

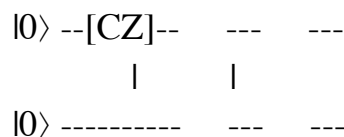
### Question

Consider a quantum system made of two qubits. The system is in the initial state  $|\psi\rangle = |00\rangle$ . A Control Z gate is applied on the system with the first qubit as target qubit and the second qubit as control qubit. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

### Answer 1

The Control Z gate, also known as the Controlled-Phase gate, is a two-qubit gate that applies a phase factor to the target qubit if and only if the control qubit is in the state  $|1\rangle$ . In the case you mentioned, the Control Z gate is applied with the first qubit as the target qubit (T) and the second qubit as the control qubit (C).

Graphical representation of the circuit:



To understand the action of the gate, let's write down the state of the system at each step.

- 1 Initial state:  $|\psi\rangle = |00\rangle$
- 2 Apply the Control Z gate: The gate does not introduce any changes to the system since the control qubit is in the state  $|0\rangle$ .
- 3 Final state: The system remains in the state  $|00\rangle$ .

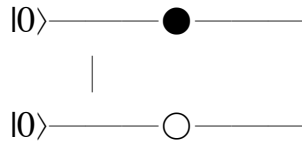
So, after applying the Control Z gate, the final state of the system is still  $|00\rangle$ . The gate does not modify the state of the qubits in this case because the control qubit is in the state  $|0\rangle$ , and the gate only introduces a phase change when the control qubit is in the state  $|1\rangle$ .

### Answer 2

The Control Z gate is a two-qubit gate that applies a phase shift of  $-1$  to the state  $|11\rangle$  and leaves all other basis states unchanged. The gate acts on the target qubit if and only if the control qubit is in the state  $|1\rangle$ .

In this case, the Control Z gate is applied to the qubits in the state  $|00\rangle$ , with the first qubit as target qubit and the second qubit as control qubit. Since the second qubit is in the state  $|0\rangle$ , the gate has no effect on the state of the system.

The circuit diagram for this operation is:



where the  $\bullet$  represents the Control Z gate, and the wires represent the qubits. The top wire is the first qubit (the target qubit), and the bottom wire is the second qubit (the control qubit).

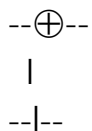
The final state of the system after the Control Z gate is applied is still  $|00\rangle$ , since the gate did not change the state of the system.

### Answer 3

The Control Z gate, also known as the Controlled-Z gate or CZ gate, is a two-qubit gate in quantum computing. It applies a phase flip to the target qubit (the first qubit in this case) if and only if the control qubit (the second qubit) is in the state  $|1\rangle$ . The gate does nothing if the control qubit is in the state  $|0\rangle$ .

In this specific scenario, the initial state of the system is  $|\psi\rangle = |00\rangle$ , where the first qubit is in the state  $|0\rangle$  and the second qubit is also in the state  $|0\rangle$ .

The graphical representation of the circuit with the Control Z gate can be shown as follows:



In this circuit diagram, the " $\oplus$ " symbol represents the Control Z gate, and the lines entering and exiting it represent the qubits. The control qubit is on top, and the target qubit is at the bottom.

The action of the Control Z gate on the system can be described as follows:

- The control qubit (the second qubit) is in the state  $|0\rangle$ , which means the gate does nothing.
- As a result, the phase of the target qubit (the first qubit) remains unchanged.

Therefore, the final state of the system remains  $|00\rangle$ , the same as the initial state.