Master MVA
Remote sensing

ENS Paris Saclay

Pointing error correction

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2 Satellite attitude error effects on stereo images

What is pointing error ? Sensiblity of roll, pitch and yaw. Scheme (roll and pitch depend of the height of the satellite whereas yaw only the angle)

Fig. 46

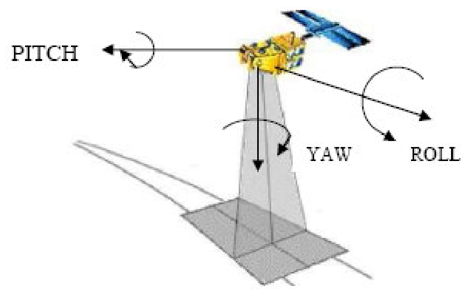

Figure 1: Roll, pitch and yaw angles.

3 Correction of Relative Pointing Error

[?]

A simple way to correct the relative pointing error is to transform one of the two images, in such a way that the corresponding points fall on the respective epipolar curves: given two images u, v and a set of correspondences $(\mathbf{x}_i, \mathbf{x}'_i)_{i=1\dots N}$, we search for a transformation f such that, for all i, the transformed point $f(\mathbf{x}'_i)$ lies on the epipolar curve $\mathrm{epi}_{uv}^{\mathbf{x}_i}(R)$. The desired transformation f^* minimises the relative pointing error:

$$f^* = \underset{f}{\mathrm{argmin}} \, \frac{1}{N} \sum_{i=1}^{N} d(f(\mathbf{x}'_i), \mathrm{epi}_{uv}^{\mathbf{x}_i}(R)) \tag{1}$$

3.0.).1 Affine fundamental matrix approximation

From [?], we know that, for each correspondence i, the epipolar curve $\mathrm{epi}_{uv}^{\mathbf{x}_i}(R)$ is approximated up to 0.05 pixels by the straight line $F\mathbf{x}_i$, where F is the affine fundamental matrix between the two views for the considered tile. Since the fundamental matrix, due to the approximation, is *affine*, all the lines $(F\mathbf{x}_i)_{i=1\dots N}$ are parallel.

3.1 Roll and Pitch Angles

Because of sensitivity issues, we first can take only roll and pitch error into account. Therefore, according to section 2, we search for a transformation f such that

$$f(\mathbf{x}) = T\mathbf{x}$$

where T is a translation:

$$T = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

We can write the transformation f, for $\mathbf{x} = (x\ y\ 1)^T$, as:

$$f(\mathbf{x}) = T\mathbf{x} = \begin{pmatrix} x + t_1 \\ y + t_2 \\ 1 \end{pmatrix}$$

Without any additional restriction, we may assume that these lines are horizontal (otherwise just do a change of coordinates). This change of coordinates is called *rectification*. We find

After rectification, for each point i, the horizontal line $F\mathbf{x}_i$ can be written as

$$F\mathbf{x}_i = [0\ 1\ c_i]$$

With these notations, for each point correspondence $(\mathbf{x}_i, \mathbf{x}'_i)$, the pointing error e is:

$$e(\mathbf{x}_i, \mathbf{x}'_i) = d(\mathbf{x}'_i, epi_{uv}^{\mathbf{x}_i}(R)) = d(\mathbf{x}'_i, F\mathbf{x}_i) = |y'_i + c_i|$$

Here the error e is invariant to any horizontal translation, thus the search for a translation minimizing the relative pointing error of formula (1) can be restricted to vertical translations only. With a vertical translation of parameter t, the global pointing error becomes

$$E = \frac{1}{N}\sum_{i=1}^{N} d(T\mathbf{x}'_i, F\mathbf{x}_i) = \frac{1}{N}\sum_{i=1}^{N} |y'_i + t + c_i|$$

The translation that minimizes this sum is given by the geometric median (Weiszfeld, 1937) of the vector $(-y'_i - c_i)_{i=1\ldots N}$. The pointing error can thus be minimized by applying a translation to one of the images. Note that the median is robust against outliers, thus this correction procedure works well even in the presence of false matches.

