Technische Universität Kaiserslautern
Fachbereich Mathematik
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19.01.2021

Exercises to the lecture

"Mathematical Statistics"

-Sheet 10-

Due Date: 26.01.2021 until 10 a.m.

Always submit the executable R files! Zip your submission if necessary. Name your submission and all .pdfs as follows:
{**Last name of exercise partners**}-**ExerciseSheet**-{**Number of exercise sheet**}
E.g. Sass-Hauch-ExerciseSheet-10

10.1 According to a simple genetic principle, if both the mother and the father of a child have genotype Aa, then there is probability $\frac{1}{4}$ that the child will have genotype AA, probability $\frac{1}{2}$ that it will have genotype Aa, and probability $\frac{1}{4}$ that it will have genotype aa. In a random sample of 24 children having both parents with genotype Aa, it is found that 10 have genotype AA, 10 have genotype Aa, and 4 have genotype aa. Test whether the simple genetic principle is correct. Choose $\alpha = 5\%$.

10.2 **R:** Accident at the lab. You have been working on a sample material for 4 weeks. After waiting another 6 weeks it is your turn to put it into the `measuretron2000`. After you get the results you notice that your sample seems to be contaminated. (It must have happened when your colleague sneezed into your Petri dish!)

You do not have the time to redo the experiment. You waited weeks and wish to hand in a paper.

You can find the contaminated data in `measurement.csv`.

a) Look at the contaminated data and come up with a simple method to obtain your original data. You expect your measurements to be small.

b) After you obtain your true data, you assume it to be either log-normally distributed or exponentially distributed. Check this using graphical methods. Which distribution do you choose? Give a reason! *Hint:* Maybe a qq-plot could be helpful.

c) After you hand in the paper with your results an anonymous referee tells you to do a χ^2 goodness-of-fit test for the two distributions. Choose $\alpha = 0.05$. Do this to get your paper published!

10.3 In the wonderful state of Bananarepública the members of parliament were asked about their stance on a tax reform. You can see the data in Table 1.

a) Test whether party membership has an influence on the stance on the tax reform. Choose $\alpha = 5\%$.

b) **R:** Check your result using $chisq.test()$.

10.4 Let $X_1, \ldots, X_n$ be iid random variables with $\mathcal{L}(X_1) = \text{Poisson}(\lambda)$.

	favor	indifferent	opposed
Partido Comunista de Bananarepública(PCdB)	68	56	32
Partido Demócrata Cristiano(PDC)	52	72	20

Table 1: Stances of the members of parliament

a) Let $\lambda_0 > 0$. Find the level α Wald test for

$$H_0 : \lambda = \lambda_0 \text{ vs. } H_1 : \lambda \neq \lambda_0$$

Hint: Use $I(P_{\bar{X}_n}) = \frac{1}{\bar{X}_n}$.

b) **R:** Let $\lambda_0 = 1$ and $\alpha = 0.05$. Use simulation techniques to check which sample size n is big enough to get a level α test.

For this implement the test. Sample n values from a Poisson distribution with $\lambda = \lambda_0$. You do this $m = 10000$ times and count how often you reject H_0. Use this to estimate the size of the test. You do this for $n = 5, 10, 15, 20, \ldots, 200$. How big does n have to be until the estimated size is below the chosen α?

Hint: Adjust m if you are not sure of the results.