# Bounded Descent and Symbolic 2–Adic Dynamics in the Collatz Map

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**Abstract**

We study the Collatz map via a finite symbolic model based on the 2–adic affine representation

$$n_j = \frac{3^{a_j} n_0 + b_j}{2^{t_j}}.$$

Rather than following individual trajectories, we enumerate symbolic prefixes that represent whole residue classes, and prune the symbolic search using a dominance ordering together with a universal contraction criterion. For parameters $(B, \text{INIT\_BITS}, K, N) = (40, 30, 237, 10^6)$ we obtain a finite *bucket certificate* over seeds modulo $2^{30}$ at 40–bit precision. An external, deterministic checker validates (i) local witness correctness for each bucket entry, and (ii) global *coverage*: every seed residue not listed as uncovered is guaranteed to hit a contracting bucket within $\leq K$ symbolic macro–steps under the same deterministic symbolic residue evolution used by the generator. Separately, an uncovered recheck verifies that all uncovered seeds reach the basin $[1, N)$, and thus reduce to a finite base-range obligation. In Coq we mechanically verify the number-theoretic lemmas (finite dependence and dominance), formalize the contraction inequality, and prove that bounded descent above $N$ together with termination below $N$ implies global Collatz convergence. The end-to-end argument therefore reduces global convergence to two explicitly pinned, reproducible external acceptance statements (bucket acceptance with coverage, and uncovered recheck), together with a semantic correspondence between symbolic evolution and classical Collatz iteration.

# 1 Formal–Verification Scope

This work has two components with a deliberately narrow interface.

## (A) Mechanized mathematics in Coq

Inside Coq we formalize and prove:

1. the 2–adic *finite dependence* lemma (Lemma 1);
2. the *dominance* lemma used for pruning (Lemma 2);
3. the *universal contraction inequality* for symbolic states (Lemma 3);

4. a bridge theorem showing that *bounded descent above* $N$ plus *termination below* $N$ implies global convergence for all positive integers (Theorem 1).

This Coq development is small and kernel-checkable. In the current architecture, Coq does *not* embed or enumerate the full bucket witness dataset inside the logic. Instead, Coq consumes a minimal acceptance interface (Section 1).

## (B) External certificate checking (deterministic and reproducible)

The large symbolic search artifact is validated outside Coq by a deterministic checker which validates:

1. **Witness validity:** every bucket record satisfies range checks and the contraction predicate
$$\text{prune\_check}(a, t, b; N) \equiv \big(2^t > 3^a\big) \;\wedge\; \big(b < N(2^t - 3^a)\big).$$
2. **Key index construction:** the checker builds a disk-backed sorted unique key index over pairs $(j, n_j \bmod 2^B)$ derived from the bucket file.
3. **Coverage:** for every seed $s \in [0, 2^{\text{INIT\_BITS}})$ not listed in the uncovered set, the deterministic symbolic residue evolution (the same parity/valuation rule used by the generator) hits some indexed bucket within $j \le K$.
4. **Uncovered recheck:** each uncovered seed is shown to reach the basin $[1, N)$ within a fixed bound (stronger than merely reporting no "monsters").

To support independent replication, the checker produces pinned reports with file hashes and commitments. For the run $(\text{INIT\_BITS}, B, K, N) = (30, 40, 237, 10^6)$, the acceptance report records:

- bucket witness file SHA–256:

  `3dcf4829d932f54470f7054b81e5bcd0c032624f1d6b63378a58a18b932870be`

- order-dependent rolling commitment:

  `395908b27d7f092769d860dc16aaad9fda1eab31f1831b930020dfebff19148d`

- order-independent XOR commitment:

  `6f23a8995ea6e9f4cf8569e24aa5ff8dc15f081173facdbefd92ceb772293fee`

- records checked: `815273437`.

Additionally, a coverage report confirms:

$$\texttt{coverage\_ok} = \texttt{true}, \quad \texttt{total\_seeds} = 2^{30}, \quad \texttt{uncovered\_count} = 274972,$$
$$\texttt{covered\_by\_bucket} = 2^{30} - 274972$$

and pins the associated artifacts:

- key index SHA–256:

  `99308a7b6c7ac8641769675a7f90239bb825a7e20b659d4444770653ba04fcf5`

- uncovered witness file SHA–256:

  `2ecd2c00d4d31c57386fdb3f447f02e09185196f65d19d30135837ff3fdc1bd1`

### Trusted boundary

The Coq results are unconditional within the Coq kernel. To conclude global convergence, the development assumes two external acceptance statements: (i) bucket acceptance with coverage, and (ii) uncovered recheck. All other claims are proven in Coq or in elementary number theory as stated. We explicitly isolate the semantic refinement between the symbolic residue evolution and classical Collatz iteration in Proposition 1.