3.2 Roll, Pitch and Yaw Angles

If we assume that the scene is located at infinity with respect to the satellite, an error in the sensor attitude measurement can be modeled in image space as a translation composed with a rotation. Therefore we have

$$f(\mathbf{x}) = TR\mathbf{x}$$

where R is a rotation and T a translation:

$$R = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix},\ T = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

3.2.a) Approximation of the Rotation

To correct the pointing error in the rectified setting we find R^* and T^* minimizing the pointing error defined as follow :

$$(R^*, T^*) = \underset{R,T}{\operatorname{argmin}} \sum_{i=1}^{N} d(TR\boldsymbol{x}'_{\boldsymbol{i}}, F\boldsymbol{x}_i)^2 \tag{2}$$

As in [**?**]

After rectification, the horizontal line $F\boldsymbol{x}_i$ can be written, in homogeneous coordinates, as

$$F\boldsymbol{x}_i = \begin{bmatrix} 0 & 1 & y_i \end{bmatrix}$$

With these notations, if the model now includes a rotation R, for each point correspondence $(\boldsymbol{x}_i, \boldsymbol{x}'_i)$, we have

$$e(\mathbf{x}_i, \mathbf{x}'_i) = d(TR\boldsymbol{x}'_i, \mathrm{epi}_{uv}^{\boldsymbol{x}_i}(R)) = d(TR\boldsymbol{x}'_i, F\boldsymbol{x}_i) = |x'_i \sin\theta + y'_i \cos\theta + t - y_i|$$

where θ is the angle of the rotation.

We can consider that θ is small enough compare to 2π. In fact on satellites such as Sentinel or Pleiades (the precision of the yaw angle is around 50 μrad. Even if Planet's sensor would be 100 less precise, we will still have an yaw angle less than 1 degree. With this approximation, we can write:

$$e(\mathbf{x}_i, \mathbf{x}'_i) = |x'_i\theta + y'_i + t - y_i|$$

Correcting the global pointing error is to minimize:

$$(\theta^*, t^*) = \underset{\theta,t}{\operatorname{argmin}} \sum_{i=1}^{N} d(TR\boldsymbol{x'_i}, F\boldsymbol{x}_i)^2 = \sum_{i=1}^{N} (x'_i\theta + y'_i + t - y_i)^2$$

We can write this optimization problem as :

$$X^* = \underset{X}{\operatorname{argmin}} \|AX + b\|^2$$

where

$$A = \begin{pmatrix} x'_1 & 1 \\ x'_2 & 1 \\ \vdots & \vdots \\ x'_n & 1 \end{pmatrix} \quad X = \begin{pmatrix} \theta \\ t \end{pmatrix} \quad b = \begin{pmatrix} y'_1 - y_1 \\ y'_2 - y_2 \\ \vdots \\ y'_1 - y_1 \end{pmatrix}$$

The solution of this optimization problem is the solution of the normal equation

$$A^T AX = -A^T b$$

Here,

$$A^T A = \begin{pmatrix} \sum\limits_{i=1}^{N} x_i'^2 & \sum\limits_{i=1}^{N} x'_i \\ \sum\limits_{i=1}^{N} x'_i & N \end{pmatrix}$$

So $A^T A$ is inversible, and the optimal correction in the rectified images is:

$$\begin{pmatrix} \theta^* \\ t^* \end{pmatrix} = X^* = -(A^T A)^{-1} A^T b$$

3.2.b) Solving Numerically an Optimization Problem

To correct the pointing error in the rectified setting we find R^* and T^* minimizing the pointing error defined as follow :

$$(R^*, T^*) = \underset{R,T}{\operatorname{argmin}} \sum_{i=1}^{N} d(RT\boldsymbol{x'_i}, F\boldsymbol{x}_i) \tag{3}$$

Since we are not able to solve this optimization problem analytically, we solve it numerically thanks to the L-BFGS algorithm. As input are the correspondences on the images and the affine approximation matrix and the optimization algorithm outputs the optimal angle $\theta \in [0, 2\pi]$ corresponding to the error on the yaw and a translation $t \in \mathbb{R}$ corresponding to a translation in the normal direction w.r.t. the epipolar lines.

3.2.b).1 Limited-memory BFGS

Limited-memory BFGS is an optimization algorithm in the family of quasi-Newton methods that approximates the BFGS algorithm using a limited amount of computer memory. In our case we do not need to compute the gradient analytically since the algorithm approximates it using finite difference. We use this algorithm as a black box to solve the minimization problem.

