decreases by that constant

b) 60 c) 52 d) 420 e) Yes, **If you multiply/ divide each value by a constant, the mean is also multiplied or divided by the same constant.** Mean: 4.2

2. a)

Mass(kg) x	$t = x - 50$	f	ft
47	-3	5	-15
48	-2	1	-2
49	-1	3	-3
50	0	5	0
51	1	10	10
52	2	3	6
53	3	5	15
54	4	8	32
55	5	10	50

$$\frac{\sum ft}{\sum f} = \frac{93}{50} = 1.86$$

c)

$$Mean = 50 + 1.86 = 51.86$$

3.

Mass(kg)	x	$t = x - 110.5$	f	ft
100-103	101.5	-9	1	-9
104-107	105.5	-5	15	-75
108-111	109.5	-1	42	-42

112-115	113.5	3	31	93
116-119	117.5	7	8	56
120-123	121.5	11	3	33

$$Mean = 110.5 + \frac{56}{100} = 116.1$$

Cumulative Frequency, Quartiles and Percentiles

Cumulative Frequency

Cumulative frequency is defined as a running total of frequencies. The frequency of an element in a set refers to how many of that element there are in the set. Cumulative frequency can also defined as the sum of all previous frequencies up to the current point.

The cumulative frequency is important when analyzing data, where the value of the cumulative frequency indicates the number of elements in the data set that lie below the current value. The cumulative frequency is also useful when representing data using diagrams like histograms.

Cumulative Frequency Table

The cumulative frequency is usually observed by constructing a cumulative frequency table. The cumulative frequency table takes the form as in the example below.

Example 1

The set of data below shows the ages of participants in a certain summer camp. Draw a cumulative frequency table for the data.

Age (years)	Frequency
10	3
11	18
12	13
13	12
14	7
15	27

Solution:

The cumulative frequency at a certain point is found by adding the frequency at the present point to the cumulative frequency of the previous point.

The cumulative frequency for the first data point is the same as its frequency since there is no cumulative frequency before it.

Age (years)	Frequency	Cumulative Frequency
10	3	3
11	18	3+18 = 21
12	13	21+13 = 34
13	12	34+12 = 46
14	7	46+7 = 53
15	27	53+27 = 80

Cumulative Frequency Graph (Ogive)

A cumulative frequency graph, also known as an Ogive, is a curve showing the cumulative frequency for a given set of data. The cumulative frequency is plotted on the y-axis against the data which is on the x-axis for un-grouped data. When dealing with grouped data, the Ogive is formed by plotting the cumulative frequency against the upper boundary of the class. An Ogive is used to study the growth rate of data as it shows the accumulation of frequency and hence its growth rate.

Example 2

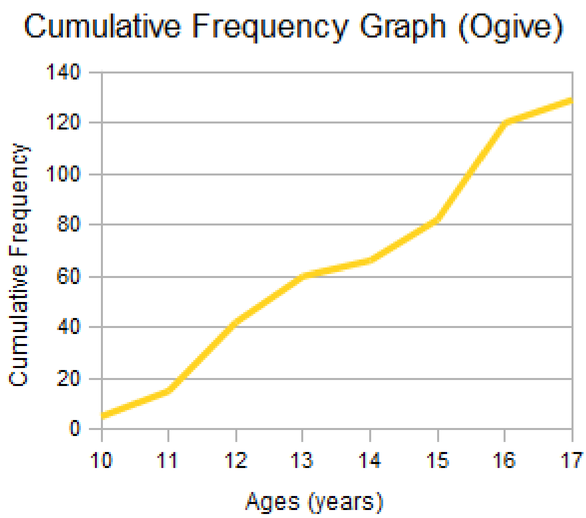
Plot the cumulative frequency curve for the data set below

Age (years)	**Frequency**
10	5
11	10
12	27
13	18
14	6
15	16
16	38
17	9

Solution:

Age (years)	**Frequency**	**Cumulative Frequency**
10	5	5
11	10	5+10 = 15
12	27	15+27 = 42
13	18	42+18 = 60
14	6	60+6 = 66

15	16	66+16 = 82
16	38	82+38 = 120
17	9	120+9 = 129



Percentiles

A percentile is a certain percentage of a set of data. Percentiles are used to observe how many of a given set of data fall within a certain percentage range; for example; a thirtieth percentile indicates data that lies the 13% mark of the entire data set.