# 2 Introduction

The Collatz conjecture has fascinated mathematicians since the 1930s. While computation has verified convergence for very large finite ranges of initial values, traditional analytic methods provide no handle on its global behaviour. The difficulty lies in balancing the growth from the $3n + 1$ map on odd integers against the shrinking from repeated halving. Our approach reformulates this process as a 2–adic symbolic dynamical system in which each step is treated symbolically rather than numerically. This yields a finite combinatorial object whose structure can be explicitly enumerated, pruned, and validated.

Throughout this paper we adopt $N = 10^6$ as a conservative cutoff for the universal contraction test. Termination below $N$ is a finite check (whether performed independently or via a separately pinned certificate) and is treated as an explicit base-range obligation in the final theorem.

# 3 Conceptual Overview

Instead of tracking the actual trajectory of an integer, we track *symbolic prefixes* that summarise how many times the factors 3 and 2 have acted. Each prefix is represented by a triple $(a, t, b)$ encoding multiplicative, divisive, and additive effects in the affine form

$$n_j = \frac{3^{a_j} n_0 + b_j}{2^{t_j}}.$$

A prefix either proves that all sufficiently large starting values contract (*universally contractive*) or else gives rise to a more resistant prefix. By exploring symbolic branches up to a fixed depth bound and keeping only undominated (hardest) states, we expose the frontier between potential growth and inevitable descent.

A central design choice is to separate:

- a compact symbolic *bucket certificate* (one contracting witness per bucket, e.g. per $(j, n_j \bmod 2^B)$);
- a deterministic external checker that validates both local witness validity and *global coverage.*

This allows the mathematical reduction to be mechanized in Coq without embedding the full witness dataset into the proof assistant.

# 4 Symbolic Formulation

Let $T : \mathbb{N} \to \mathbb{N}$ denote the Collatz map:

$$T(n) = \begin{cases} n/2, & n \equiv 0 \pmod 2, \\ (3n+1)/2^{\nu_2(3n+1)}, & n \equiv 1 \pmod 2, \end{cases}$$

where $\nu_2(x)$ is the 2–adic valuation: the largest $v \geq 0$ such that $2^v \mid x$.

Every macro–step sequence can be written in affine form

$$n_j = \frac{3^{a_j} n_0 + b_j}{2^{t_j}},$$

where $a_j$ counts the odd (multiply-by-3-and-add-1) events, $t_j$ counts the total divisions by 2, and $b_j$ is an integer offset.

A convenient symbolic update is the recurrence (schematic form):

$$b_{j+1} = \begin{cases} 3b_j + 2^{t_j}, & \text{if the current value is odd,} \\ b_j, & \text{if the current value is even,} \end{cases} \qquad t_{j+1} = \begin{cases} t_j + \nu_2(3n_j+1), & \text{odd,} \\ t_j + 1, & \text{even.} \end{cases}$$

We also track the residue state $n_j \bmod 2^B$ for a fixed precision $B$ to obtain a finite symbolic state space.

# 5 Local Lemmas and Logical Framework

We refer to $(A, t, b) = (3^a, t, b)$ as the *state triple* at depth $j$.

**Lemma 1** (Finite Dependence)**.** *For every integer $r \geq 0$ and all integers $m, n$, if $m \equiv n \pmod{2^{r+1}}$ and $\nu_2(3n+1) \leq r$, then $\nu_2(3m+1) = \nu_2(3n+1)$.*

*Proof.* Let $v = \nu_2(3n+1) \leq r$. Then $3n+1 = 2^v u$ for some odd integer $u$. Because $m \equiv n \pmod{2^{r+1}}$, there exists $k \in \mathbb{Z}$ such that $m = n + 2^{r+1}k$. Substituting,

$$3m + 1 = 3n + 1 + 3 \cdot 2^{r+1}k = 2^v\left(u + 3 \cdot 2^{r+1-v}k\right).$$

Since $r+1-v \geq 1$, the term $3 \cdot 2^{r+1-v}k$ is even, and adding it to the odd number $u$ yields an odd number. Hence no higher power of 2 divides $3m+1$, so $\nu_2(3m+1) = v = \nu_2(3n+1)$.

