SEQUENTIAL MACHINE LEARNING FOR ADAPTIVE EDUCATIONAL SYSTEMS

PhD defense of Julien SEZNEC, the 15th of December 2020

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inventors for the digital world

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OUTLINE

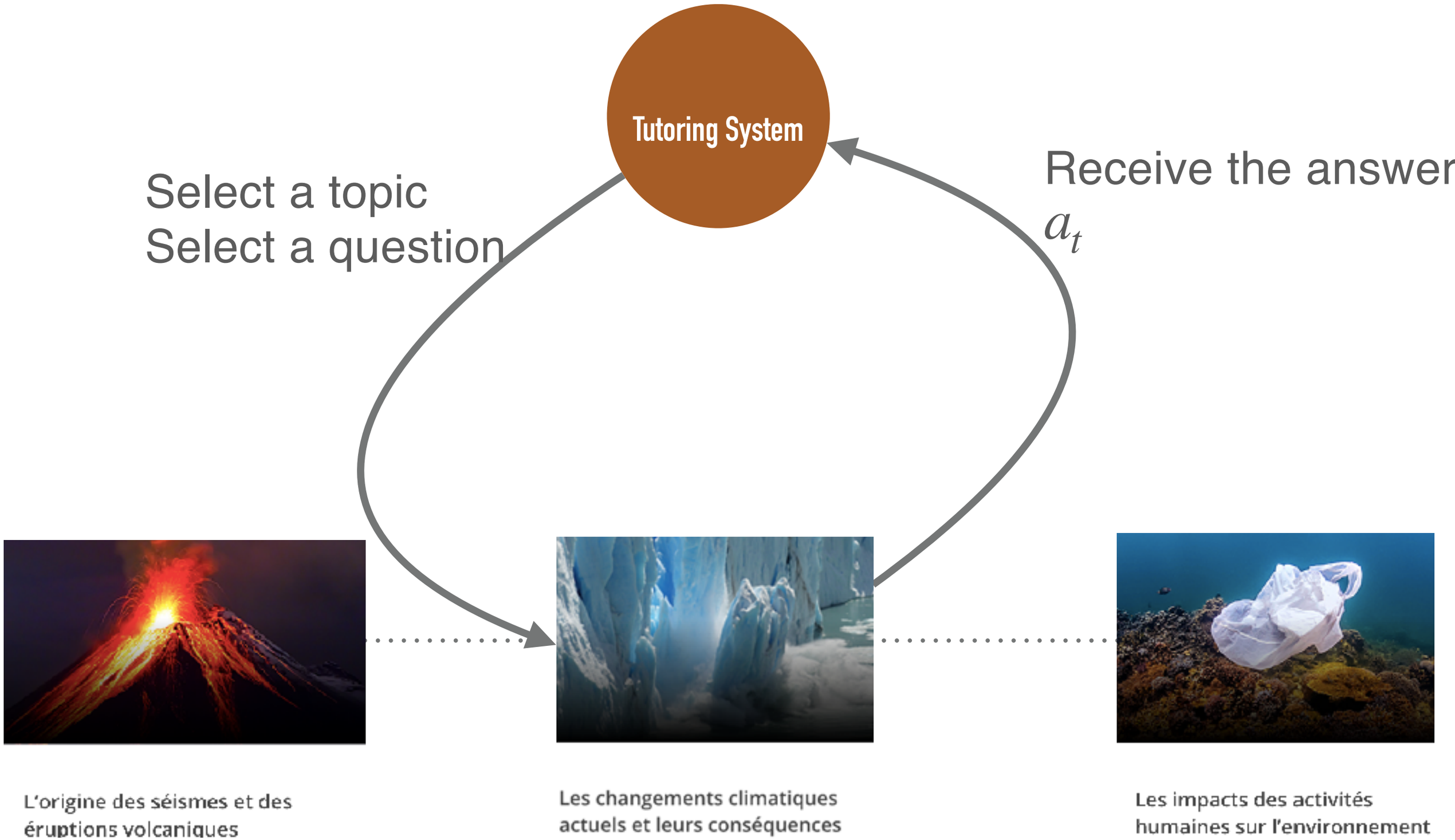
1. Bandits for Adaptive Educational Systems

