Pointing error correction

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1 Planet data

1.1 Structure

- data
  - s03_20161003T161107Z
    - panchromatic
      - s03_20161003T161107Z_pan_d1_0001_rpc.txt
      - s03_20161003T161107Z_pan_d1_0001.tif
    - pansharp
      - s03_20161003T161107Z_pansharp_bgrn_d1_0001_rpc.txt
      - s03_20161003T161107Z_pansharp_bgrn_d1_0001.tif
  - s03_20161003T161148Z
    - panchromatic
      - s03_20161003T161148Z_pan_d1_0001_rpc.txt
      - s03_20161003T161148Z_pan_d1_0001.tif
    - pansharp
      - s03_20161003T161148Z_pansharp_bgrn_d1_0001_rpc.txt
      - s03_20161003T161148Z_pansharp_bgrn_d1_0001.tif
  - s03_20161003T161231Z
    - panchromatic
      - s03_20161003T161231Z_pan_d1_000_rpc.txt
      - s03_20161003T161231Z_pan_d1_0001.tif
    - pansharp
      - s03_20161003T161231Z_pansharp_bgrn_d1_0001_rpc.txt
      - s03_20161003T161231Z_pansharp_bgrn_d1_0001.tif

1.2 Geolocalisation

2 Attitude of a Satellite

3 Satellite attitude error effects on stereo images

4 Correction of Relative Pointing Error

[1]

A simple way to correct the relative pointing error is thus to transform one of the two images, in such a way that the corresponding points fall on the respective epipolar curves: given two images u, v and a set of correspondences $(\mathbf{x}_i, \mathbf{x}'_i)_{i=1\ldots N}$, we search for a translation f such that, for all i, the transformed point $f(\mathbf{x}'_i)$ lies on the epipolar curve $epi_{uv}^{\mathbf{x}_i}(R)$. The desired transformation f^* minimises the relative pointing error defined by:

$$f^* = \underset{f}{\operatorname{argmin}} \frac{1}{N} \sum_{i=1}^{N} d(f(\mathbf{x}'_i), epi_{uv}^{\mathbf{x}_i}(R)) \tag{1}$$

From [1], we know that the epipolar curve $epi_{uv}^{\mathbf{x}_i}(R)$ is approximated up to 0.05 pixels by the straight line $F\mathbf{x}_i$, where F is the affine fundamental matrix between the two views for the considered tile. As this fundamental matrix is an *affine* fundamental matrix, all the lines $F\mathbf{x}_i$ are parallel.

4.1 Roll and Pitch Angles

Because of sensitivity issues, we can take only roll and pitch error into account. Therefore according to section 3, we search for a transformation f such that

$$f(\mathbf{x}) = T\mathbf{x}$$

where T is a translation:

$$T = \begin{pmatrix} 1 & 0 & t_1 \\ 0 & 1 & t_2 \\ 0 & 0 & 1 \end{pmatrix}$$

So we have, for $\mathbf{x} = (x\ y\ 1)^T$,

$$f(\mathbf{x}) = T\mathbf{x} = \begin{pmatrix} x + t_1 \\ y + t_2 \\ 1 \end{pmatrix}$$

4.1.a) Pointing error after rectification

Without any additional restriction, we may assume that these lines are horizontal (otherwise just do a change of coordinates). The horizontal line $F\mathbf{x}_i$ can be written, in homogeneous coordinates, as

$$F\mathbf{x}_i = [0\ 1\ c_i]$$

With these notations, for each point correspondence $(\mathbf{x}_i, \mathbf{x}'_i)$, we have

$$e(\mathbf{x}_i, \mathbf{x}'_i) = d(\mathbf{x}'_i, epi_{uv}^{\mathbf{x}_i}(R)) = d(\mathbf{x}'_i, F\mathbf{x}_i) = |y'_i + c_i|$$

Here the error e is invariant to any horizontal translation, thus the search for a translation minimizing the relative pointing error of formula (1) can be restricted to vertical translations only. With a vertical