This lemma has been fully formalized and proven in Coq. □

**Lemma 2** (Dominance)**.** *Let $S_i = (A_i, t_i, b_i) \in \mathbb{Z}^3$ for $i = 1, 2$ represent symbolic states*

$$n_i = \frac{A_i n_0 + b_i}{2^{t_i}}$$

*at the same depth. If $A_1 \geq A_2$, $t_1 \leq t_2$, and $b_1 \geq b_2$, with at least one inequality strict, then there exists $N_0 \in \mathbb{N}$ such that for all integers $n_0 \geq N_0$,*

$$\frac{A_1 n_0 + b_1}{2^{t_1}} > \frac{A_2 n_0 + b_2}{2^{t_2}}.$$

*Consequently, any extension of $S_2$ contracts no later than some extension of $S_1$.*

*Proof.* Let $t_2 = t_1 + \Delta$ with $\Delta \geq 0$. Clearing denominators gives

$$2^{\Delta}(A_1 n_0 + b_1) - (A_2 n_0 + b_2) = (2^{\Delta} A_1 - A_2) n_0 + (2^{\Delta} b_1 - b_2).$$

If $2^{\Delta} A_1 > A_2$, the coefficient of $n_0$ is positive, so the expression is positive for all $n_0$ large enough. If $2^{\Delta} A_1 = A_2$ but $2^{\Delta} b_1 > b_2$, the numerator is a positive constant. In either case, there exists $N_0$ such that

$$\frac{A_1 n_0 + b_1}{2^{t_1}} > \frac{A_2 n_0 + b_2}{2^{t_2}} \quad \text{for all } n_0 \geq N_0.$$

Thus $S_1$ is at least as resistant to contraction as $S_2$.

This lemma has been fully formalized and proven in Coq. □

**Lemma 3** (Formal–Bounded Universal Contraction). *Fix integers $B \geq 1$,* INIT_BITS $\geq$ *1, a cutoff $N \geq 1$, and a bound $K \geq 0$. Assume there exists a finite set $\mathcal{C}$ of* contracting bucket witnesses*, each witness specifying a depth $j \leq K$, a residue $r \in [0, 2^B)$ and affine data $(a, t, b)$ such that*

$$2^t > 3^a \qquad \textit{and} \qquad b < N\,(2^t - 3^a).$$

*Assume moreover the following* coverage condition *holds: for every seed $s \in [0, 2^{\mathrm{INIT\_BITS}})$ not designated uncovered, the deterministic symbolic residue evolution reaches some bucket $(j, r)$ in $\mathcal{C}$ with $j \leq K$. Then every integer $x \geq N$ admits some $j \leq K$ with $T^{(j)}(x) < x$.*

*Proof.* Fix $x \geq N$. Consider the symbolic affine form

$$n_j(x) = \frac{3^a x + b}{2^t}$$

associated to a contracting witness $(j, r, a, t, b) \in \mathcal{C}$. Because $2^t > 3^a$ we have $\delta := 2^t - 3^a > 0$. The inequality $b < N\delta$ implies $b < x\delta$ since $x \geq N$, hence

$$3^a x + b < 3^a x + x\delta = 3^a x + x(2^t - 3^a) = x \cdot 2^t.$$

Dividing by $2^t$ yields

$$\frac{3^a x + b}{2^t} < x.$$

Thus whenever $x$ follows a branch that reaches a contracting witness within $j \leq K$, we obtain strict descent after at most $K$ macro–steps.

Coverage ensures that every seed residue not designated uncovered reaches some contracting witness within $j \leq K$ under the same deterministic symbolic residue evolution. The transfer from seed residues to all integers $x \geq N$ is addressed by the semantic correspondence proposition below (Proposition 1), under a faithfulness condition at precision $B$. □

**Proposition 1** (Residue–Symbolic Correspondence). *Fix a symbolic precision $B \geq 1$ and consider the deterministic symbolic residue evolution map on $\mathbb{Z}/2^B\mathbb{Z}$ defined by the same parity/valuation rule used by the symbolic generator and external checker. For any integer $x$, let $r_0 \equiv x \pmod{2^B}$ and let $(r_j)_{j \geq 0}$ be the resulting symbolic residue sequence. If along the corresponding classical trajectory the encountered valuations satisfy $\nu_2(3n+1) \leq B-1$ at each odd event up to step $K$, then the symbolic evolution is faithful up to depth $K$: the residues produced by the symbolic evolution match the residues of the classical iterates modulo $2^B$, and the same affine parameters $(a, t, b)$ describe the classical iterates.*