2. Rested Rotting Bandits

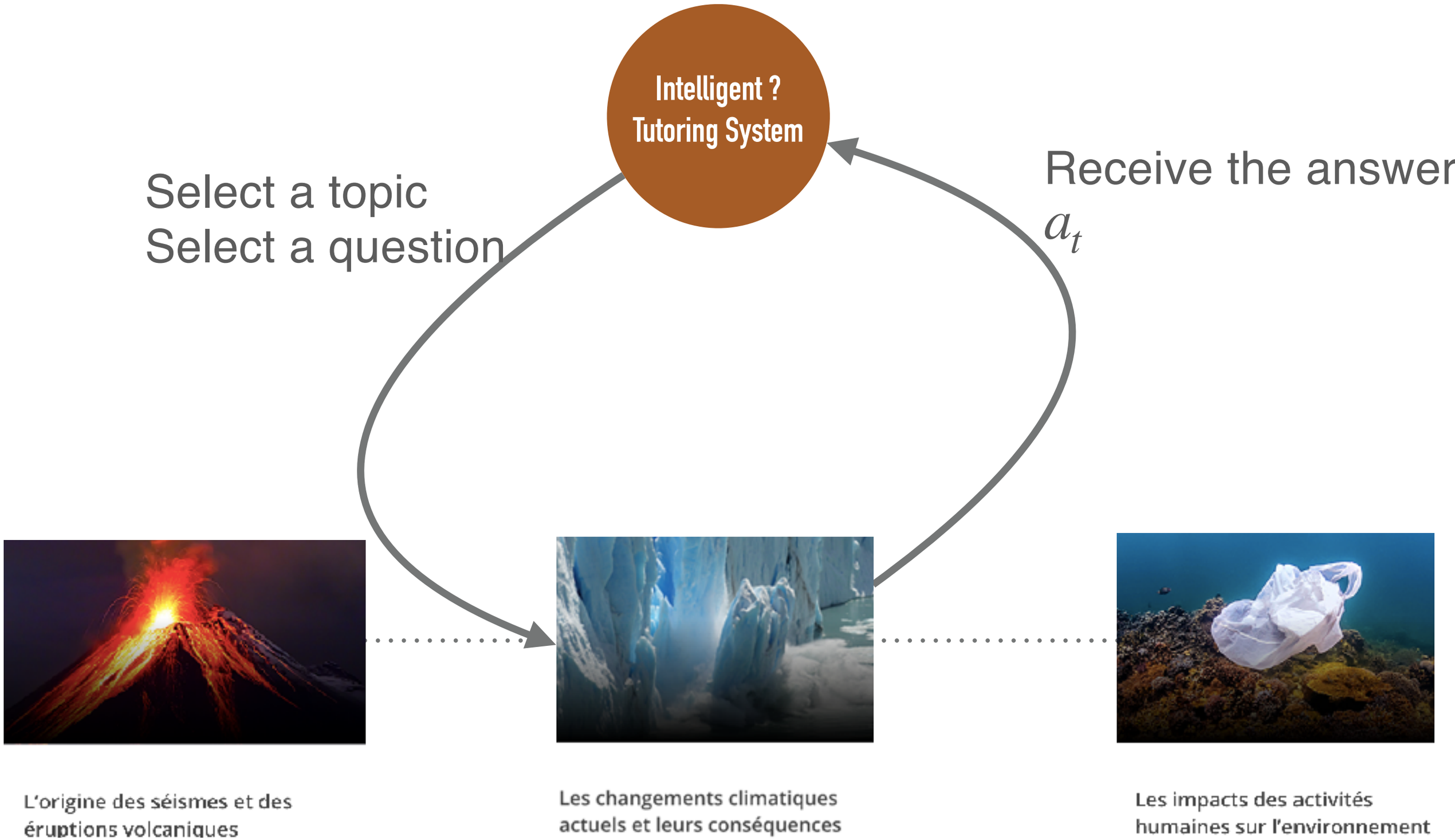
3. Restless Rotting Bandits

AFTERCLASSE

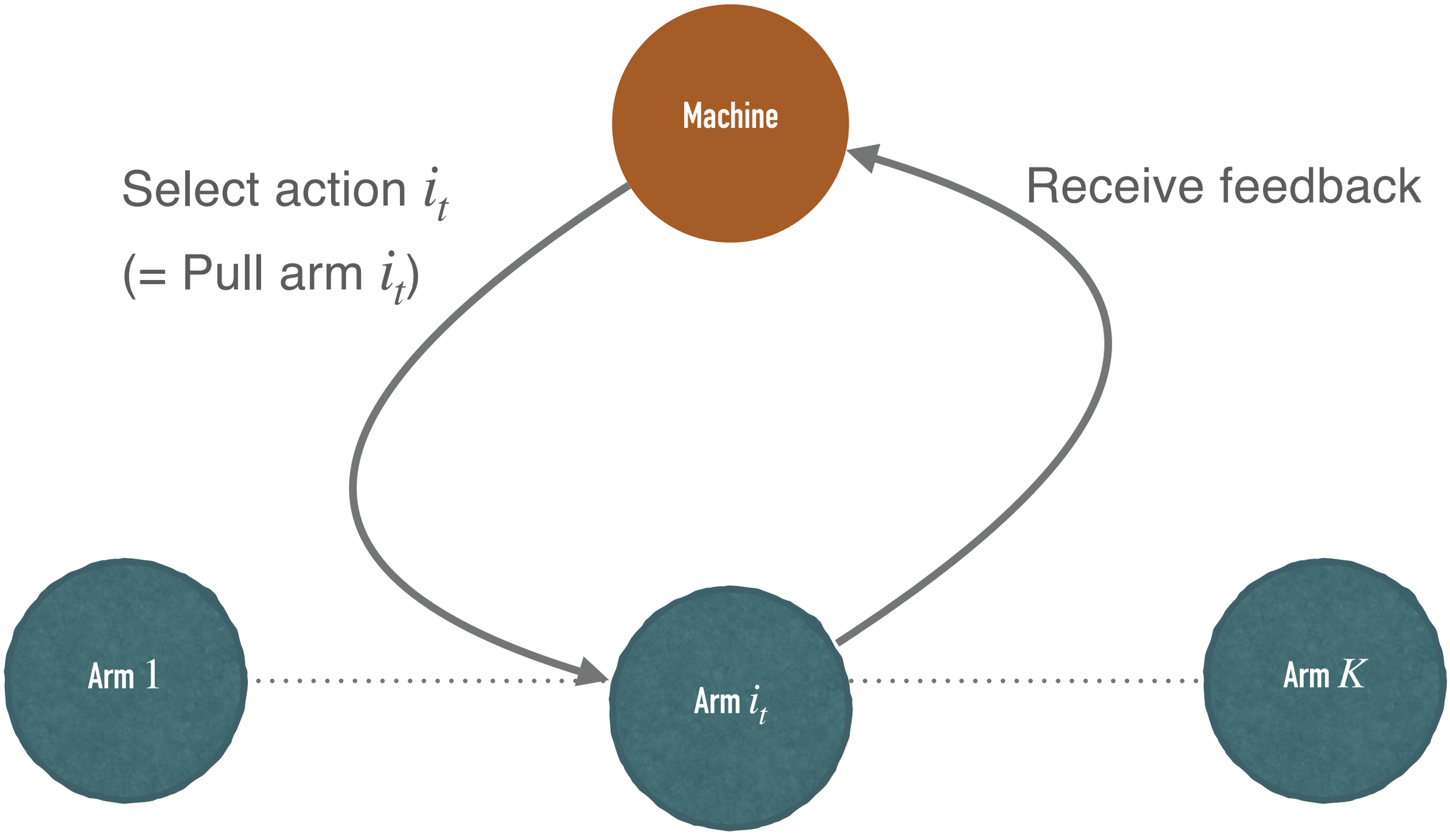
Sequential Adaptive Educational Systems



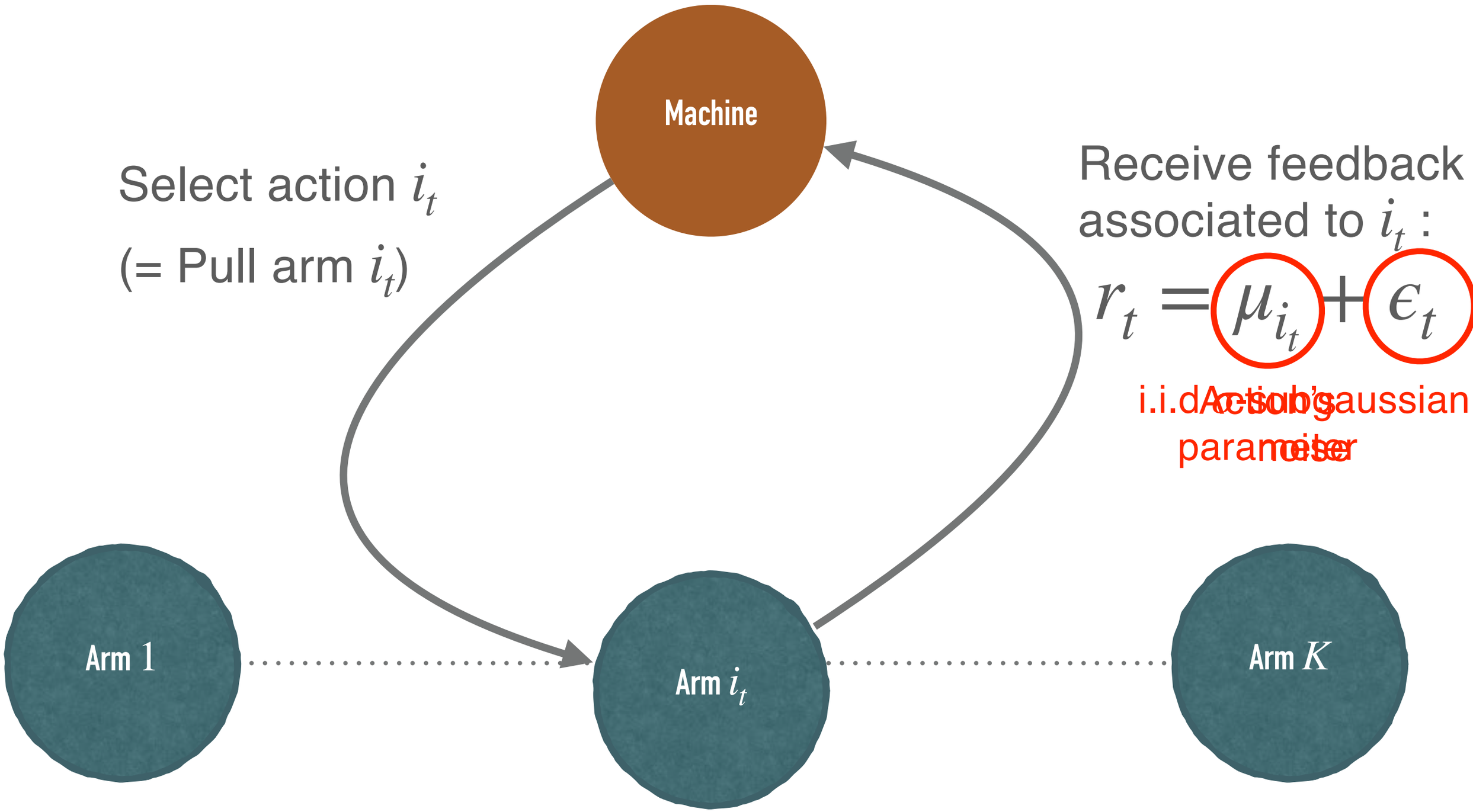
Sequential Adaptive Educational Systems



Sequential Exploration: Bandits Feedback



Stationary Bandits: Feedback



Stationary Bandits: Objective

- Standard view : Feedback = reward
- Objective : Maximize cumulative reward in expectation at the horizon T

$$J_T(\pi) = \mathbb{E}\left[\sum_{t<T} r_t\right] = \mathbb{E}\left[\sum_{t<T} \mu_{i_t}\right]$$