4 Planet data

4.1 Structure

- data
  - s03_20161003T161107Z
    - panchromatic
      - s03_20161003T161107Z_pan_d1_0001_rpc.txt
      - s03_20161003T161107Z_pan_d1_0001.tif
    - pansharp
      - s03_20161003T161107Z_pansharp_bgrn_d1_0001_rpc.txt
      - s03_20161003T161107Z_pansharp_bgrn_d1_0001.tif
  - s03_20161003T161148Z
    - panchromatic
      - s03_20161003T161148Z_pan_d1_0001_rpc.txt
      - s03_20161003T161148Z_pan_d1_0001.tif
    - pansharp
      - s03_20161003T161148Z_pansharp_bgrn_d1_0001_rpc.txt
      - s03_20161003T161148Z_pansharp_bgrn_d1_0001.tif
  - s03_20161003T161231Z
    - panchromatic
      - s03_20161003T161231Z_pan_d1_000_rpc.txt
      - s03_20161003T161231Z_pan_d1_0001.tif
    - pansharp
      - s03_20161003T161231Z_pansharp_bgrn_d1_0001_rpc.txt
      - s03_20161003T161231Z_pansharp_bgrn_d1_0001.tif

4.2 Geolocalisation

A Rotations and translation

In this part, we consider any point $(x, y) \in \mathbb{R}^2$. As translation and rotation do not commute, we will consider two cases: rotation followed by translation and translation followed by rotation, with R denoting the rotation of angle θ and T denoting the translation of vector (t_x, t_y). Naming f the resulting function.

$$T = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix}$$
$$R = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

A.1 Rotation followed by a translation

This happens when correcting the pointing error first with the rotation.

$$\begin{aligned} f(\boldsymbol{x}) &= TR\boldsymbol{x} \\ &= \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\theta x - \sin\theta y \\ \sin\theta x + \cos\theta y \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} \cos\theta x - \sin\theta y + t_x \\ \sin\theta x + \cos\theta y + t_y \\ 1 \end{pmatrix} \end{aligned}$$

A.2 Translation followed by a rotation

This happens when correcting the pointing error first with the translation

$$\begin{aligned} f(\boldsymbol{x}) &= RT\boldsymbol{x} \\ &= \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x + t_x \\ y + t_y \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} \cos\theta(x + t_x) - \sin\theta(y + t_y) \\ \sin\theta(x + t_x) + \cos\theta(y + t_y) \\ 1 \end{pmatrix} \end{aligned}$$