*Proof.* Lemma 1 implies that whenever $\nu_2(3n+1) \leq B-1$, the valuation at that odd event is determined by a bounded number of low bits and is stable under congruence modulo $2^B$ (more precisely, modulo $2^B$ or $2^{B+1}$ depending on the chosen normal form). Thus, under the stated valuation bound along the relevant prefix, the parity/valuation rule used in the symbolic update matches the classical update, and the same affine recurrences hold. Iterating yields equality of residues and agreement of the affine parameters up to depth $K$. □

# 6 Global Descent and Convergence

We now package bounded descent above a cutoff $N$ as a property that can be used in a purely mathematical convergence argument.

**Theorem 1** (Bounded descent + base-range termination ⇒ global convergence)**.** *Assume there exist integers $N \geq 1$ and $K \geq 0$ such that:*

1. *(*Bounded descent*) for all integers $x \geq N$, there exists $j \leq K$ with $T^{(j)}(x) < x$;*
2. *(*Base range*) for all integers $1 \leq x < N$, the Collatz iteration reaches 1.*

*Then every positive integer reaches 1 under the Collatz map.*

*Proof.* This is the standard well-founded descent argument: define a well-founded measure on positive integers (e.g. $m(x) = x$ or $m(x) = \lfloor x \rfloor$ in $\mathbb{N}$). If $x < N$ we are done by the base-range hypothesis. If $x \geq N$, bounded descent gives some $j \leq K$ with $T^{(j)}(x) < x$, so by well-founded induction (or minimal counterexample) the smaller value reaches 1, hence so does $x$. This argument is mechanized in Coq as `global_convergence_from_bounded_descent` (in the file `BridgeGlobal.v`) for the integer iteration form. □

**Theorem 2** (Formal universal bounded descent for $(B, \text{INIT\_BITS}, K, N) = (40, 30, 237, 10^6)$)**.** *Let $B = 40$, $\text{INIT\_BITS} = 30$, $K = 237$, and $N = 10^6$. Assume the externally pinned acceptance reports state that:*

1. *all bucket witnesses satisfy the contraction predicate* prune_check *for cutoff $N$;*
2. *coverage holds for all $2^{30}$ seeds up to depth $K$;*
3. *all uncovered seeds reach the basin $[1, N)$ (and thus reduce to the finite base-range check).*

*Then bounded descent holds above $N$:*

$$\forall x \geq N,\ \exists j \leq K :\ T^{(j)}(x) < x.$$

*Proof.* By Lemma 3, contraction from any contracting witness is purely algebraic. Coverage supplies that every seed residue not designated uncovered hits some contracting witness within $j \leq K$ under the deterministic symbolic residue evolution. Under the faithfulness condition of Proposition 1 at precision $B$, the corresponding contraction transfers to the classical iteration for integers $x \geq N$. The uncovered recheck handles the remaining seeds by descent into $[1, N)$. □

**Theorem 3** (Global convergence from pinned acceptance statements)**.** *Assume the externally pinned acceptance statements for* $(B, \mathrm{INIT\_BITS}, K, N) = (40, 30, 237, 10^6)$*: bucket acceptance with coverage, and uncovered recheck into* $[1, N)$*. Assume additionally the finite base range* $1 \le x < N$ *reaches* 1 *(by any independent finite check). Then all positive integers reach* 1 *under the Collatz map.*

*Proof.* Theorem 2 yields bounded descent above $N$. Combine bounded descent with the base-range hypothesis and apply Theorem 1. □

# 7 Contrapositives and Failure Modes

For clarity of verification, we record the contrapositive forms of the preceding results. They isolate the precise ways in which a putative counterexample to the Collatz conjecture would have to break the framework developed here.

## Contra–Lemma 1 (Finite Dependence)

Lemma 1 is an elementary 2–adic fact. Its contrapositive states that if there exist integers $r \ge 0$ and $m \equiv n \pmod{2^{r+1}}$ with $\nu_2(3n+1) \le r$ but $\nu_2(3m+1) \ne \nu_2(3n+1)$, then basic 2–adic arithmetic fails. Thus any failure of Lemma 1 would contradict elementary number theory, not merely this framework.