- Optimal oracle policy

$$\pi^\star(t) \in \operatorname{argmax}_{i \leq K} \mu_i$$

- Regret

$$R_T(\pi) = J_T(\pi^\star) - J_T(\pi) = \sum_{i \in \mathcal{K}} N_{i,T} \Delta_i$$

$\Delta_i = \max_j \mu_j - \mu_i$

$N_{i,T}$ Number of pulls of arm i at round T

Upper Confidence Bound Policy

Select the arm with the largest index:

$$\hat{\mu}_i + \sqrt{\frac{2\sigma^2 \log 1/\delta}{N_{i,t}}}$$

Expected average reward + cost of information

Empirical average + bonus for confidence level

$$\delta = \max\left(\frac{KN_{i,t}}{t}, 1\right)$$

Optimal problem-independent rate:

$$\Theta\left(\sigma\sqrt{KT}\right)$$

Optimal problem-dependent rate:

$$\sum_{i,\Delta_i>0} \frac{2\sigma \log(T)}{\Delta_i}$$

Bandits in Educational Systems

1. What is the objective ? What is the reward ?

2. What is the « best » action ?

3. How do actions impact observations ?

4. What can we learn with ~100 samples ?

Bandits In Educational Systems

Clément et al. (2016)	EXP3	Target the topic with the largest improvement	$r_t = \frac{1}{h}\sum_{i=0}^{h-1}\left(a_{t-i} - a_{t-h-i}\right)$
Melesko et al. (2019)	UCB	Target the most difficult topic	$r_t = -a_t$
Lin et al. (2020)	Thomson sampling	Target the most difficult topic	$r_t = -a_t$

Bandits In Educational Systems

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3. Restless Rotting Bandits

When Bandits Go Rotting …

$$a_t \uparrow \Longrightarrow \quad r_t = -\, a_t \downarrow$$

L'origine des séismes et des éruptions volcaniques

Les changements climatiques actuels et leurs conséquences

Les impacts des activités humaines sur l'environnement

Rested vs. Restless Bandits

Rested :

- Actions trigger non-stationarity
- No action = no change = rested
- $\mu_i(N_{i,t})$

Restless :

- Changes happen independently of the actions
- Action can change when they are not selected
- $\mu_i(t)$

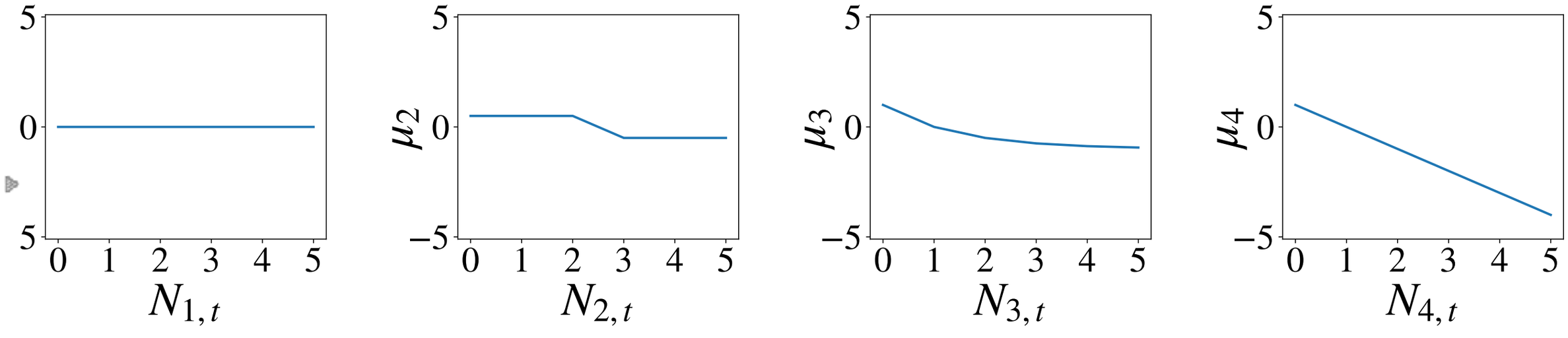
Rested Rotting Bandits are …

Stochastic bandits …

- K arms
- At each round t, agent pulls arm i and receives a noisy reward $r_t \leftarrow \mu_i + \varepsilon_t$ *(ε_t i.i.d. ; σ-sub-gaussian)*
- Maximize cumulative reward: $J_T(\pi) = \mathbb{E}\left[\sum_{t \leq T} r_t\right]$

… with rotting arms

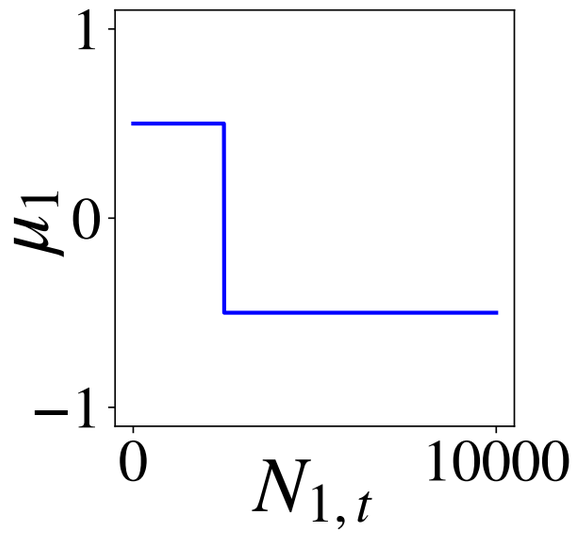
- $\{\mu_i\}$ are ***non-increasing*** functions of $N_{i,t}$ the ***number of pulls of arm*** i at time t
- $L \triangleq \max_{i \in K} \max_{n \leq T} \mu_i(n) - \mu_i(n+1)$

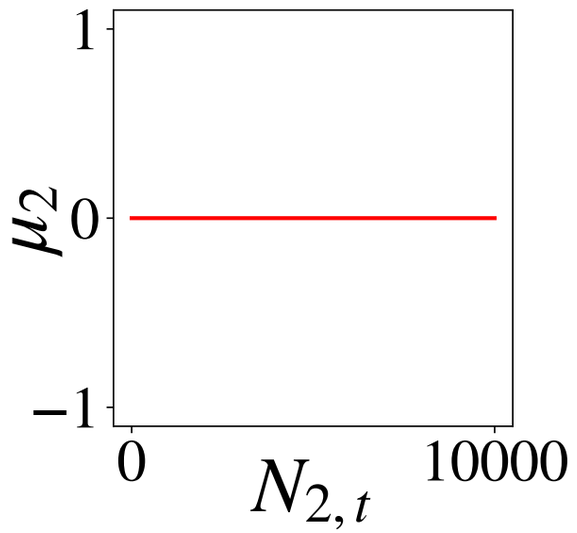


Oracle & Regret

Optimal Oracle:

$$\pi^\star(t) = \arg\max_i \mu_i\left(N_{i,t}\right)$$

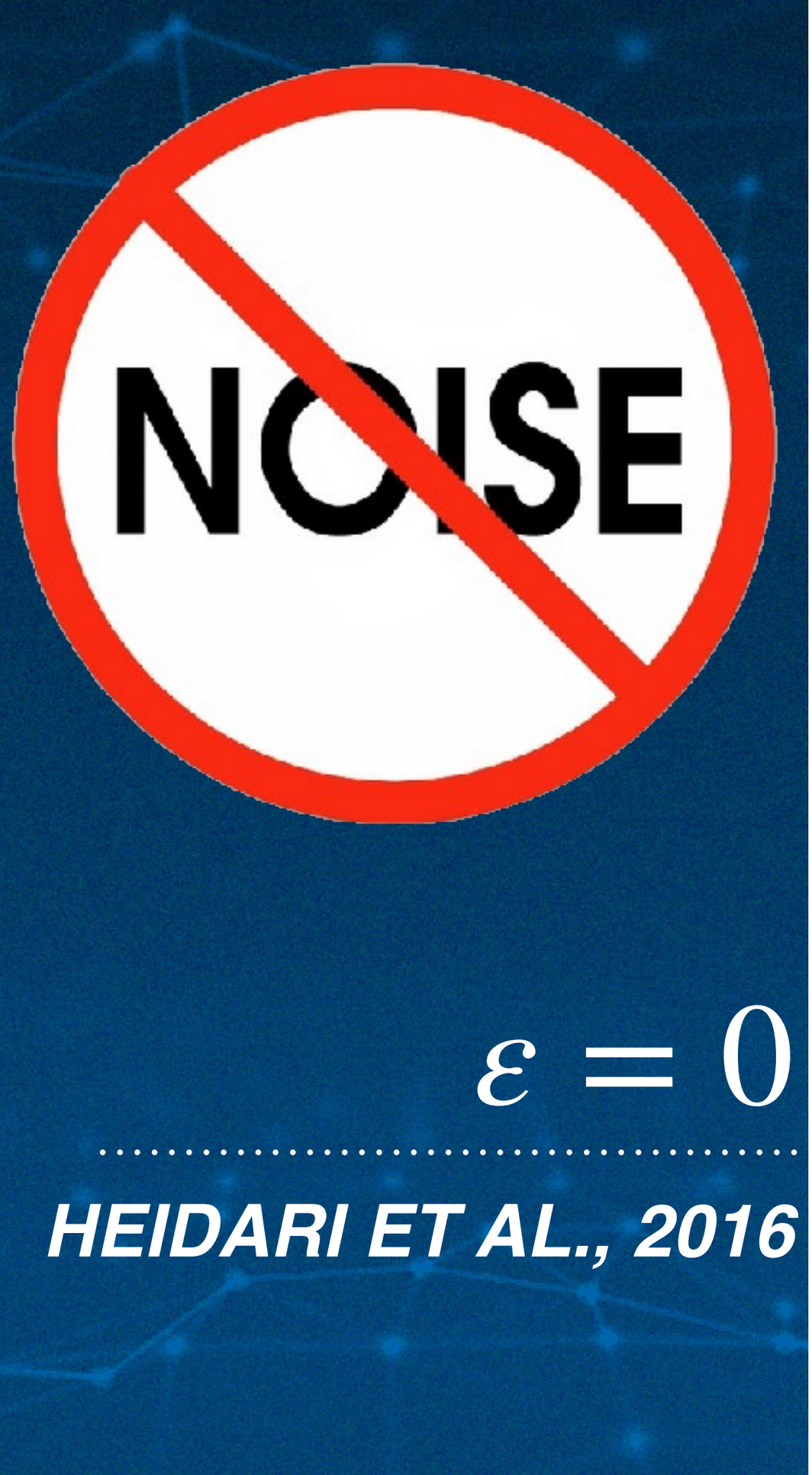




$$R_T(\pi) = J_T(\pi^\star) - J_T(\pi)$$

$$= \sum_{i \in \mathrm{UP}} \sum_{s=N^\pi_{i,T}+1}^{N^\star_{i,T}} \mu_i(s) - \sum_{i \in \mathrm{OP}} \sum_{s=N^\star_{i,T}+1}^{N^\pi_{i,T}} \mu_i(s)$$

$$\begin{cases} \mathrm{UP} : \text{underpulled arms by } \pi \text{ w.r.t } \pi^\star \\ \mathrm{OP} : \text{overpulled arms by } \pi \text{ w.r.t } \pi^\star \end{cases}$$