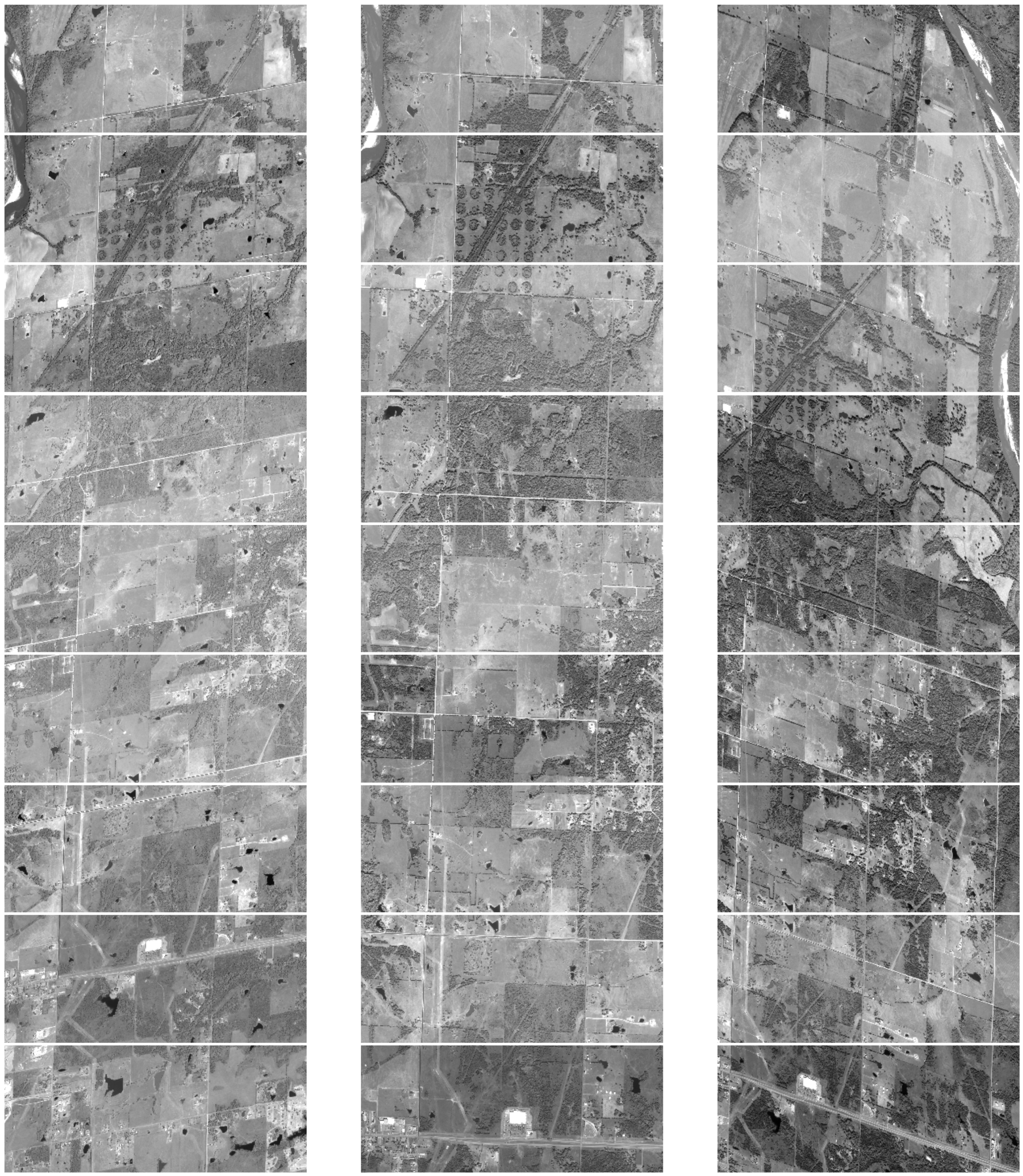

Figure 2: Mosaic of d1. From left to right: 1Z, 7Z, 8Z

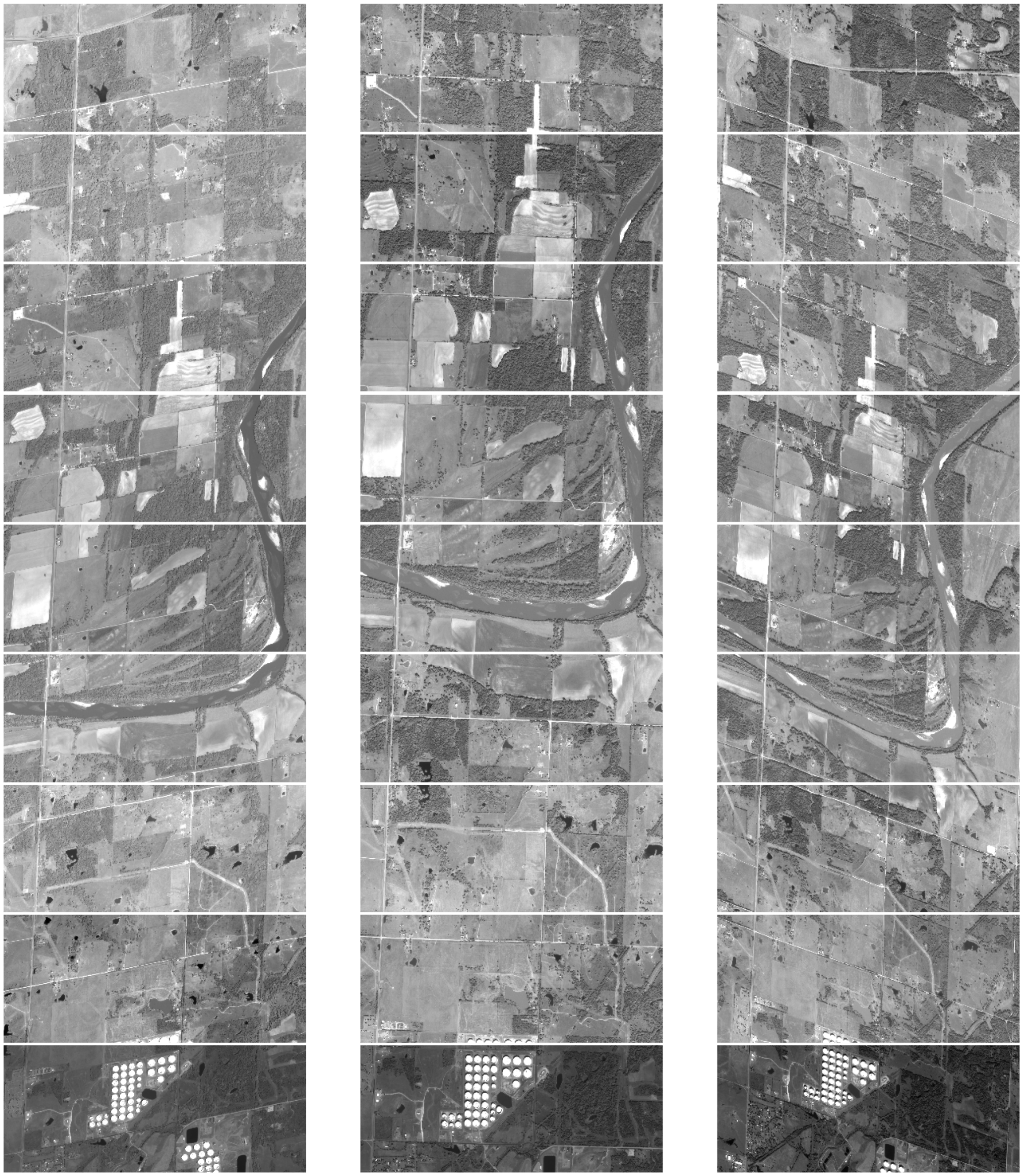