Calculating Percentiles

Let designate a percentile as $\mathbf{P_m}$ where **m** represents the percentile we're finding, for example for the tenth percentile, **m**} would be 10. Given that the total number of elements in the data set is **N**

$$\mathrm{P}_m = \frac{m}{100} \times \mathrm{N}$$

Quartiles

The term quartile is derived from the word quarter which means one fourth of something. Thus a quartile is a certain fourth of a data set. When you arrange a date set increasing order from the lowest to the highest, then you divide this data into groups of four, you end up with quartiles. There are three quartiles that are studied in statistics.

First Quartile (Q_1)

When you arrange a data set in increasing order from the lowest to the highest, then you proceed to divide this data into four groups, the data at the lower fourth ($^1/_4$) mark of the data is referred to as the First Quartile.

The First Quartile is equal to the data at the 25th percentile of the data. The first quartile can also be obtained using the Ogive whereby you section off the curve into four parts and then the data that lies on the last quadrant is referred to as the first quartile.

Second Quartile (Q_2)

When you arrange a given data set in increasing order from the lowest to the highest and then divide this data into four groups, the data value at the second fourth ($^2/_4$) mark of the data is referred to as the Second Quartile.

This is the equivalent to the data value at the half way point of all the data and is also equal to the data value at the 50th percentile.

The Second Quartile can similarly be obtained from an Ogive by sectioning off the curve into four and the data that lies at the second quadrant mark is then referred to as the second data. In other words, all the data at the half way line on the cumulative frequency curve is the second quartile. The second quartile is also equal to the median.

Third Quartile (Q_3)
When you arrange a given data set in increasing order from the lowest to the highest and then divide this data into four groups, the data value at the third fourth (¾) mark of the data is referred to as the Third Quartile.

This is the equivalent of the the data at the 75th percentile. The third quartile can be obtained from an Ogive by dividing the curve into four and then considering all the data value that lies at the ¾ mark.

Calculating the Different Quartiles

The different quartiles can be calculated using the same method as with the median.

First Quartile
The first quartile can be calculated by first arranging the data in an ordered list, then finding then dividing the data into two groups. If the total number of elements in the data set is odd, you exclude the median (the element in the middle).

After this you only look at the lower half of the data and then find the median for this new subset of data using the method for finding median described in the section on averages. This median will be your First Quartile.

Second Quartile
The second quartile is the same as the median and can thus be found using the same methods for finding median described in the section on averages.

Third Quartile
The third quartile is found in a similar manner to the first quartile. The difference here is that after dividing the data into two groups, instead of considering the data in the lower half, you consider the data in the upper half and then you proceed to find the Median of this subset of data using the methods described in the section on Averages.
This median will be your Third Quartile.

Calculating Quartiles from Cumulative Frequency

As mentioned above, we can obtain the different quartiles from the Ogive, which means that we use the cumulative frequency to calculate the quartile.

Given that the cumulative frequency for the last element in the data set is given as f_c, the quartiles can be calculated as follows:

$$Q_1 \text{ position} = \frac{(f_c + 1)}{4}$$

$$Q_2 \text{ position} = \frac{2(f_c + 1)}{4}$$

$$Q_3 \text{ position} = \frac{3(f_c + 1)}{4}$$

The quartile is then located by matching up which element has the cumulative frequency corresponding to the position obtained above.

Example 3

Find the First, Second and Third Quartiles of the data set below using the cumulative frequency curve.