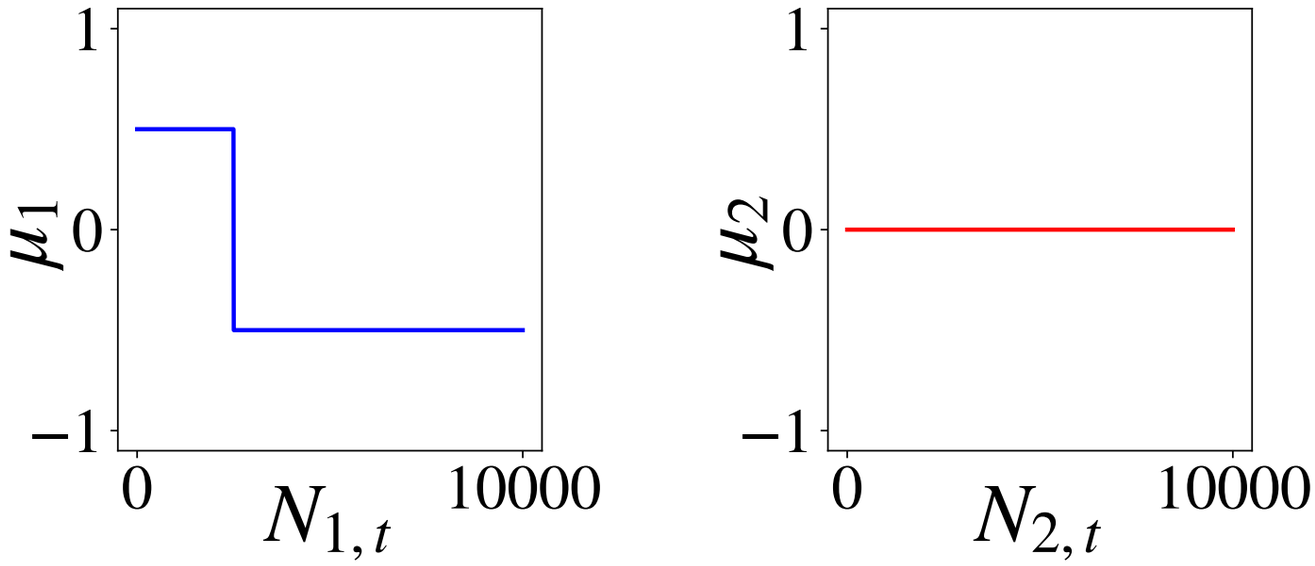


Optimal Oracle:
$\pi_O(t) = \arg\max_i \mu_i \left(N_{i,t}\right)$

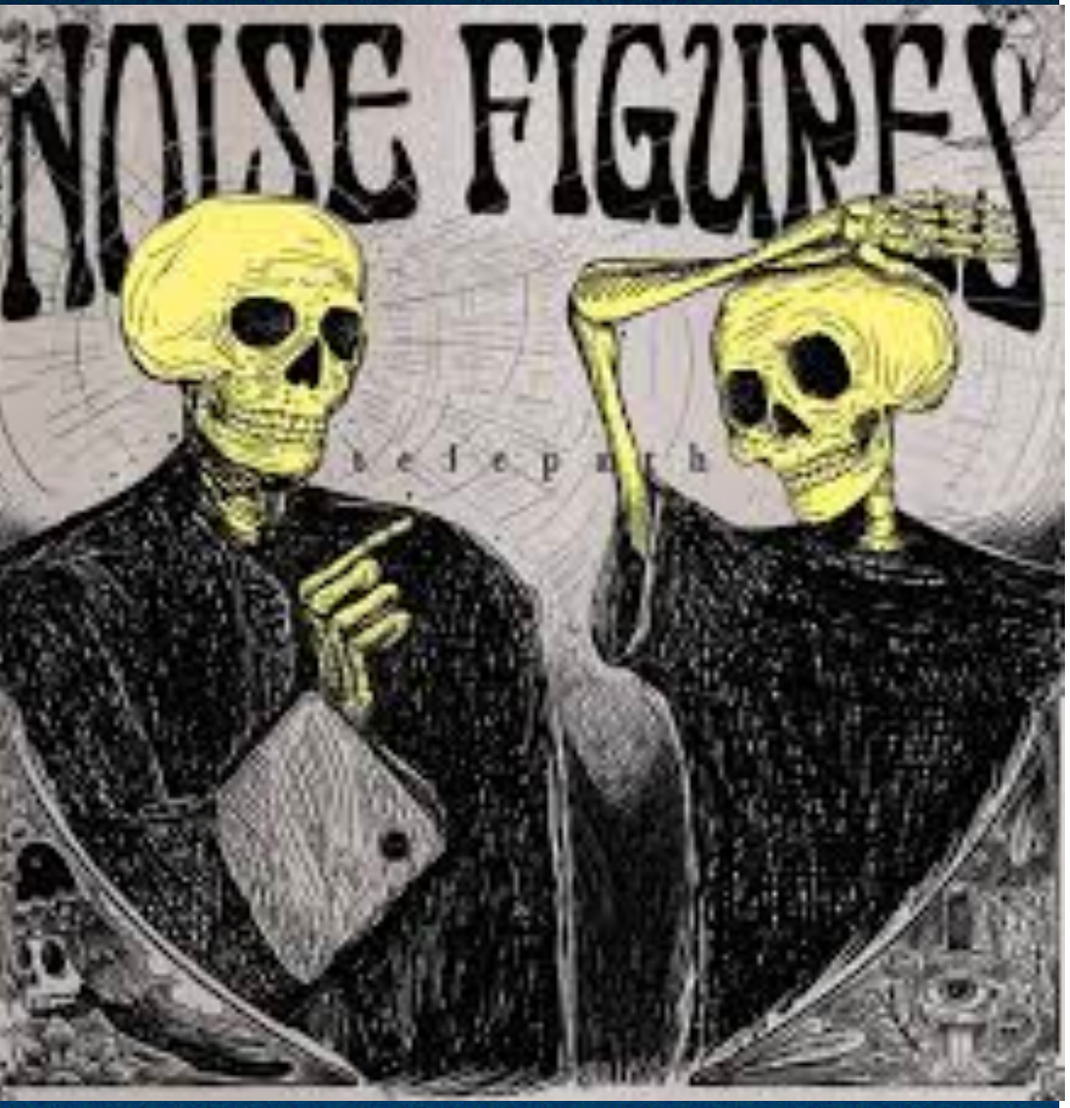
Online bandit policy:
$\pi_B(t) = \arg\max_i \mu_i \left(N_{i,t} - 1\right)$

Minimax regret rate: $R_T(\pi_B) \leq (K-1)\,L$

$\rightarrow$ At most,1 mistake per arm.



Rested : New mistake cancels the old one !



NOISE

LEVINE ET AL., 2017

NOISE = AVERAGE

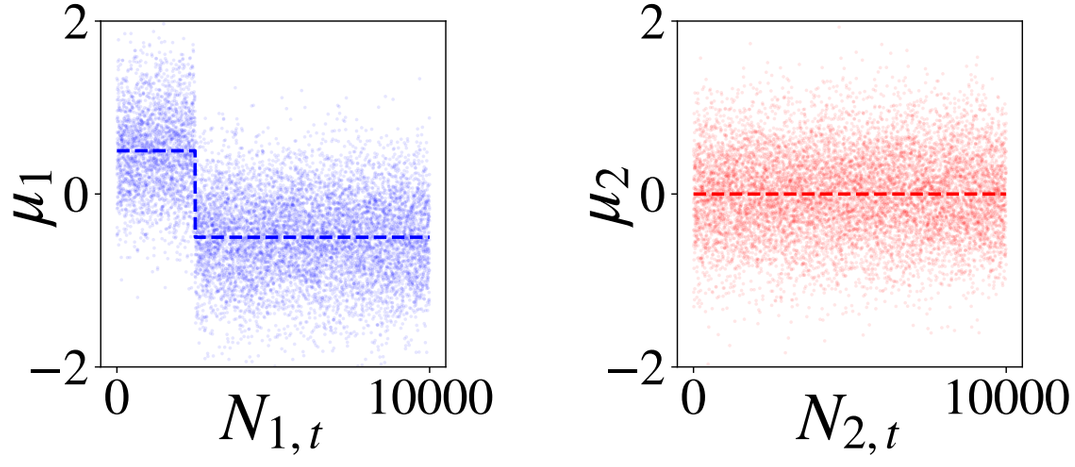
(Which samples? How many?)

wSWA [LEVINE, 2017]

Algorithm 3 SWA (Levine et al., 2017)

Input: K, L, T, σ

1: $h \leftarrow \widetilde{\mathcal{O}}\left(\left(\frac{\sigma T}{KL}\right)^{2/3}\right)$
2: **for** $t \leftarrow Kh+1, Kh+2, \ldots$ **do**
3: SELECT : $\arg\max_{i \in \mathcal{K}} \widehat{\mu}_i^h(N_{i,t})$
4: **end for**



Average of the h last samples

Regret due to bias: $\widetilde{O}\left(LKh\right)$

Regret due to variance : $\widetilde{O}\left(\sigma T\sqrt{h}^{-1}\right)$

Worst case regret : $\widetilde{O}\left(K^{1/3}T^{2/3}\right)$

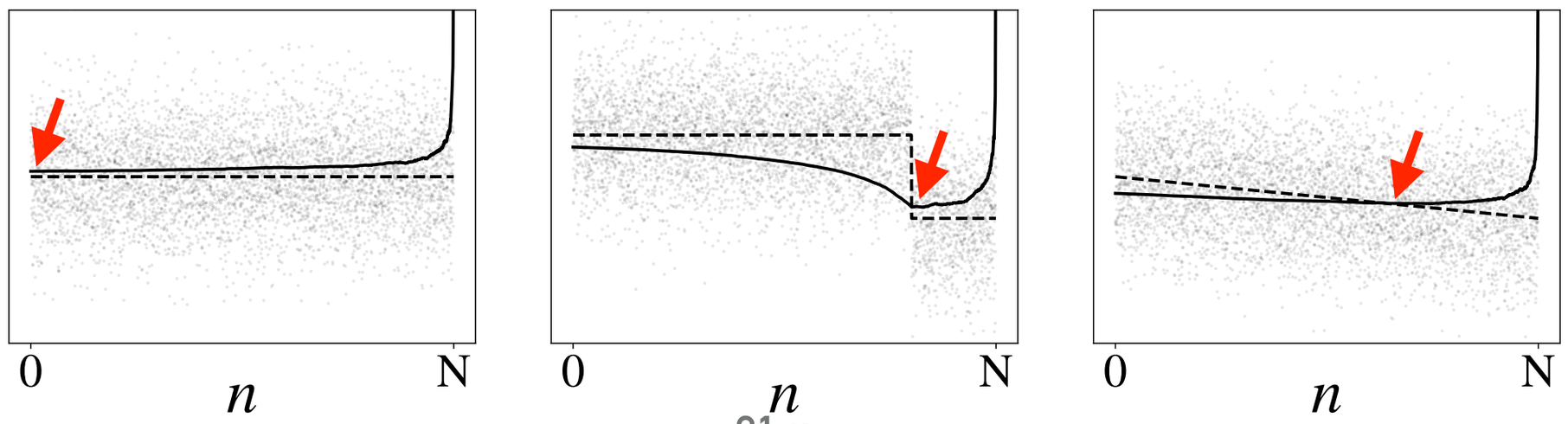
ROTTING ADAPTIVE WINDOW UCB

Algorithm 5 RAW-UCB

Input: K, σ, α

1: **for** $t \leftarrow K+1, K+2, \ldots$ **do**
2: $\quad \delta_t \leftarrow \frac{1}{t^\alpha}$
3: $\quad$ SELECT : $\arg\max_{i \in K} \min_{h \leq N_{i,t}} \left(\widehat{\mu}_i^h(N_{i,t}) + c(h, \delta_t) \right)$
4: **end for**

