

Question

Consider a quantum system made of one qubit. The system is in the initial state $|\psi\rangle = 1/\sqrt{N} (a|0\rangle + b|1\rangle)$, where a and b are complex numbers and N is a normalization factor. A quantum gate Pauli Y is applied on $|\psi\rangle$. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

The Pauli Y gate is a single-qubit gate that acts on the state vector of the qubit by performing a rotation around the y-axis of the Bloch sphere. Specifically, it maps the state $|0\rangle$ to the state $i|1\rangle$ and the state $|1\rangle$ to the state $-i|0\rangle$. Therefore, the action of the Pauli Y gate on the initial state $|\psi\rangle$ is:

$$\begin{aligned} Y|\psi\rangle &= 1/\sqrt{N} (a Y|0\rangle + b Y|1\rangle) \\ &= 1/\sqrt{N} (a i|1\rangle - b i|0\rangle) \end{aligned}$$

To represent this circuit graphically, we can use a circuit diagram where the initial state $|\psi\rangle$ is input into the Pauli Y gate and the output state is measured:

$|\psi\rangle \rightarrow Y \rightarrow \text{measurement}$

The final state of the system after the application of the Pauli Y gate is $1/\sqrt{N} (a i|1\rangle - b i|0\rangle)$.

Answer 2

The Pauli Y gate is a single-qubit gate in quantum computing that acts as a rotation around the y-axis of the Bloch sphere, which is a geometric representation of the state space of a single qubit. The Pauli Y gate is defined as:

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

When the Pauli Y gate is applied to a qubit in the state $|0\rangle$, it transforms the

state into $i|1\rangle$. Similarly, when it is applied to a qubit in the state $|1\rangle$, it transforms the state into $-i|0\rangle$. Thus, the Pauli Y gate introduces a phase shift of π radians between the $|0\rangle$ and $|1\rangle$ states.

Now, let's consider the initial state $|\psi\rangle = 1/\sqrt{N} (a|0\rangle + b|1\rangle)$. The action of the Pauli Y gate on this state can be represented by the following circuit:

$$|\psi\rangle \xrightarrow{Y} \text{---} \rangle$$

The Y gate acts on the input state $|\psi\rangle$ to give the output state:

$$\begin{aligned} Y|\psi\rangle &= 1/\sqrt{N} (aY|0\rangle + bY|1\rangle) \\ &= 1/\sqrt{N} (a(0 - i)|1\rangle + b(i|0\rangle)) \\ &= 1/\sqrt{N} (-ai|1\rangle + bi|0\rangle) \end{aligned}$$

Therefore, the final state of the system after the application of the Pauli Y gate is $1/\sqrt{N} (-ai|1\rangle + bi|0\rangle)$. This state can be represented as a point on the Bloch sphere located on the negative y-axis.

Answer 3

The Pauli Y gate, denoted as Y, is one of the fundamental single-qubit gates in quantum computing. It is represented by the following matrix:

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix},$$

where i is the imaginary unit.

To determine the action of the Pauli Y gate on the initial state $|\psi\rangle = 1/\sqrt{N} (a|0\rangle + b|1\rangle)$, we simply multiply the state vector by the Y gate matrix:

$$\begin{aligned} Y|\psi\rangle &= Y(1/\sqrt{N} (a|0\rangle + b|1\rangle)) \\ &= 1/\sqrt{N} (Y(a|0\rangle) + Y(b|1\rangle)) \\ &= 1/\sqrt{N} (aY|0\rangle + bY|1\rangle). \end{aligned}$$

The action of the Pauli Y gate on the basis states $|0\rangle$ and $|1\rangle$ is as follows:

$$Y|0\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i|1\rangle$$

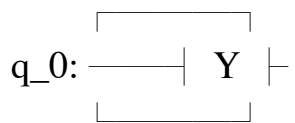
$$\begin{aligned}
& [i, 0] |0\rangle \\
& = [[0, -i], \\
& [i, 0]] |1\rangle, \\
& [0] \\
& = [[0], \\
& [i]] \\
& = i|1\rangle,
\end{aligned}$$

$$\begin{aligned}
Y|1\rangle &= [[0, -i], \\
& [i, 0]] |1\rangle \\
&= [[0, -i], \\
& [i, 0]] |0\rangle, \\
& [1] \\
&= [-i], \\
& [0] \\
&= -i|0\rangle.
\end{aligned}$$

Therefore, we can express the final state after applying the Pauli Y gate as:

$$\begin{aligned}
Y|\psi\rangle &= 1/\sqrt{N} (aY|0\rangle + bY|1\rangle) \\
&= 1/\sqrt{N} (a(i|1\rangle) + b(-i|0\rangle)) \\
&= 1/\sqrt{N} (ia|1\rangle - ib|0\rangle).
\end{aligned}$$

Graphical representation of the circuit:



The final state of the system is $|\psi\rangle = 1/\sqrt{N} (ia|1\rangle - ib|0\rangle)$, where a and b are complex numbers and N is the normalization factor.