Assignment 01 - Guidance

Spacecraft Guidance and Navigation

AY 2022-2023

POLITECNICO MILANO 1863

Exam

Assign. 1

Assign. 2

Oral on assign. 1

Oral on assign. 2

Oral on course

geometric mean

arithmetic mean

Final mark

Exam

JUST JOKING!

Exam

Assign. 1

Assign. 2

Oral on course

arithmetic mean

optional

Proposed final mark

- All of you will be asked to come in person during the official exam sessions to check our evaluation and discuss any doubts you/we might have
- In case you are not satisfied with the proposed final mark, you can take an optional oral exam (which will take place during the official exam sessions)

Exam

- Remarks:
  - For those who will not submit the report by the deadlines, you'll be asked to do it before the official exam sessions
  - Once you receive the evaluation of the assigmments, please wait for the in-person meeting during the exam sessions to discuss about the evaluation

Laboratory sessions - Assignments

Goals ⇒

- Contribute to the final grade
- Relief the workload of the project by spreading it
- Assets for MSc thesis: Matlab, SPICE, and Latex

Assignment evaluation

Weights	Fail	Poor	Good	Excellent
Report	• Major mismatches w.r.t. the assignment • Report awfully written	• Minor mismatches w.r.t. the assignment • Figures and tables not clear • English is poor	• Answers lengthy, but correct • Figures and tables clear • Good English	• Answers concise and clear • Figures and table clear and meaningful • Good English
Code	• Code does not run • Major algorithmic errors • Code not complete	• Minor algorithmic errors • Code not documented • Code takes unnecessary long to run	• Code runs smoothly • Code is fairly documented • Computational efficiency improvable	• Code runs smoothly • Code is well documented • Care is taken to account computational efficiency

Laboratory sessions

- Laboratory sessions **will not be recorded**
  - we are here to give you answers while you are working

- When writing in the forum:
  - Reply to the proper thread
  - Check if your question was already posted
  - Be patient (do not re-ask a question that still has to receive an answer)
  - Do **not** attach/send code asking for debugging

Timetable		Room
18 Oct	16.15-18.15	BL.27.12 (+ Morselli virtual room)
19 Oct	14.15-16.15	BL.28.22 (+ Morselli virtual room)
21 Oct	09.15-12.15	BL.28.21 (+ Morselli virtual room)
25 Oct	16.15-18.15	BL.27.12 (+ Morselli virtual room)
26 Oct	14.15-16.15	BL.28.22 (+ Morselli virtual room)
28 Oct	14.15-16.15	BL.28.21 (Lecture)
02 Nov	16.15-18.15	BL28.22 (Lecture)
08 Nov	**23.30 CET**	**Assignment 1 deadline**

WeBeep forum

- Organization
    - 1 sub-thread per problem point
    - 1 thread for generic questions (integrators and solvers)

Delivery of assignments

Assigment will be delivered through **WeBeep**:

DEADLINE ⇨ Nov 08, 2022, 23:30:00 CET

1) Click on the link to **load Assignment 1** in your Overleaf

   https://bit.ly/SGN_22_Assignment1

2) **Fill the report** and be sure it is compiled properly
3) **Download the PDF** and merge it in a **zipped file** with MATLAB code. Rename it `lastname123456_Assign1.zip`
4) **Submit the compressed file** by uploading it on Webeep
   - **Max Size**: 10 Mb

Assignment 01 - Topics

3 Exercises

- Periodic Orbits
- Shooting Methods
- Continuous guidance

Suggested MATLAB Version: R2021b (or newer)

Report

1 Impulsive guidance

Exercise 1

Let $\mathbf{x}(t) = \varphi(\mathbf{x}_0, t_0; t)$ be the flow of the geocentric two-body model. 1) Using one of Matlab's built-in integrators, implement and validate* a propagator that returns $\mathbf{x}(t)$ for given $\mathbf{x}_0$, t_0, t, and μ. 2) Given the pairs $\{\mathbf{r}_1, \mathbf{r}_2\}$ and $\{t_1, t_2\}$, develop a solver that finds $\mathbf{v}_1$ such that $\mathbf{r}(t_2) = \mathbf{r}_2$, where $(\mathbf{r}(t), \mathbf{v}(t))^\top = \varphi((\mathbf{r}_1, \mathbf{v}_1)^\top, t_1; t_2)$ (Lambert's problem). To compute the derivatives of the shooting function, use either a) finite differences or b) the state transition matrix $\Phi = \mathrm{d}\varphi/\mathrm{d}\mathbf{x}_0$. Validate the algorithms against the classic Lambert solver. 3) Using the propagator of point 1) in the heliocentric case, and reading the motion of the Earth and Mars from SPICE, solve the shooting problem