- Rewards are decreasing: $\widehat{\mu}_i^h\left(N_{i,t}\right) + c(h, \delta_t)$ is a UCB for the future value
- RAW-UCB selects the minimum (tightest) one (on h) as index of the arm



Upper Bounds

Worst-case upper bound

$$\mathbb{E}\left[R_T\left(\pi_{\mathrm{R}}\right)\right] \leq C\sigma\sqrt{KT\log(T)} + KL$$

Comparison w/ wSWA

$$\mathbb{E}\left[R_T\left(\pi_{\mathrm{wSWA}}\right)\right] = \tilde{O}\left(L^{1/3}\sigma^{2/3}K^{1/3}T^{2/3}\right)$$

Problem-dependent upper bound

$$\mathbb{E}\left[R_T\left(\pi_{\mathrm{R}}\right)\right] \leq \sum_{i\in\mathcal{K}} O\left(\frac{\log(T)}{\Delta_{i,h^+_{i,T}-1}}\right)$$

Comparison w/ wSWA

Pure worst-case strategy

$\Delta_{i,h}$ Difference between the average of the h first overpulls of arm i and the worst reward pulled by the optimal policy

$h^+_{i,T}$ High-probability upper bound on the number of overpulls for RAW-UCB

Upper Bounds

Worst-case upper bound

$$\mathbb{E}\left[R_T\left(\pi_{\mathrm{R}}\right)\right] \leq C\sigma\sqrt{KT\log(T)} + KL$$

Comparison w/ wSWA

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Problem-dependent upper bound

$$\mathbb{E}\left[R_T\left(\pi_{\mathrm{R}}\right)\right] \leq \sum_{i\in\mathcal{K}} O\left(\frac{\log(T)}{\Delta_{i,h^+_{i,T}-1}}\right)$$

Comparison w/ wSWA

Pure worst-case strategy

$\Delta_{i,h} = \Delta_i$ on a stationary bandits problem

$\Delta_{i,h^+_{i,T}-1}$ is a problem-dependent quantity

Proof Sketch

1. Decompose the regret

$$R_T(\pi) = \sum_{i \in \mathrm{UP}} \sum_{s=N^{\pi}_{i,T}+1}^{N^{\star}_{i,T}} \mu_i(s) - \sum_{i \in \mathrm{OP}} \sum_{s=N^{\star}_{i,T}+1}^{N^{\pi}_{i,T}} \mu_i(s)$$

$$\leq \sum_{i \in \mathrm{OP}} \sum_{s=N^{\star}_{i,T}+1}^{N^{\pi}_{i,T}} \left(\mu_T^+ - \mu_i(s)\right)$$

μ_T^+ : largest underpulled values

$\sim$ "number of mistakes" $\times$ "average gap"

2. Use the adaptive window mechanism to get

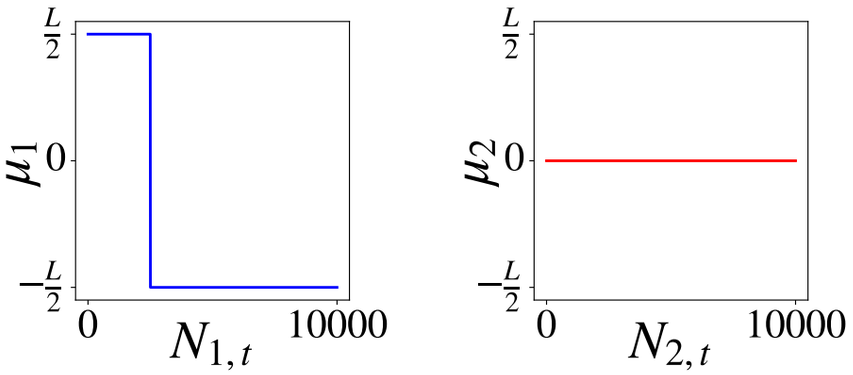
$$\text{"number of mistakes"} \sim \frac{\log T}{\text{"average gap"}^2}$$

3. Work on the gap to be problem-dependent

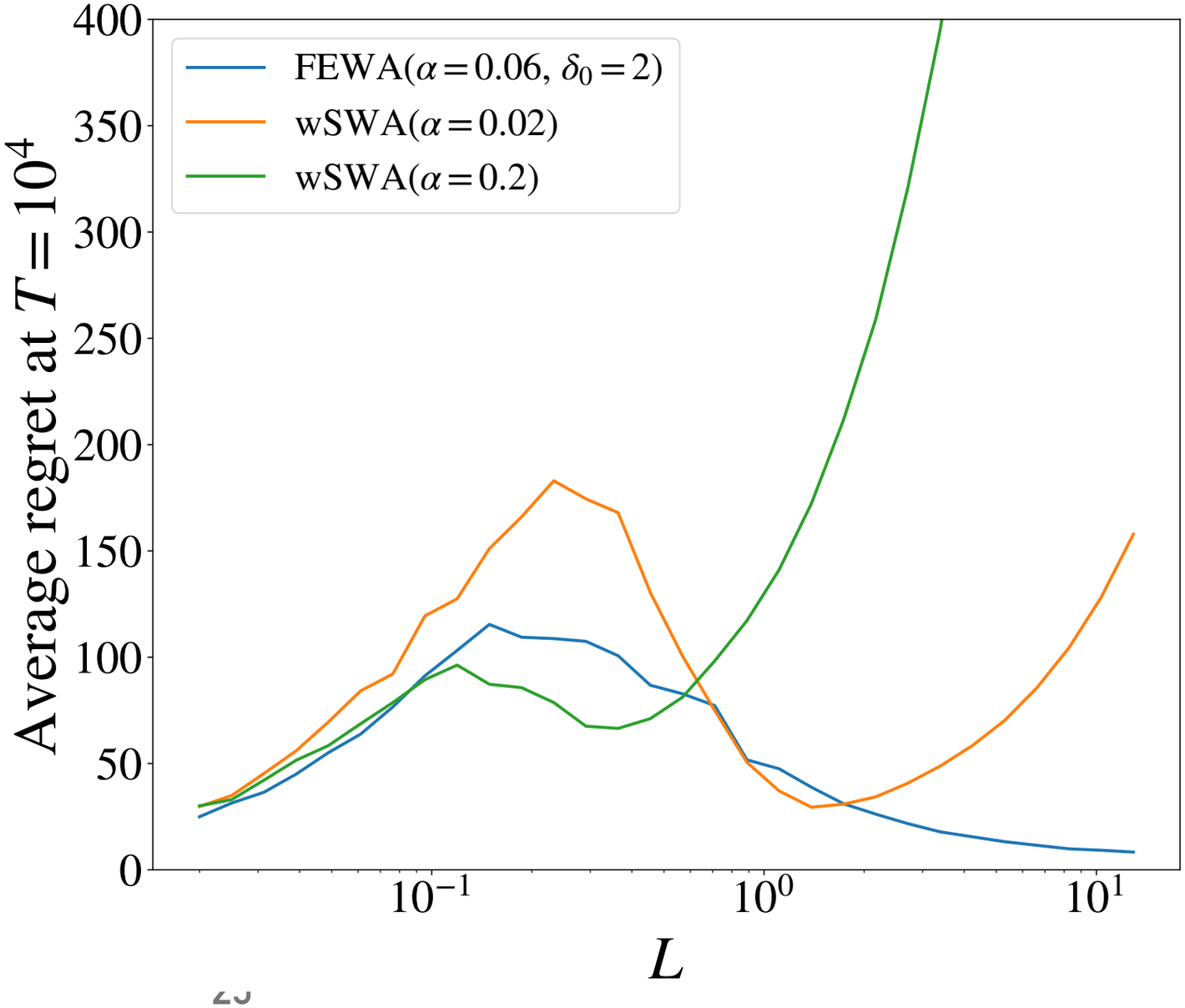
"number of mistakes" $\rightarrow$ "maximal number of mistakes with high probability"

