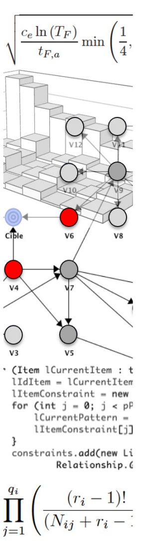
Graphes et Applications

Plan



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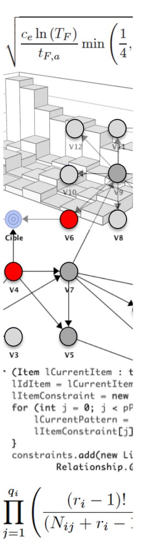
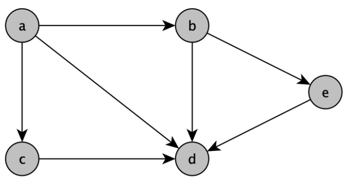
Graphes et Applications

Représentation d'un graphe en machine

Matrice d'adjacence



Graphe orienté



$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Matrice $M = (m_{ij})$:

$$m_{ij} = \begin{cases} 1 & \text{si } (i,j) \in E \\ 0 & \text{si } (i,j) \notin E \end{cases}$$

Quelle serait la complexité d'un algorithme qui ferait la liste de tous les successeurs de tous les sommets ?

O(n^2)

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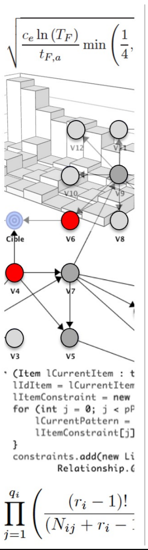
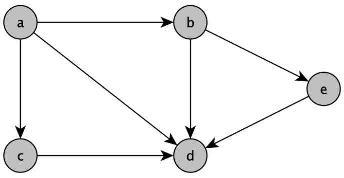
Graphes et Applications

Représentation d'un graphe en machine

MATRICE D'ADJACENCE

Graphe orienté





$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$M^2 = \begin{pmatrix} 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M^3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad M^4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$M^n = (m_{ij})^n$: nombre de chemins de longueur n allant de i à j

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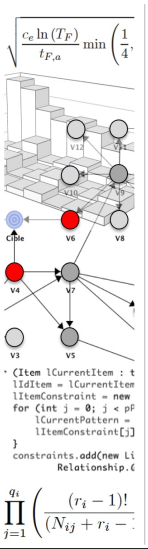
Représentation d'un graphe en machine

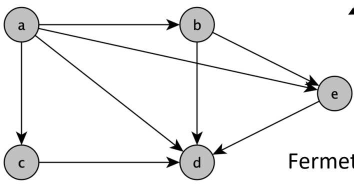
MATRICE D'ADJACENCE

Graphe orienté



$$M + M^2 + M^3 = \begin{pmatrix} 0 & 1 & 1 & 4 & 1 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \longrightarrow M' = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Matrice booléenne



Fermeture transitive du graphe initial

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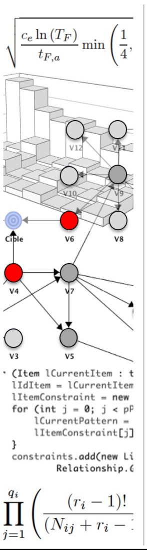
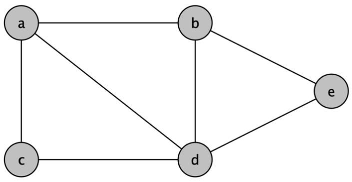
Graphes et Applications

Représentation d'un graphe en machine

MATRICE D'ADJACENCE



Graphe non-orienté



$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Matrice symétrique

Matrice $M = (m_{ij})$:

$$m_{ij} = \begin{cases} 1 & \text{si } (i,j) \in E \\ 0 & \text{si } (i,j) \notin E \end{cases}$$

Que vaut M^2 M^3 M^4 ... ?

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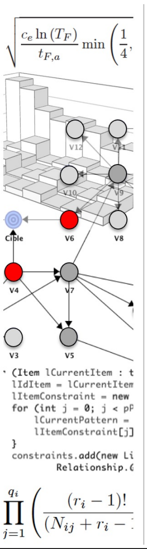
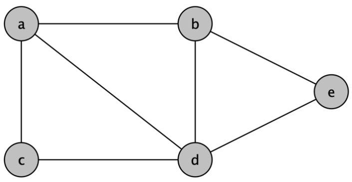
Graphes et Applications

Représentation d'un graphe en machine

MATRICE D'ADJACENCE



Graphe non-orienté



$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$M^2 = \begin{pmatrix} 3 & 1 & 1 & 2 & 2 \\ 1 & 3 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 & 1 \\ 2 & 2 & 1 & 4 & 1 \\ 2 & 1 & 1 & 1 & 2 \end{pmatrix} \quad M^3 = \begin{pmatrix} 4 & 7 & 5 & 7 & 3 \\ 7 & 4 & 3 & 7 & 5 \\ 5 & 3 & 2 & 6 & 3 \\ 7 & 7 & 6 & 6 & 6 \\ 3 & 5 & 3 & 6 & 2 \end{pmatrix} \quad M^4 = \begin{pmatrix} 19 & 14 & 11 & 19 & 14 \\ 14 & 19 & 14 & 19 & 11 \\ 11 & 14 & 11 & 13 & 9 \\ 19 & 19 & 13 & 26 & 13 \\ 14 & 11 & 9 & 13 & 11 \end{pmatrix}$$

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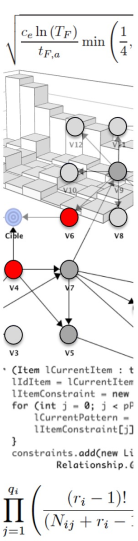
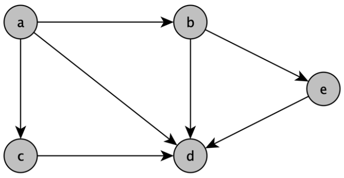
Graphes et Applications

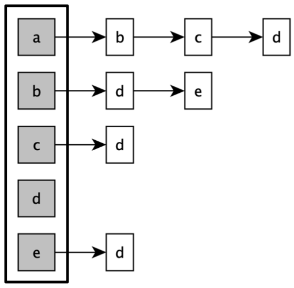
Représentation d'un graphe en machine

LISTE D'ADJACENCE

Graphe orienté







Quelle serait la complexité d'un algorithme qui ferait la liste de tous les successeurs de tous les sommets ?

O(n+m)

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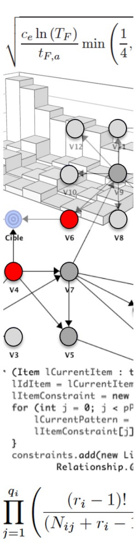
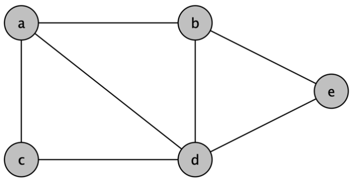
Graphes et Applications

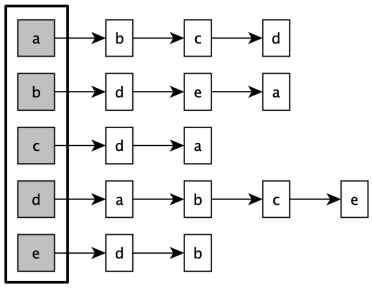
Représentation d'un graphe en machine

LISTE D'ADJACENCE

Graphe non-orienté







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