$$\min_{\mathbf{x}_1, t_1, t_2} \Delta v \quad \text{s.t.} \quad \begin{cases} \mathbf{r}_1 = \mathbf{r}_E(t_1) \\ \mathbf{r}(t_2) = \mathbf{r}_M(t_2) \\ t_1^L \leq t_1 \leq t_1^U \\ t_2^L \leq t_2 \leq t_2^U \\ t_2 \geq t_1 \end{cases} \tag{1}$$

where $\Delta v = \Delta v_1 + \Delta v_2$, $\boldsymbol{\Delta}\mathbf{v}_1 = \mathbf{v}_1 - \mathbf{v}_E(t_1)$, $\boldsymbol{\Delta}\mathbf{v}_2 = \mathbf{v}(t_2) - \mathbf{v}_M(t_2)$. $\mathbf{x}_1 = (\mathbf{r}_1, \mathbf{v}_1)^\top$, and $(\mathbf{r}(t), \mathbf{v}(t))^\top = \varphi(\mathbf{x}_1, t_1; t_2)$. Define lower and upper bounds, and make sure to solve the problem stated in Eq. (1) for different initial guesses.

Write your answer here

- Develop the exercises in one Matlab script; name the file `lastname123456_Assign1.m`
- Organize the script in sections, one for each exercise; use local functions if needed.
- Download the PDF from the Main menu.
- Create a single .zip file containing both the report in PDF and the MATLAB file. The name shall be `lastname123456_Assign1.zip`.
- Red text indicates where answers are needed; be sure there is no red stuff in your report.
- In your answers, be concise: to the point.
- **Deadline for the submission: Nov 11 2021, 23:30.**
- **Load the compressed file to the Assignments folder on Webeep.**

Exercise

Answer here

NB: The code is not your report!

Answer the question **in the report** and add any plot you think is relevant for your answers there.

Any description missing in the report but present in the code **will not** contribute to the evaluation!

Script

LastName123456_Assign1.m

```matlab
% Modeling and Simulation of Aerospace Systems (2020/2021)
% Assignment # 1
% Author: XXX YYY

%% Ex 1
clearvars; close all; clc;

% Develop the exercise here

%% Ex 2
clearvars; close all; clc;

% Develop the exercise here

%% Ex N
clearvars; close all; clc;

% Develop the exercise here

%% Functions
% Define your functions at the end of the script

function y = MyFunction(x)

% Content...
y = 2*x;

end
```

Name of
the script
(one per exercise)

One section
per exercise point

Functions at the
end of the script

Exercise 1 – Periodic Orbits

Exercise 1

Consider the 3D Sun–Earth Circular Restricted Three-Body Problem with $\mu = 3.0359 \times 10^{-6}$.

1) Find the x-coordinate of the Lagrange point L_2 in the rotating, adimensional reference frame with at least 10-digit accuracy.

Solutions to the 3D CRTBP satisfy the symmetry

$$\mathcal{S} : (x, y, z, \dot{x}, \dot{y}, \dot{z}, t) \rightarrow (x, -y, z, -\dot{x}, \dot{y}, -\dot{z}, -t).$$

Thus, a trajectory that crosses perpendicularly the $y = 0$ plane twice is a periodic orbit.

2) Given the initial guess $\mathbf{x}_0 = (x_0, y_0, z_0, v_{x0}, v_{y0}, v_{z0})$, with

$$\begin{aligned} x_0 &= 1.008296144180133 \\ y_0 &= 0 \\ z_0 &= 0.001214294450297 \\ v_{x0} &= 0 \\ v_{y0} &= 0.010020975499502 \\ v_{z0} &= 0 \end{aligned}$$

Find the periodic halo orbit that passes through z_0; that is, develop the theoretical framework and implement a differential correction scheme that uses the STM either approximated through finite differences or achieved by integrating the variational equation.

The periodic orbits in the CRTBP exist in families. These can be computed by continuing the orbits along one coordinate, e.g., z_0. This is an iterative process in which one component of the state is varied, while the other components are taken from the solution of the previous iteration.

3) By gradually increasing z_0 and using numerical continuation, compute the families of halo orbits until $z_0 = 0.0046$.

8 points

Exercise 2

Consider the two-impulse transfer problem stated in Section 3.1 (Topputo, 2013)*.

1) Using the procedure in Section 3.2, produce a first guess solution using $\alpha = 1.5\pi$, $\beta = 1.41$, $\delta = 7$, and $t_i = 0$. Plot the solution in both the rotating frame and Earth-centered inertial frame (see Appendix 1 in (Topputo, 2013)).

2) Considering the first guess in 1) and using $\{\mathbf{x}_i, t_i, t_f\}$ as variables, solve the problem in Section 3.1 with simple shooting in the following cases

   a) without providing any derivative to the solver, and

   b) by providing the derivatives and by estimating the state transition matrix with variational equations.