Age (years)	**Frequency**
10	5
11	10
12	27
13	18
14	6

15	16
16	38
17	9

Solution:

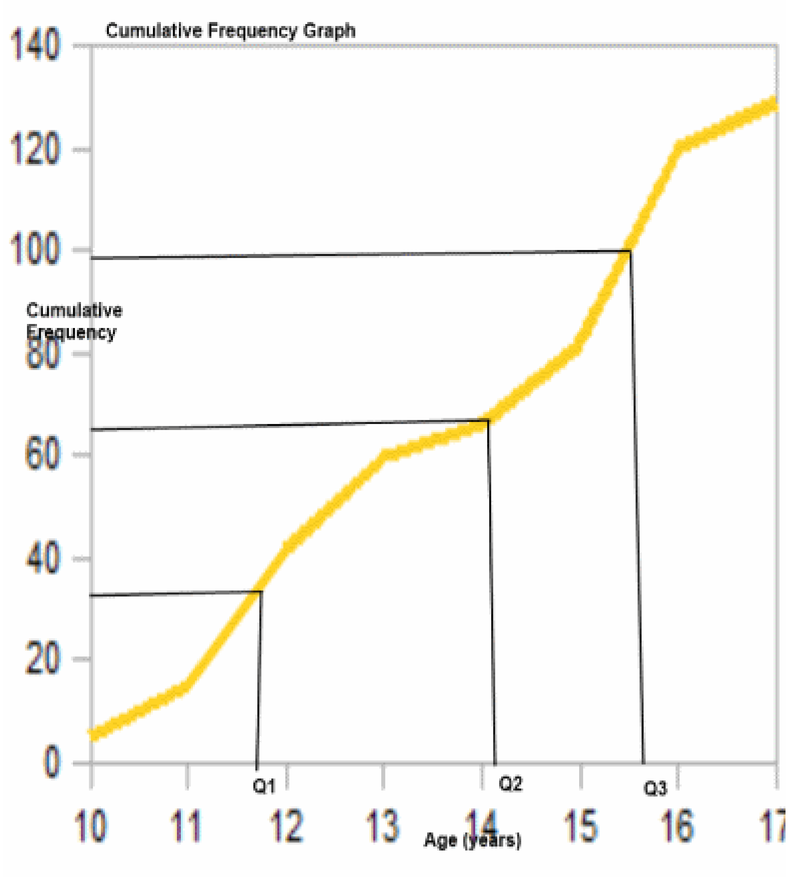
Age (years)	Frequency	Cumulative Frequency
10	5	5
11	10	15
12	27	42
13	18	60
14	6	66
15	16	82
16	38	120
17	9	129

$$f_c = 129$$

$$\mathrm{Q}_1 \text{ position} = \frac{(129+1)}{4} = 32.5$$

$$\mathrm{Q}_2 \text{ position} = \frac{2(129+1)}{4} = 65$$

$$\mathrm{Q}_3 \text{ position} = \frac{3(129+1)}{4} = 97.5$$



From the Ogive, we can see the positions where the quartiles lie and thus can approximate them as follows

$$Q_1 = 11.5$$

$$Q_2 = 14.5$$

$$Q_3 = 15.5$$

Interquartile Range

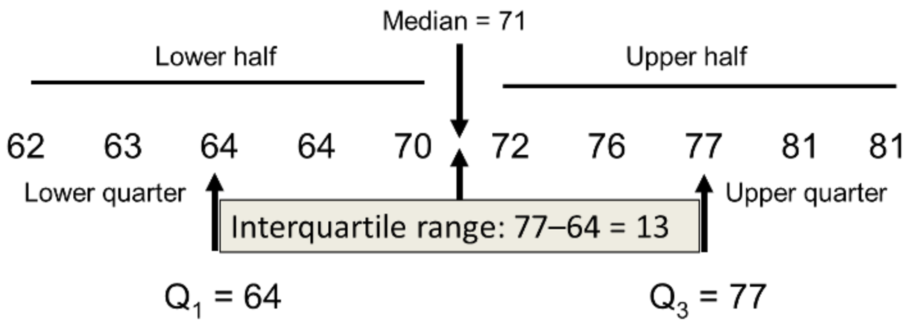
The interquartile range is the difference between the third quartile and the first quartile.