Figure 3: Mosaic of d2. From left to right: 1Z, 7Z, 8Z

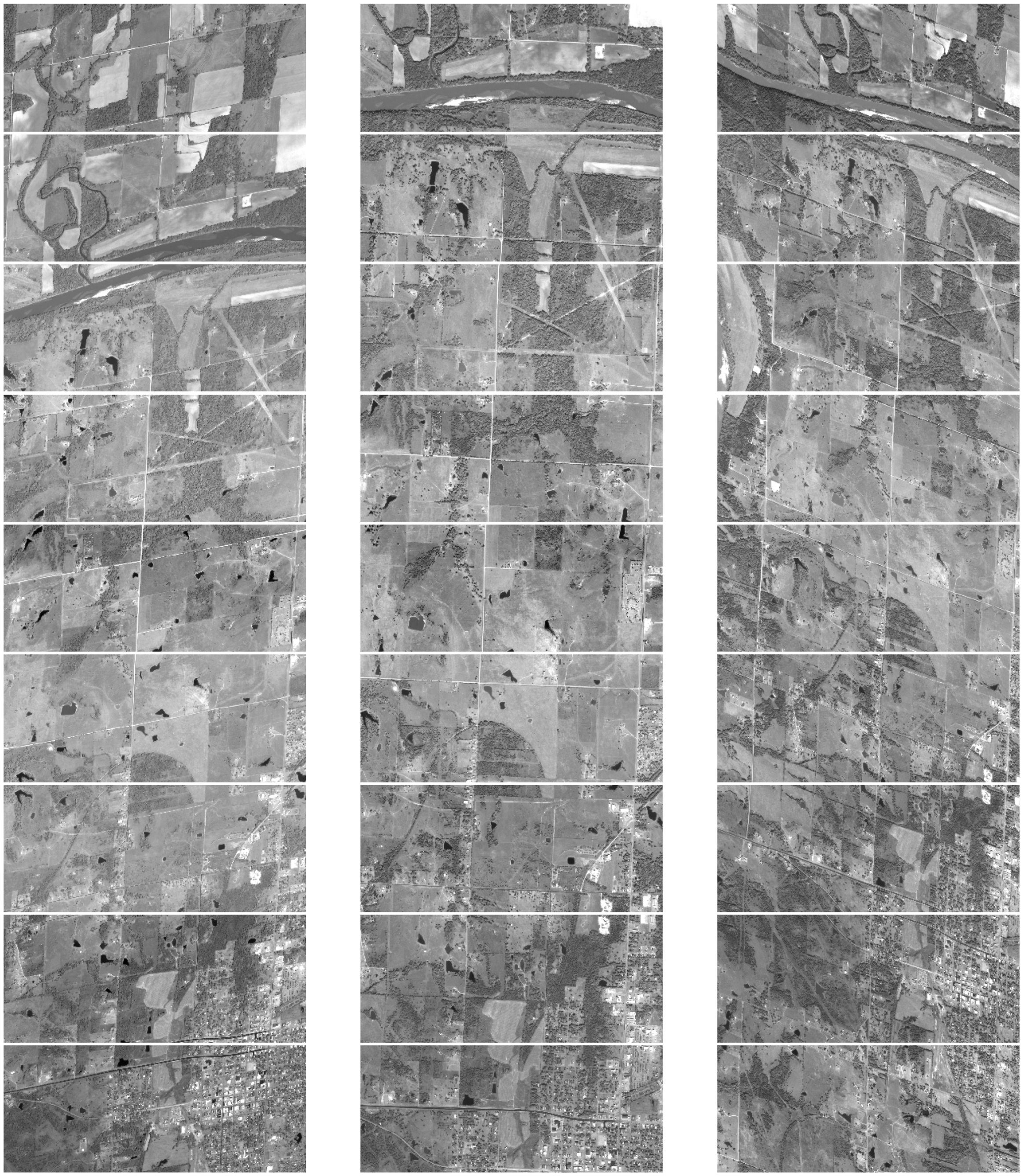

Figure 4: Mosaic of d3. From left to right: 1Z, 7Z, 8Z