3) Considering the first guess solution in 1) and the procedure in Section 3.3, solve the problem with multiple shooting taking $N = 4$ and using the variational equation to compute the Jacobian of the nonlinear equality constraints.

11 points

Exercise 3 – Continuous guidance

Exercise 3

A low-thrust option is being considered for an Earth-Mars transfer[†]. Provide a *time-optimal* solution under the following assumptions: the spacecraft moves in the heliocentric two-body problem, Mars instantaneous acceleration is determined only by the Sun's gravitational attraction, the departure date is fixed, and the spacecraft initial and final states are coincident with those of the Earth and Mars, respectively.

1) Write down the spacecraft equations of motion, the costate dynamics, and the zero-finding problem for the unknowns $\{\boldsymbol{\lambda}_0, t_f\}$ with the appropriate transversality condition.

2) Solve the problem considering the following data:

   – Launch date: `2022-08-03-12:45:20.000 UTC`
   – Spacecraft mass: $m_0 = 1500$ kg
   – Electric properties: $T_{\max} = 150$ mN, $I_{sp} = 3000$ s
   – Number of thrusters: 4

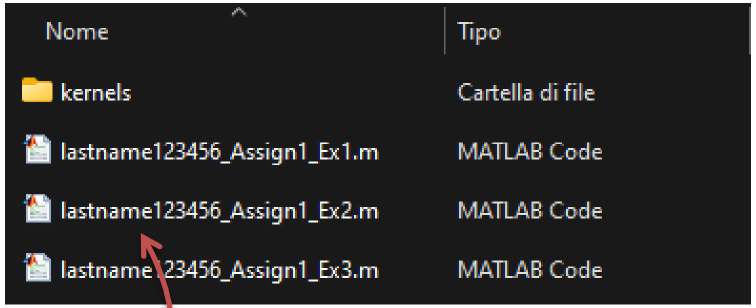
   Report the obtained solution in terms of $\{\boldsymbol{\lambda}_0, t_f\}$ and the error with respect to the target. Validate your results exploiting the properties of time-optimal solutions.

3) Solve the problem for a degraded configuration with only 3 thrusters available, assuming that the failure occurs immediately after launch. Plot the thrust angles and compare them to the nominal case in 2).

11 points

Supporting material

- Available on **WeBeep**, inside the folder Laboratories
- Compressed folder **Assignment01.zip** contains:
    - Subfolder `kernels` with all required SPICE kernels



lastname123456_Assign1_Ex1.m

lastname123456_Assign1_Ex2.m

lastname123456_Assign1_Ex3.m

How to prepare your script

- Unzip the folder and work on your Matlab scripts inside it
- Remember to **load the meta-kernel** (if you created it)
    - Suggestion: use relative paths
- **Obs:** no need to upload the supporting material when preparing your delivery on WeBeep

General hints

Report content

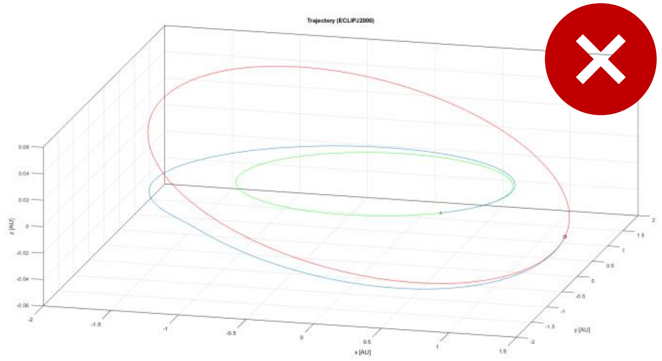
- **State vectors** must be provided with corresponding **reference frame and origin**
- Always put the (correct!) units
- Do not mix position and velocities when reporting errors

r_x [km]	r_y [km]	r_z [km]
1234.567	123.456	234.567

Satellite position (@Sun ECLIPJ2000)

Plots

- Make sure that axis labels and titles are clearly readable with page zoom at 100%
- Remember to put the (correct!) units on each axis
- When plotting trajectories specify the **reference frame and origin** in the plot title or caption



General hints

LaTeX

- Do not put multiplication symbols, especially as asterisks
- Clearly distinguish between vectors and scalar: write vector using underline, arrow or bold
    - Latin alphabet: `\mathbf{x}` or `\mathbf{r}`
    - Greek symbols : `\boldsymbol{\lambda}`
    - **Obs:** No need to use norm with this notation $\|\boldsymbol{\lambda}_x\| \leftrightarrow \lambda_x$
- Write scalar product using `\cdot`
- Prefer the use of `\dfrac{}{}`over`\frac{}{}` when writing equations
- When inserting text in an equation remember to use`\mathrm{}`

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