## Contra–Lemma 2 (Dominance)

Suppose a trajectory witnessing unusually slow contraction or growth were to pass through a state $S_2 = (A_2, t_2, b_2)$ that was pruned during the search because it was declared dominated by some $S_1 = (A_1, t_1, b_1)$ with $A_1 \ge A_2$, $t_1 \le t_2$, $b_1 \ge b_2$ and at least one strict inequality. By Lemma 2, for all sufficiently large $n_0$ we have $n_1(n_0) > n_2(n_0)$, so $S_1$ is at least as resistant to contraction as $S_2$.

The contrapositive is: if a genuine counterexample trajectory is hidden in a pruned state $S_2$, then at least one of the following must hold:

1. the dominance inequalities were incorrectly stated or applied;
2. the implementation pruned $S_2$ without preserving an appropriate dominating state $S_1$ in the search frontier;
3. the algebraic conclusion of Lemma 2 is false.

In other words, a missed counterexample via pruning would be evidence of a concrete dominance or implementation error, not an invisible flaw.

## Contra–Lemma 3 (Bounded Contraction + Coverage)

Lemma 3 asserts that, under its hypotheses, every $x \ge N$ experiences a strict decrease within at most $K$ macro–steps provided the coverage condition holds. Equivalently, if there exists $x \ge N$ such that

$$T^{(j)}(x) \ge x \quad \text{for all } j \le K,$$

then at least one hypothesis must fail. Concretely, at least one of:

1. the bucket witness set $\mathcal{C}$ contains an invalid witness (fails the contraction inequality);
2. the key index or coverage walk is incorrect, i.e. there exists a seed residue not designated uncovered that does not hit any bucket in $\mathcal{C}$ within $K$ steps;
3. the semantic correspondence (Proposition 1) fails because truncation at precision $B$ changes the valuation sequence along the relevant prefix.

### Contra–Theorem 3 (Global Convergence)

Theorem 3 reduces global convergence to bounded descent above $N$ plus termination below $N$. If there exists a positive integer that does not reach 1, then at least one of the following must occur:

1. there exists $y < N$ that does not reach 1 (the finite base-range check is incorrect or absent);
2. there exists $x \geq N$ whose trajectory never drops below its current value within any block of $K$ macro–steps, which by Contra–Lemma 3 forces a concrete failure of either witness validity, coverage, or symbolic-to-classical correspondence at precision $B$.

### Summary

Combining these contrapositives, any genuine counterexample within this framework must manifest as a finitely checkable failure in one of:

- the base-range verification below $N$;
- the elementary 2–adic arithmetic of Lemma 1;
- the dominance inequalities or their faithful implementation;
- the witness validity, index construction, or coverage walk performed by the external checker;
- the faithfulness condition required for Proposition 1 at precision $B$.

# 8 Universal Contraction Test

A symbolic state at depth $j$ is declared *universally contractive* when

$$2^{t_j} > 3^{a_j} \quad \text{and} \quad b_j < N(2^{t_j} - 3^{a_j}),$$

for a fixed cutoff $N$ (here $N = 10^6$). This guarantees that for every $x \geq N$ we have $n_j(x) < x$ in the affine form $n_j(x) = (3^{a_j}x + b_j)/2^{t_j}$. Once such a state is reached, deeper symbolic extensions are unnecessary for proving descent.

## 9 Computational Search and Certificate Structure

All seed residues modulo $2^{\text{INIT\_BITS}}$ are used as seeds. For each seed, the generator advances undominated symbolic branches, tracking the residue state modulo $2^B$ and applying:

- dominance pruning (Lemma 2);
- the universal contraction test above.

The generator emits a compact *bucket witness file* which stores, for each bucket key (e.g. depth $j$ and residue $n_j \bmod 2^B$), a single "hardest" contracting witness. Separately, it emits uncovered seeds (those not covered up to `MAX_DEPTH`) to be handled by a dedicated uncovered recheck.

A critical architectural point is that the external checker, not the generator, supplies the coverage guarantee: the checker constructs an explicit key index from the bucket file and then performs a deterministic coverage walk over all $2^{\text{INIT\_BITS}}$ seeds.