$$\mu_T^+ \rightarrow \min_i \min_{s \leq N^{\star}_{i,T}} \mu_i(s)$$

Experimental Results



$$\sigma = 1 \; ; \; L$$



RESTED CONTRIBUTIONS

Algorithm	Sliding Window Average (Levine et al., 2017)	FEWA & RAW-UCB
Average(s)	Fixed single window	Multiple windows
Problem Independent Bound	$\tilde{\mathcal{O}}\left(L^{1/3}\sigma^{2/3}K^{1/3}T^{2/3}\right)$	$\tilde{\mathcal{O}}\left(\sigma\sqrt{KT}\right)+\mathcal{O}\left(KL\right)$
Problem-Dependent Bound	✗	$\mathcal{O}\left(\sum_{i\in\mathcal{K}}\frac{\log T}{\Delta_i^h}\right)$
Knowledge	T L σ	σ
Complexity per round	$\mathcal{O}\left(K^{1/3}T^{2/3}\right)$	$\mathcal{O}\left(T\right)$ efficient algo $\mathcal{O}\left(K\log T\right)$

OUTLINE

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3. Restless Rotting Bandits

Non-Stationary Restless Bandits

- $\{\mu_i\}$ are functions of round t - restless
- Minimize cumulative regret w.r.t. the optimal strategy: $\sum_{t \le T} \mu_{i_t^\star}(t) - \mu_{i_t}(t)$
- Unlearnable if μ_i can change significantly at every round.
- Common settings
  - μ_i is piece-wise stationary with Υ_T pieces *(Garivier & Moulines, 2011)*
  - μ_i has a permitted amount of change V_T *(Besbes et al., 2014)*

$$\sum_{t \le T} \max_{i \in \mathcal{K}} |\mu_i(t) + 1 - \mu_i(t)| \le V_T$$

Piece-wise stationary rate:

$$\Theta\left(\sqrt{K \Upsilon_T T}\right)$$

(Garivier & Moulines, 2011)

Variational budget rate:

$$\Theta\left(K^{1/3} V_T^{1/3} T^{2/3}\right)$$

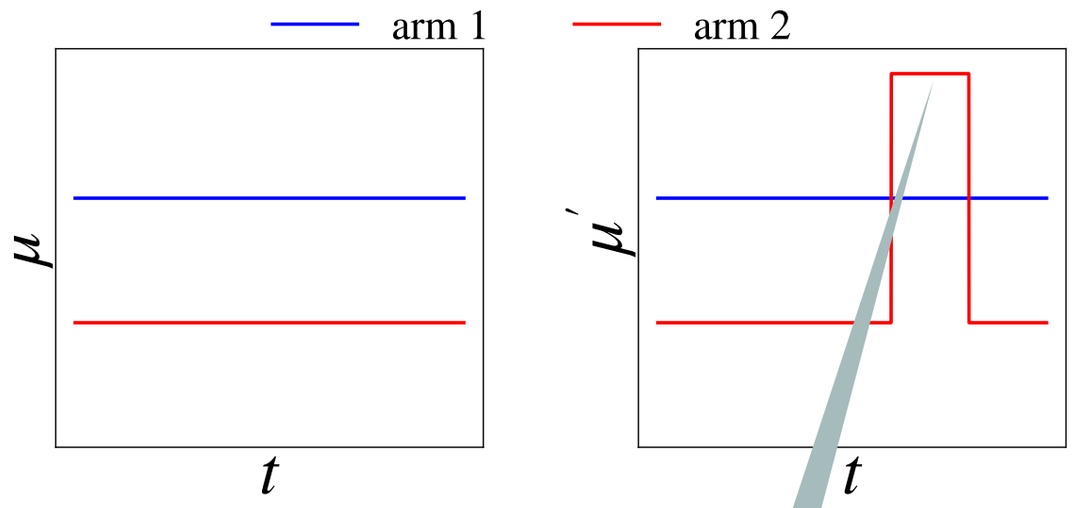
(Besbes et al. , 2014)

Problem Dependent Guarantees

EXP3.S (Auer et al. 2002b), an adversarial algorithm, matches the two minimax rates.
Can we get a problem-dependent bound?

Theorem (Garivier et al. 2011): **Let π a policy suffering $R_T(\mu)$ on a 2-arms stationary bandits problem μ. Then, for *T* large enough, there exists a piece-wise stationary problem μ' such that π suffers:**

$$R_T(\mu') \geq \frac{T}{22 R_T(\mu)}$$



Quick ***increase***

$$\tau = \frac{T}{R_T(\mu)/\Delta}$$

Corollary : No !

$\mathcal{O}\left(\sqrt{T}\right)$ **anywhere ⇨** $\mathcal{O}\left(\sqrt{T}\right)$ **everywhere**

RESTLESS ROTTING

Do we need to spend $\mathcal{O}\left(\sqrt{T}\right)$ exploration budget when rewards *decay* ?

RAW-UCB on Rotting Restless Bandits

RAW-UCB without knowing T, V_T nor Υ_T

Rotting Variational budget

$$\mathbb{E}\left[R_T\left(\pi_{\mathrm{R}}\right)\right] \leq \tilde{\mathcal{O}}\left(\sigma^{2/3} K^{1/3} V_T^{1/3} T^{2/3}\right)$$

