

### Question

Consider a quantum system made of two qubits. The system is in the initial state  $|\psi\rangle = 1/\sqrt{N} (a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)$ . A CNOT gate is applied on the system with the first qubit as target qubit and the second qubit as control qubit. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

### Answer 1

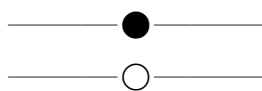
Certainly! In quantum computing, a CNOT (Controlled-NOT) gate is a two-qubit gate that performs a NOT operation (bit flip) on the target qubit if and only if the control qubit is in the state  $|1\rangle$ . In this case, the CNOT gate is applied with the first qubit (denoted as qubit 1) as the target qubit and the second qubit (denoted as qubit 2) as the control qubit.

The action of the CNOT gate on the system can be described as follows:

If the control qubit (qubit 2) is in the state  $|0\rangle$ , then the target qubit (qubit 1) remains unchanged.

If the control qubit (qubit 2) is in the state  $|1\rangle$ , then the target qubit (qubit 1) undergoes a bit flip (NOT operation).

To represent this operation graphically, we can use the following circuit diagram:



In this diagram, the circle represents the control qubit (qubit 2), and the target qubit (qubit 1) is represented by a cross. The line connecting them represents the CNOT gate.

Now, let's calculate the final state of the system after applying the CNOT gate.

The initial state of the system is given as  $|\psi\rangle = 1/\sqrt{N} (a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)$ .

When the CNOT gate is applied with qubit 1 as the target and qubit 2 as the control, the action of the gate can be described as:

$$\text{CNOT}(|xy\rangle) = |x, x \oplus y\rangle,$$

where  $\oplus$  represents the XOR operation. In this case,  $x$  represents the state of qubit 1 (target qubit), and  $y$  represents the state of qubit 2 (control qubit).

Let's apply this operation to each term in the initial state:

$$\text{CNOT}(a|00\rangle) = a|00\rangle,$$

$$\text{CNOT}(b|01\rangle) = b|01\rangle,$$

$$\text{CNOT}(c|10\rangle) = c|11\rangle,$$

$$\text{CNOT}(d|11\rangle) = d|10\rangle.$$

The final state of the system after applying the CNOT gate is:

$$1/\sqrt{N} (a|00\rangle + b|01\rangle + c|11\rangle + d|10\rangle).$$

So, the final state of the system is given by  $|\psi\rangle = 1/\sqrt{N} (a|00\rangle + b|01\rangle + c|11\rangle + d|10\rangle)$ .

## Answer 2

Sure! The CNOT gate is a two-qubit gate in quantum computing that is used for conditional operations. It acts as a controlled NOT gate, where the target qubit is flipped if and only if the control qubit is in the state  $|1\rangle$ .

In this case, the CNOT gate is applied to the two qubits in the system, with the first qubit as the target qubit and the second qubit as the control qubit. The graphical representation of the circuit looks like this:

$$|\psi\rangle \xrightarrow{\text{CNOT}} |\psi'\rangle$$

where the  $|\psi\rangle$  represents the initial state of the system, and the  $|\psi'\rangle$  represents the final state of the system after the CNOT gate has been applied.

To understand the action of the CNOT gate on the system, let's first write out the matrix representation of the gate in the standard basis:

$$\text{CNOT} = |00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 11| + |11\rangle\langle 10|$$

When we apply this gate to the system, the CNOT gate acts on both qubits simultaneously. The control qubit (the second qubit) acts as a switch, determining whether or not the target qubit (the first qubit) gets flipped.

So let's look at what happens to each of the basis states in the system:

If the control qubit is in state  $|0\rangle$ , then the CNOT gate has no effect on the system, since the target qubit is not flipped. So, for example, if the system is initially in state  $|00\rangle$ , the final state will be the same as the initial state:  $|00\rangle$ .

If the control qubit is in state  $|1\rangle$ , then the CNOT gate flips the target qubit. So, for example, if the system is initially in state  $|01\rangle$ , the final state will be  $|11\rangle$ .

Using this logic, we can write out the final state of the system after the CNOT gate has been applied. Let's look at each of the basis states in the initial state:

$$\begin{aligned} &\text{CNOT}(a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle) \\ &= a|00\rangle + b|11\rangle + c|10\rangle + d|01\rangle \end{aligned}$$

So the final state of the system is:

$$|\psi'\rangle = 1/\sqrt{N} (a|00\rangle + b|11\rangle + c|10\rangle + d|01\rangle)$$

In words, the CNOT gate has flipped the second qubit if and only if it was in state  $|1\rangle$ , resulting in the final state where the first and second qubits are either both flipped or both unchanged.

### Answer 3

The CNOT gate, also known as the Controlled-NOT gate, is a two-qubit gate that performs a NOT operation on the target qubit (the second qubit) if and only if the control qubit (the first qubit) is in the state  $|1\rangle$ . The gate leaves the target qubit unchanged if the control qubit is in the state  $|0\rangle$ .

In this case, the initial state of the system is given as  $|\psi\rangle = 1/\sqrt{N} (a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)$ . To describe the action of the CNOT gate, let's consider the possible states of the control and target qubits individually:

- If the control qubit is in the state  $|0\rangle$ , the target qubit remains unchanged.
- If the control qubit is in the state  $|1\rangle$ , the target qubit undergoes a NOT operation, flipping its state.

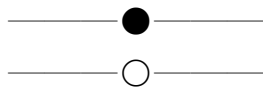
Thus, when applying the CNOT gate to the given initial state, we can describe the action as follows:

$$\text{CNOT gate } (|0\rangle \text{ control}, |1\rangle \text{ target}) = |0\rangle \text{ control} \otimes I \text{ target}$$

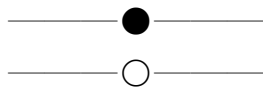
$$\text{CNOT gate } (|1\rangle \text{ control}, |0\rangle \text{ target}) = |1\rangle \text{ control} \otimes X \text{ target}$$

Here,  $I$  represents the identity gate (no operation), and  $X$  represents the Pauli-X gate (NOT gate).

To create a graphical representation of the circuit, we can use the following convention:



The " $\bullet$ " represents the control qubit, and the " $\circ$ " represents the target qubit. The line connecting them represents the CNOT gate. In this case, the circuit would look like:



Finally, to determine the final state of the system, we apply the CNOT gate to the initial state  $|\psi\rangle$ :

$$1/\sqrt{N} (a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)$$

$$= 1/\sqrt{N} (a|00\rangle + b|01\rangle + c|11\rangle + d|10\rangle)$$

The final state of the system is given as:

$$|\psi'\rangle = 1/\sqrt{N} (a|00\rangle + b|01\rangle + c|11\rangle + d|10\rangle)$$