$$\text{Interquartile range} = Q_3 - Q_1$$

With an Even Sample Size:

For the sample (n=10) the median diastolic blood pressure is 71 (50% of the values are above 71, and 50% are below). The quartiles can be determined in the same way we determined the median, except we consider each half of the data set separately.

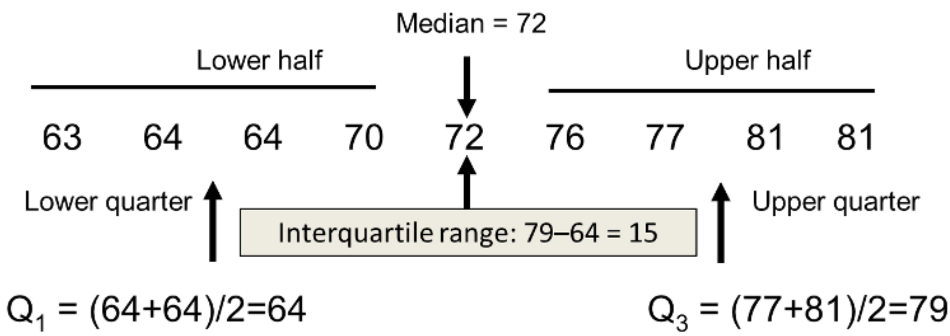
Interquartile Range with Even Sample Size



There are 5 values below the median (lower half), the middle value is 64 which is the first quartile. There are 5 values above the median (upper half), the middle value is 77 which is the third quartile. The interquartile range is 77 – 64 = 13; the interquartile range is the range of the middle 50% of the data.

With an Odd Sample Size:
When the sample size is odd, the median and quartiles are determined in the same way. Suppose in the previous example, the lowest value (62) were excluded, and the sample size was n=9. The median and quartiles are indicated below.

Interquartile Range with Odd Sample Size



When the sample size is 9, the median is the middle number 72. The quartiles are determined in the same way looking at the lower and upper halves, respectively. There are 4 values in the lower half, the first quartile is the mean of the 2 middle values in the lower half ((64+64)/2=64). The same approach is used in the upper half to determine the third quartile ((77+81)/2=79).

Discrete or continuous variables

Variables in any calculation can be characterized by the value assigned to them. A discrete variable consists of separate, indivisible categories. No values can exist between a variable and its neighbour. For example, if you were to observe a class attendance registered from day-to-day, you may discover that the class has 29 students on one day and 30 students on another. However, it is impossible for student attendance to be between 29 and 30. (There is simply no room to observe any values between these two values, as there is no way of having 29 and a half students.)

Not all variables are characterized as discrete. Some variables (such as time, height and weight) are not limited to a fixed set of indivisible categories. These variables are called continuous variables, and they are divisible into an infinite number of possible values. For example, time can be measured in fractional parts of hours, minutes, seconds and milliseconds. So, instead of finishing a race in 11 or 12 minutes, a jockey and his horse can cross the finish line at 11 minutes and 43 seconds.

It is essential to know the difference between the two types of variables in order to properly calculate their cumulative frequency.

Exercise 3

Ensure students follow the steps. Cumulative frequency is marked against the upper boundary of each class. Check for understanding on upper boundaries.

Students should join points with a smooth curve and choose an appropriate scale from which they can read values.

Have the students discuss the **quartiles** and **percentiles** and their meaning. Encourage learners to use Microsoft Excel in their computations and presentations of tables and graphs.

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