minimax rate

Rotting Piecewise stationary

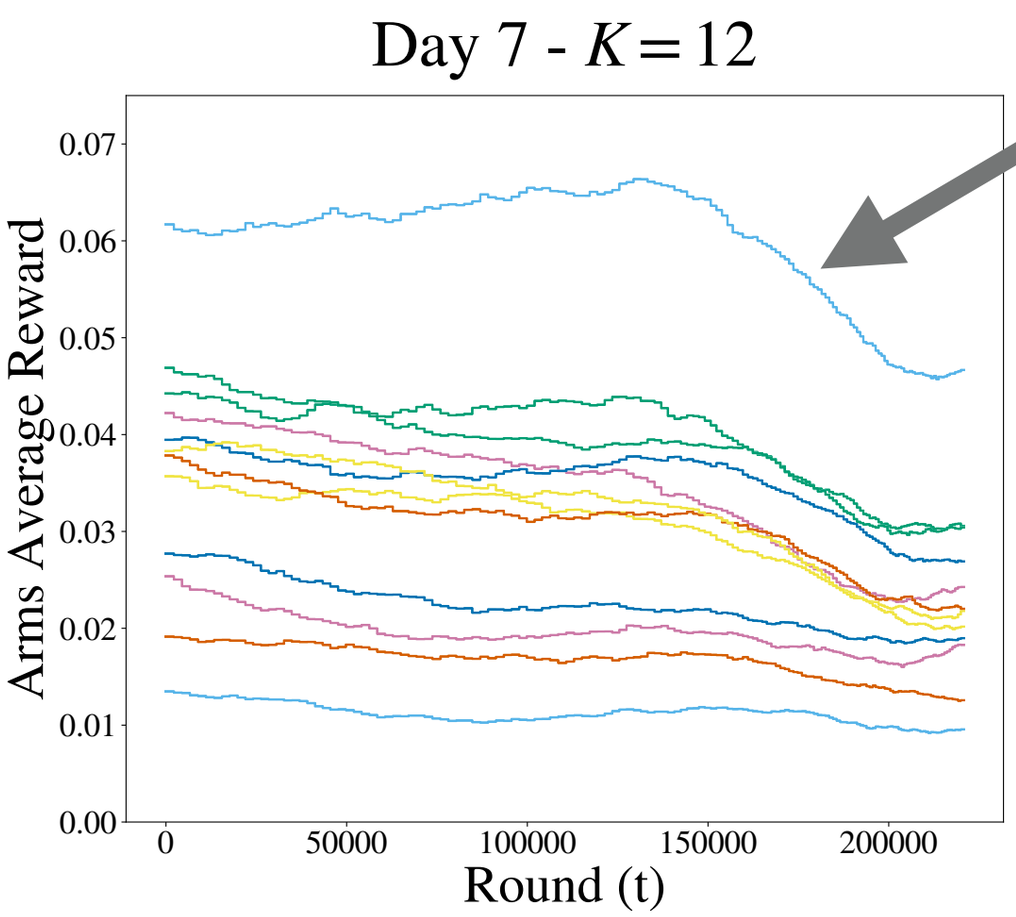
$$\mathbb{E}\left[R_T\left(\pi_{\mathrm{R}}\right)\right] \leq \tilde{\mathcal{O}}\left(\sigma\sqrt{K\Upsilon_T T}\right)$$

minimax rate

$$\mathbb{E}\left[R_T\left(\pi_{\mathrm{R}}\right)\right] \leq \sum_{i \in \mathcal{K}} \sum_{k=1}^{\Upsilon_T} \frac{32\sigma^2 \log T}{\Delta_{i,k}} + \mathcal{O}\left(\sqrt{\log T}\right)$$

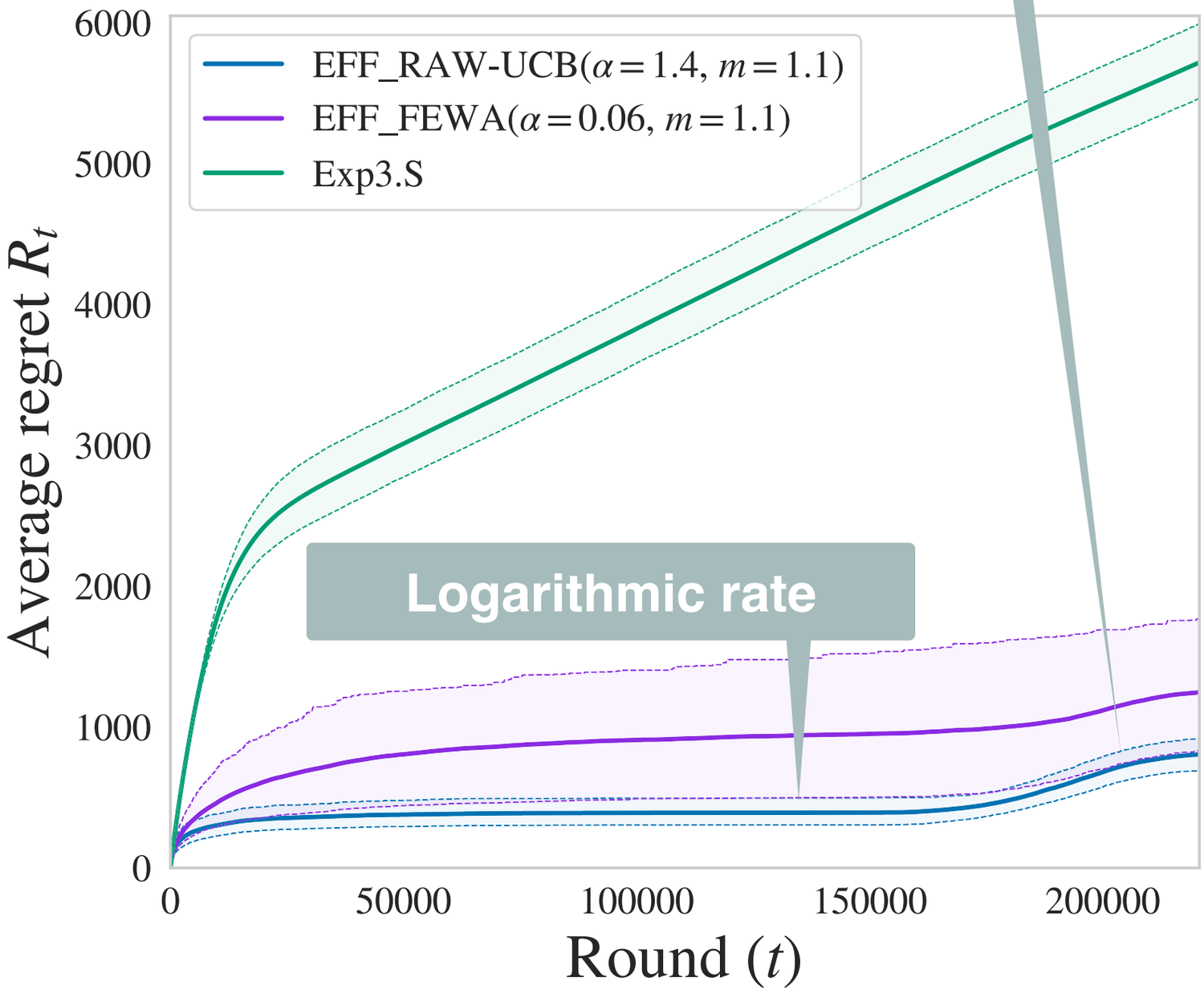
problem-dependent bound!

YAHOO! BENCHMARK



Decaying trend in the probability of click on an article from noon to 6am.

Significant decay : exploration



Restless Contributions

1 Rotting property makes restless bandits easier

✅ $\mathcal{O}\left(\log T\right)$ problem-dependent bound

2 UCB-like index is sufficient for rotting bandits

✅ No random exploration, no passive forgetting, no change-detection routine

✅ Near-optimal gap-dependent and minimax bounds

✅ Agnostic to Υ_T, V_T, T

3 RAW-UCB solves rested OR restless rotting bandits

✅ with the same tuning

❌ rested AND restless rotting bandits are **incompatible**

CONTRIBUTIONS TO EDUCATIONAL SYSTEMS

1. What is the objective ?	Target the most difficult topic	
2. What is the « best » action ?	✅ Optimal policy selects different actions when the student progresses	
3. How does action impact observation?	✅ Model the learning of the student	
4. Can we learn something relevant with ~100 samples?	✅ Not harder than stationary bandits	

CONTRIBUTIONS TO EDUCATIONAL SYSTEMS

1. What is the objective ?	Target the most difficult topic	❌ Target the easiest topic until it is mastered
2. What is the « best » action ?	✅ Optimal policy selects different actions when the student progresses	
3. How does action impact observation?	✅ Model the learning of the student	
4. Can we learn something relevant with ~100 samples?	✅ Not harder than stationary bandits	❌ RAW-UCB = Round Robin

THANK YOU !

Rotting Bandits are not Harder than Stochastic ones,
Julien Seznec, Andrea Locatelli, Alexandra Carpentier, Alessandro Lazaric, Michal Valko,
Artificial Intelligence and Statistics (2019), (with oral presentation)

A Single Algorithm for both Rested and Restless Rotting Bandits,
Julien Seznec, Pierre Ménard, Alessandro Lazaric, Michal Valko,
Artificial Intelligence and Statistics (2020)