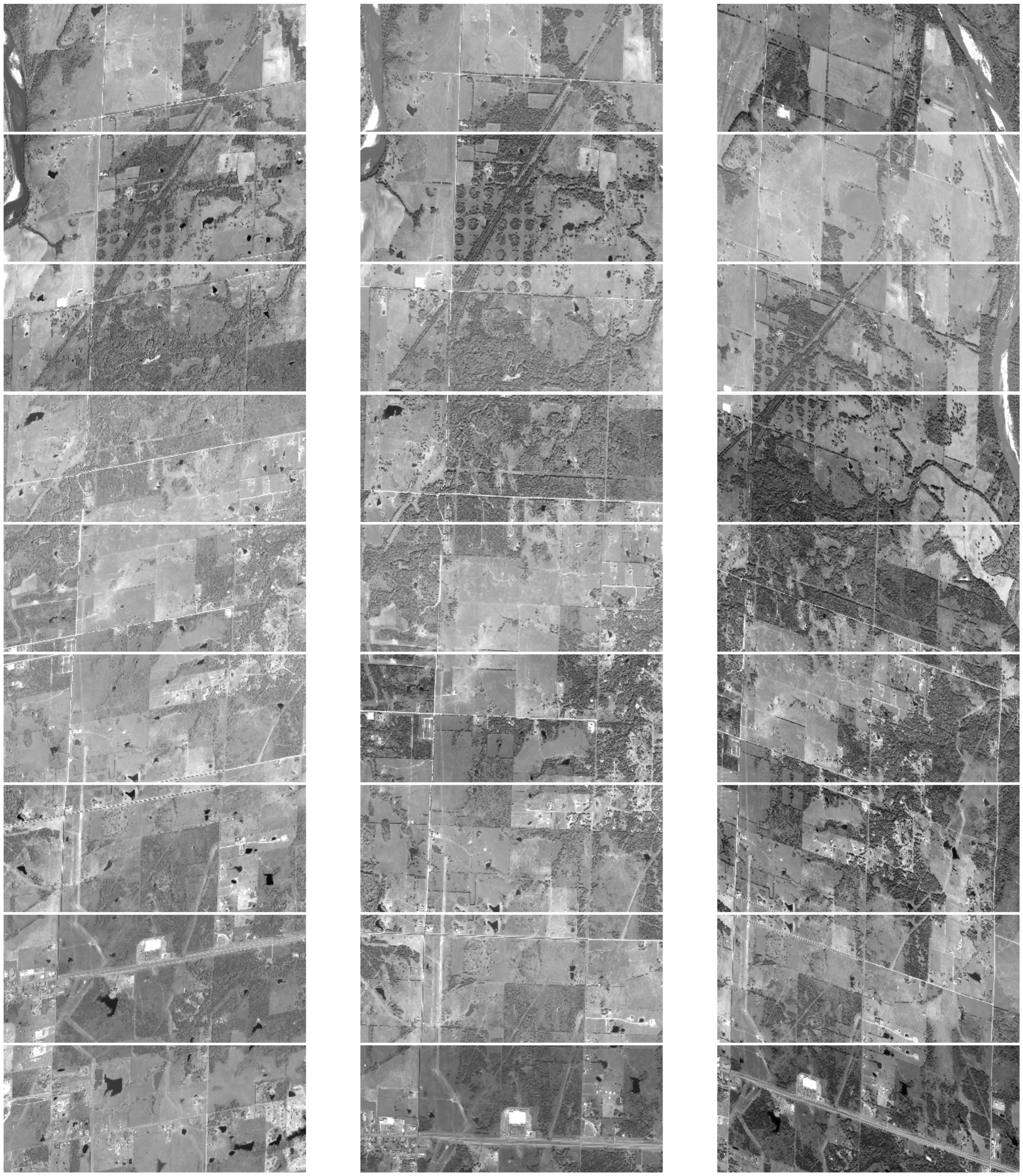

Figure 1: Mosaic of d1. From left to right: 1Z, 7Z, 8Z

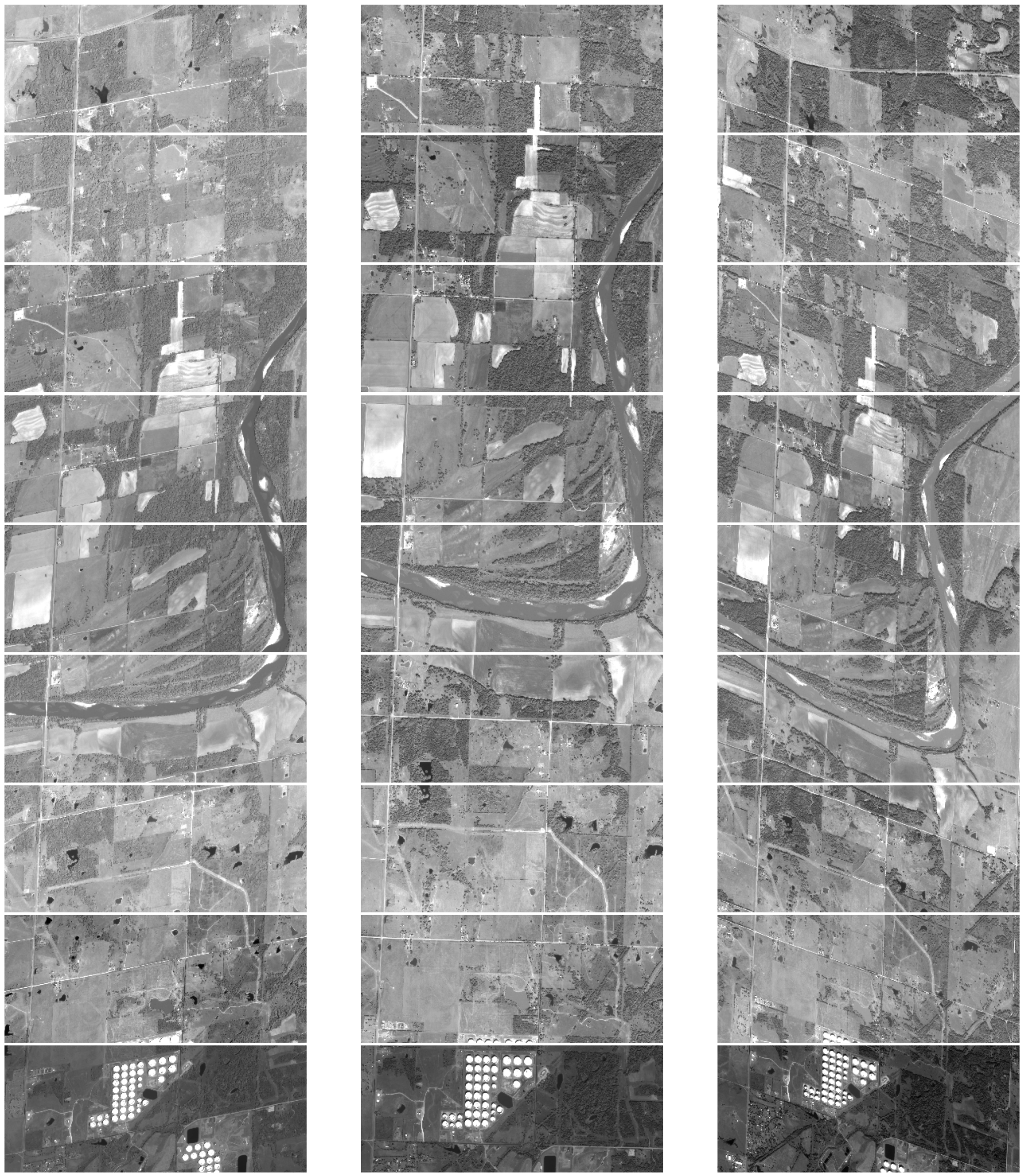

Figure 2: Mosaic of d2. From left to right: 1Z, 7Z, 8Z

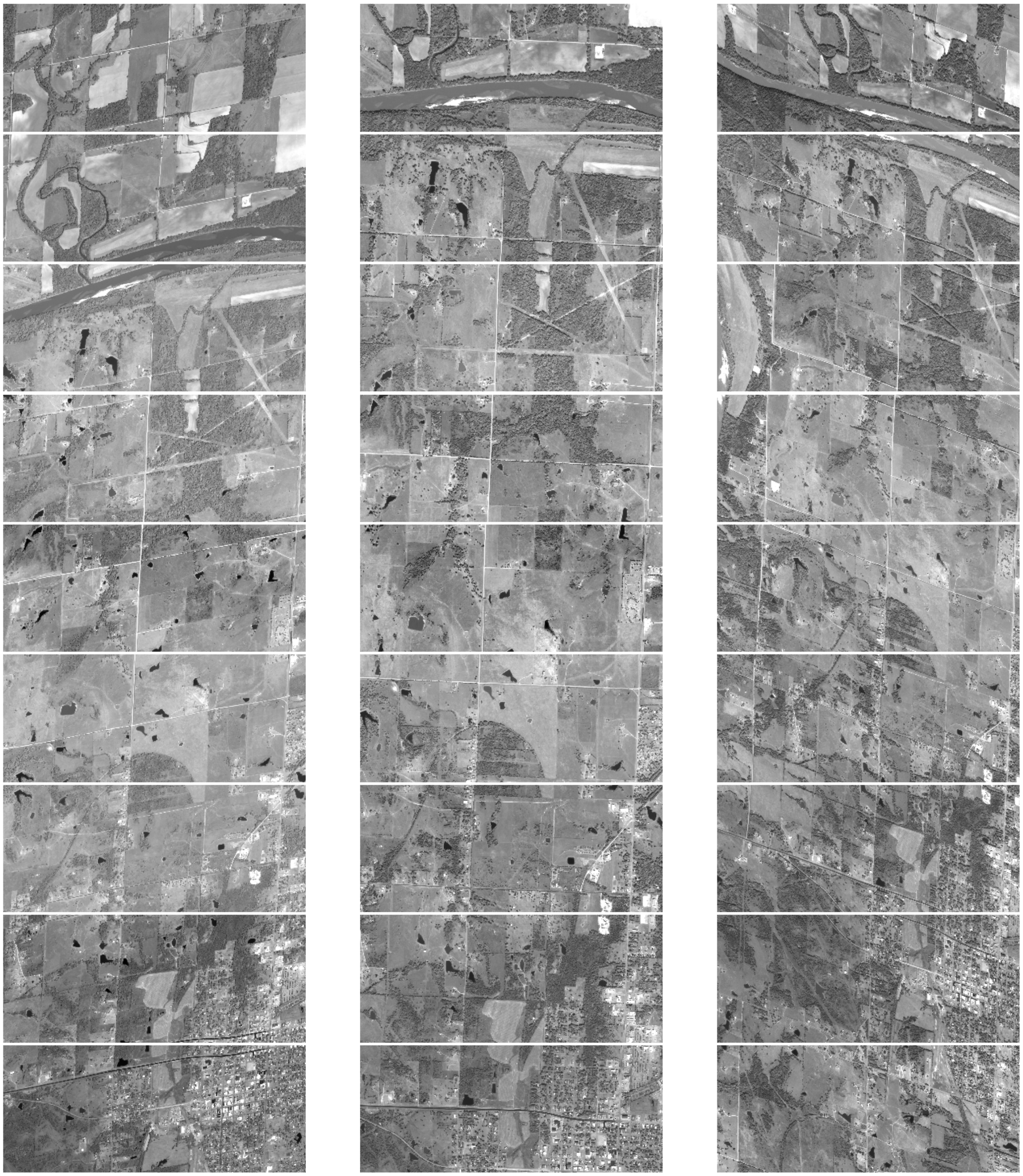

Figure 3: Mosaic of d3. From left to right: 1Z, 7Z, 8Z

Fig. 46

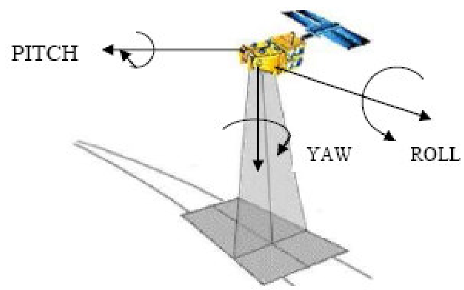

Figure 4: Roll, pitch and yaw angles.

translation of parameter t, the error becomes

$$E(T) = \frac{1}{N}\sum_{i=1}^{N} d(T\mathbf{x}'_i, F\mathbf{x}_i) = \frac{1}{N}\sum_{i=1}^{N} |y'_i + t + c_i|$$

The translation that minimizes this sum is given by the geometric median (Weiszfeld, 1937) of the vector $(-y'_i - c_i)_{i=1\ldots N}$. The relative pointing error can thus be minimized in a tile by applying a translation to one of the images. Note that the median is robust against outliers, thus this correction procedure works well even in the presence of false matches.

4.1.b) Pointing before after rectification

In the general case, we have:

$$F\mathbf{x}_i = (a_i\ b_i\ c_i)^T$$

With these notations, the error to minimize is then:

$$E(T) = \frac{1}{N}\sum_{i=1}^{N} d(T\mathbf{x}'_i, F\mathbf{x}_i) = \frac{1}{N}\sum_{i=1}^{N} \frac{|a_i(x'_i + t_1) + b_i(y'_i + t_2) + c_i|}{\sqrt{a_i^2 + b_i^2}}$$

