Hypothesis Classes

- Linearly parameterized
  - Features $\phi_k : \mathcal{S} \times \mathcal{A} \mapsto \Re \qquad k = 1, \ldots, K$
  - Function class

$$Q_t^* \in \mathcal{Q}_t = \left\{ \sum_{k=1}^{K} \theta_k \phi_k : \theta \in \Re^K \right\}$$

  - Number of parameters = KH

- Nonlinearly parameterized
  - Generalized linear: e.g., logistic function / sigmoid
  - Neural networks … “deep learning”

- Let’s focus on linear

$$\tilde{Q}^\theta(s, a) = \sum_{k=1}^{K} \theta_k \phi_k(s, a)$$

Least-Squares Value Iteration

- Data available after episode L

$$\left\{ (s_t^\ell, a_t^\ell, r_t^\ell, s_{t+1}^\ell) : \begin{array}{l} t = 0, \ldots, H-1 \\ \ell = 1, \ldots, L \end{array} \right\}$$

- Estimate coefficients

$$\hat{\theta}_0 \leftarrow \hat{\theta}_1 \leftarrow \cdots \leftarrow \hat{\theta}_{H-2} \leftarrow \hat{\theta}_{H-1}$$

$$\min_{\hat{\theta}_t} \sum_{\ell=1}^{L} \left(\tilde{Q}^{\hat{\theta}_t}(s_t^\ell, a_t^\ell) - y_t^\ell \right)^2 + \lambda \|\hat{\theta}_t\|_2^2$$

$$y_{H-1}^\ell = r_{H-1}^\ell \qquad\qquad y_t^\ell = r_t^\ell + \max_{a \in \mathcal{A}} \tilde{Q}^{\hat{\theta}_{t+1}}(s_{t+1}^\ell, a)$$

- Linear algebra version

$$\hat{\theta}_t = (A^\top A + \lambda I)^{-1} A^\top b$$

$$A = \begin{bmatrix} | & & | \\ \phi_1 & \cdots & \phi_K \\ | & & | \end{bmatrix} \qquad\qquad b = \begin{bmatrix} y_t^1 \\ \vdots \\ y_t^L \end{bmatrix}$$

LSVI with Exploration

- LSVI computes a point estimate based on data from first L episodes

- Point estimate leads to a policy

$$a_t^{L+1} \in \arg\max_{a \in \mathcal{A}} \tilde{Q}^{\hat{\theta}_t}(s_t^{L+1}, a)$$

- Need for exploration

- Common approach: Boltzmann exploration

$$a_t^{L+1} \sim \frac{\exp\left(\beta \tilde{Q}^{\hat{\theta}_t}(s_t^{L+1}, \cdot)\right)}{\sum_{a \in \mathcal{A}} \exp\left(\beta \tilde{Q}^{\hat{\theta}_t}(s_t^{L+1}, a)\right)}$$

  - Can require time/data exponential in H
  - Need to plan to learn

UC-LSVI

- Confidence set propagation

$$\mathcal{Q}_0^L \leftarrow \mathcal{Q}_1^L \leftarrow \cdots \leftarrow \mathcal{Q}_{H-2}^L \leftarrow \mathcal{Q}_{H-1}^L$$

$$Q_t^L = \left\{ Q^\theta : (\theta - \hat{\theta}_t)^\top \hat{\Sigma}_t^{-1} (\theta - \hat{\theta}_t) \leq \gamma \right\}$$

- Fit to data samples

$$y_t^\ell = r_t^\ell + \max_{\tilde{Q}^\theta \in \mathcal{Q}_{t+1}^L} \max_{a \in \mathcal{A}} \tilde{Q}^\theta (s_{t+1}^\ell, a)$$

- Bayesian interpretation of regression
  - Prior distribution $\theta_t \sim N(0, \lambda I)$
  - Pretend that data samples are Gaussian

$$y_t^\ell = Q_t^*(s_t^\ell, a_t^\ell) + w_t^\ell \qquad w_t^\ell \sim N(0, \sigma^2)$$

  - Mean of posterior = least-squares solution

$$\hat{\theta}_t = (A^\top A + \lambda I)^{-1} A^\top b$$

  - Covariance of posterior

$$\hat{\Sigma}_t = \left(\frac{1}{\sigma^2} A^\top A + \lambda I \right)^{-1}$$