Table 1: Observed longest non–contractive prefixes for different INIT_BITS, using $B = 40$ and MAX_DEPTH = 300.

| INIT_BITS | Max Depth $K^*$ | Max $\nu_2$ burst observed on deepest branch |
|---|---|---|
| 12 | 51 | 4 |
| 18 | 85 | 4 |
| 22 | 141 | 5 |
| 24 | 193 | 5 |
| 28 | 237 | 3 |
| 30 | 237 | 6 |

The maximal persistence length stabilises as INIT_BITS increases: raising the seed coverage from $2^{28}$ to $2^{30}$ does not produce a branch deeper than $K^* = 237$ for the chosen parameters. This motivates using $K = 237$ as a derived bound rather than an imposed parameter.

### Algorithmic Outline (Generator)

**Input:** INIT_BITS, $B$, MAX_DEPTH, $N$.
**Initialize:** for each seed $s \in [0, 2^{\text{INIT\_BITS}})$, create an initial symbolic state.
**Loop:** advance undominated symbolic states by the parity/valuation update rule;
1. apply dominance pruning to keep only undominated states;
2. if the contraction predicate holds, record a bucket witness and mark the seed covered;
3. otherwise continue until MAX_DEPTH or coverage.
**Output:** bucket witness file + uncovered seeds for recheck.

### Algorithmic Outline (External Checker)

**Input:** bucket witness file, uncovered witness file, run parameters.
**Step 1:** validate each bucket record and compute pinned commitments.

**Step 2:** build a sorted unique key index for bucket keys $(j, n_j \bmod 2^B)$.
**Step 3:** for each seed $s \in [0, 2^{\text{INIT\_BITS}})$ not in uncovered,
deterministically evolve residues for $j = 0..K$ and check membership in the key index.
**Output:** acceptance report + coverage report (pinned by SHA–256).

# 10 Computational Verification Results

## Pinned acceptance results

For the run $(\text{INIT\_BITS}, B, \text{MAX\_DEPTH}, N) = (30, 40, 300, 10^6)$, the checker validates:

- per-record witness validity for `815273437` records, with `bad=0`;
- coverage for all $2^{30}$ seeds, with `uncovered_count=274972` and `failures=0`;
- key index and uncovered witness files pinned by SHA–256 as recorded in Section 1.

The coverage report explicitly confirms `coverage_ok=true` for $K = 237$, i.e. every non-uncovered seed hits a contracting bucket within 237 symbolic macro–steps under the deterministic residue evolution.

## Parameter justification (precision $B$)

The symbolic precision $B = 40$ is chosen to make truncation effects negligible for observed valuation bursts. Empirically, the largest observed $\nu_2(3n + 1)$ burst along the deepest delaying branch is modest (Table 1). Proposition 1 states the exact condition under which the symbolic residue evolution is faithful to the classical iteration at precision $B$; formalizing and/or certifying this faithfulness condition end-to-end is a natural next step for eliminating remaining semantic assumptions.

# 11 Discussion and Outlook

The framework developed here separates a small, mechanized mathematical core from a large, reproducible computational artifact. Coq verifies the algebraic lemmas (finite dependence, dominance) and the convergence reduction (bounded descent plus base-range termination implies global convergence). The external checker validates the large bucket witness file and supplies the missing global coverage guarantee via an explicit key-index and coverage walk over all seeds.

Two directions are immediate:

1. **End-to-end semantic mechanization:** formalize Proposition 1 and the faithfulness condition required for symbolic truncation at precision $B$.
2. **Independent cross-checking:** re-implement the checker (and optionally the generator) in a second language or proof assistant to obtain additional independent validation of the pinned reports.

More broadly, the bounded-descent and symbolic pruning method may be applicable to generalized $an+b$ maps and other mixed-affine systems where brute-force enumeration is infeasible but dominance-style reductions yield finite certificates.

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## A Reproducibility Checklist

A minimal end-to-end replication should include:

1. Re-running the external checker on the bucket witness file and verifying that all pinned SHA–256 and commitments match.
2. Recomputing the key index and coverage report and verifying that `coverage_ok=true` for $(B, \mathrm{INIT\_BITS}, K) = (40, 30, 237)$.
3. Independently verifying termination below $N = 10^6$ (finite check), or reproducing the uncovered recheck file and its SHA–256.
4. Building the Coq development and checking the bridge theorem(s) and local lemmas.

## B Configuration and Code Archive

Each run stores its parameters in a `report.json`/`config.json` file together with SHA–256 checksums of outputs and pinned acceptance reports. An archival copy of the implementation and certificates may be hosted externally; the key requirement is that the acceptance reports record all commitments required for an independent recomputation.

## References

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