As this fundamental matrix is an *affine* fundamental matrix, all the lines $F\mathbf{x}_i$ are parallel, *i.e.*

$$F\mathbf{x}_i = (a\ b\ c_i)^T$$

We can then write the error as :

$$E(t_1, t_2) = \frac{1}{N\sqrt{a^2 + b^2}}\sum_{i=1}^{N} |ax'_i + by'_i + c_i + at_1 + bt_2|$$

4.1.b).1 Compute vectors p

We can compute the vectors going from the projection of $\mathbf{x}'_i$ on $F\mathbf{x}_i$ to $\mathbf{x}'_i$:

$$\begin{aligned} p(\mathbf{x}_i, \mathbf{x}'_i) &= d(\mathbf{x}'_i, F\mathbf{x}_i)\frac{1}{\sqrt{a_i^2 + b_i^2}}\begin{pmatrix} a_i \\ b_i \end{pmatrix} \\ &= \frac{|a_i x'_i + b_i y'_i + c_i|}{a_i^2 + b_i^2}\begin{pmatrix} a_i \\ b_i \end{pmatrix} \end{aligned}$$

And we have :

$$\begin{pmatrix} t_1^* \\ t_2^* \end{pmatrix} = -\text{median}\left[p(\mathbf{x}_i, \mathbf{x}'_i)_{1 \le i \le N}\right]$$

4.2 Roll, Pitch and Yaw Angles

If we assume that the scene is located at infinity with respect to the satellite, an error in the sensor attitude measurement can be modeled in image space as a translation composed with a rotation. Therefore we have

$$f(\mathbf{x}) = RT\mathbf{x}$$

where R is a rotation and T a translation:

$$R = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix},\ T = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

So we have, for $\mathbf{x} = (x\ y\ 1)^T$,

$$\begin{aligned} f(\mathbf{x}) = RT\mathbf{x} &= \begin{pmatrix} \cos(\theta) & -\sin(\theta) & t_x\cos(\theta) - t_y\sin(\theta) \\ \sin(\theta) & \cos(\theta) & t_x\sin(\theta) + t_y\cos(\theta) \\ 0 & 0 & 1 \end{pmatrix} \mathbf{x} \\ &= \begin{pmatrix} \cos\theta(x + t_x) - \sin\theta(y + t_y) \\ \sin\theta(x + t_x) + \cos\theta(y + t_y) \\ 1 \end{pmatrix} \end{aligned}$$

When we take the yaw into accound, the error to minimize is :

$$\begin{aligned} E(R,T) &= \frac{1}{N}\sum_{i=1}^{N} d(RT\mathbf{x}'_i, F\mathbf{x}_i) \\ &= \frac{1}{N}\sum_{i=1}^{N} \frac{|a_i(x'_i + t_x)\cos\theta - a_i(y'_i + t_y)\sin\theta + b_i(x'_i + t_x)\sin\theta + b_i(y'_i + t_y)\cos\theta + c_i|}{\sqrt{a_i^2 + b_i^2}} \end{aligned}$$

Reminder: Matrix multiplication have always the origin as a fixed point. Common workaround using homogeneous coordinates.

A Rotations and translation

In this part, we consider any point $(x, y) \in \mathbb{R}^2$. As translation and rotation do not commute, we will consider two cases: rotation followed by translation and translation followed by rotation, with R denoting the rotation of angle θ and T denoting the translation of vector (t_x, t_y). Naming f the resulting function.

$$T = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix}$$
$$R = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

A.1 Rotation followed by a translation

This happens when correcting the pointing error first with the rotation.

$$
\begin{aligned}
f(\boldsymbol{x}) &= TR\boldsymbol{x} \\
&= \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\theta x - \sin\theta y \\ \sin\theta x + \cos\theta y \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} \cos\theta x - \sin\theta y + t_x \\ \sin\theta x + \cos\theta y + t_y \\ 1 \end{pmatrix}
\end{aligned}
$$

A.2 Translation followed by a rotation

This happens when correcting the pointing error first with the translation

$$
\begin{aligned}
f(\boldsymbol{x}) &= RT\boldsymbol{x} \\
&= \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x + t_x \\ y + t_y \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} \cos\theta(x + t_x) - \sin\theta(y + t_y) \\ \sin\theta(x + t_x) + \cos\theta(y + t_y) \\ 1 \end{pmatrix}
\end{aligned}
$$

References

[1] Carlo De Franchis, Enric Meinhardt-Llopis, Julien Michel, Jean-Michel Morel, and Gabriele Facciolo. An automatic and modular stereo pipeline for pushbroom images. *ISPRS Annals of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, 2(3):49, 2014.