

### Question

Consider a quantum system made of one qubit. The system is in the initial state  $|\psi\rangle = |0\rangle$ . A T gate is applied on  $|\psi\rangle$ . Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

### Answer 1

Certainly! The T gate is a single-qubit gate in quantum computing that introduces a phase shift of  $\pi/4$  (or 45 degrees) to the state it acts upon. In matrix form, the T gate is represented as:

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

where  $e^{i\pi/4}$  is a complex number.

When the T gate is applied to the initial state  $|\psi\rangle = |0\rangle$ , the action can be described as follows:

$$T(|0\rangle) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} * \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 * 1 + 0 * 0 \\ 0 * 1 + e^{i\pi/4} * 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

So, applying the T gate to the state  $|0\rangle$  leaves it unchanged. The final state of the system remains  $|\psi\rangle = |0\rangle$ .

As for the graphical representation of the circuit, a single-qubit circuit with a T gate can be depicted as follows:

$$|0\rangle \xrightarrow{T} |0\rangle$$

This circuit diagram shows the initial state  $|0\rangle$  on the left side, the T gate in the middle, and the final state  $|0\rangle$  on the right side.

Please note that the T gate is commonly represented using the symbol "T" in quantum circuit diagrams.

### Answer 2

The T gate is a single-qubit gate in quantum computing with the following matrix representation:

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

$$[0, e^{i\pi/4}]$$

When applied to the initial state  $|0\rangle = [1, 0]$ , the T gate acts as follows:

$$\begin{aligned} T|0\rangle &= [[1, 0], [0, e^{i\pi/4}]] [[1], [0]] \\ &= [[1*1 + 0*0], [0*1 + e^{i\pi/4}*0]] \\ &= [1, 0] \\ &= |0\rangle \end{aligned}$$

So, applying the T gate to the initial state  $|0\rangle$  does not change the state at all.

The circuit diagram for this operation is:

$$|0\rangle \text{ -- T -- } \rangle$$

which represents a single qubit  $|0\rangle$  being acted upon by the T gate.

Since the T gate doesn't change the initial state of  $|0\rangle$ , the final state of the system is also  $|0\rangle$ .

### Answer 3

The T gate, also known as the  $\pi/4$  gate or the phase gate, is a single-qubit gate in quantum computing. It introduces a phase shift of  $\pi/4$  (45 degrees) to the state it operates on. In the computational basis, the T gate can be represented as follows:

T gate:

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

The T gate acts on the state  $|\psi\rangle = |0\rangle$  as follows:

T gate applied to  $|\psi\rangle$ :

$$T(|0\rangle) = T(1 * |0\rangle + 0 * |1\rangle) = 1 * |0\rangle + e^{i\pi/4} * |1\rangle$$

$$= |0\rangle + e^{i\pi/4} * |1\rangle$$

The graphical representation of the circuit would be as follows:

$|0\rangle$  --[T]-- Final State

Here, the qubit starts in the state  $|0\rangle$ , and the T gate is applied to it, resulting in the final state:

Final State:

$$|0\rangle + e^{i\pi/4} * |1\rangle$$

Please note that the final state can also be represented in terms of a normalized state vector as:

Final State:

$$(|0\rangle + e^{i\pi/4} * |1\rangle) / \sqrt{2}$$

This represents a superposition of states  $|0\rangle$  and  $|1\rangle$ , with a phase shift of  $\pi/4$  applied to the  $|1\rangle$  state.