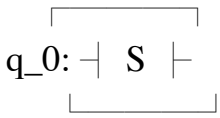
Question
Consider a quantum system made of one qubit. The system is in the initial state $|\psi\rangle = |0\rangle$. The quantum gate Phase is applied on $|\psi\rangle$. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1
The Phase gate, also known as the S gate, is a single-qubit gate in quantum computing. It introduces a phase shift of $\pi/2$ (or 90 degrees) to the state of the qubit. In the computational basis, the Phase gate is represented by the following matrix:

Phase = [[1, 0],
[0, i]]

where 'i' represents the imaginary unit.
The graphical representation of the circuit for a Phase gate acting on a qubit in the state $|0\rangle$ would be:



After applying the Phase gate to the initial state $|0\rangle$, the final state of the system can be calculated by multiplying the state vector with the Phase gate matrix:

$|\psi\rangle = |0\rangle$

Applying Phase gate:
Phase $|\psi\rangle$ = Phase($|0\rangle$)
= Phase([[1], = [[1],
[0]]) [0]])

= [[1 * 1 + 0 * 0], = [[1],
[1 * 0 + 0 * i]] [0 * 1 + 0 * 0]]

= [[1], = [[1],

[0]] [0]]

= |1⟩

Therefore, the final state of the system after applying the Phase gate on |0⟩ is |1⟩.

Answer 2
The Phase gate is a single-qubit quantum gate that applies a phase shift of 180 degrees to the state |1⟩ of the qubit, while leaving the state |0⟩ unchanged. The gate is represented by the matrix:

Phase = [1 0]
[0 -1]

Graphically, the Phase gate is represented as:

|ψ⟩ --○-- |ψ⟩
↓

where the circle represents the Phase gate.

In this case, the system is in the state |ψ⟩ = |0⟩. Applying the Phase gate to the state |0⟩ yields:

Phase(|0⟩) = [1 0] [1]
[0 -1][0]
= [1]
[0]

So the final state of the system is |ψ⟩ = |0⟩, which is unchanged by the Phase gate. Therefore, the action of the Phase gate on the system is trivial in this case.

Answer 3
The quantum gate "Phase," denoted as S, is a single-qubit gate that introduces a phase shift of π/2 (90 degrees) to the state |1⟩, while leaving the state |0⟩ unchanged. Mathematically, the gate is represented as:

S = [[1, 0], [0, i]]

where "i" represents the imaginary unit.

To describe the action of the Phase gate on the initial state $|\psi\rangle = |0\rangle$, we apply the gate to the state as follows:

$S|0\rangle = [[1, 0], [0, i]]|0\rangle = [[1, 0], [0, i]]|0\rangle = |0\rangle$

Thus, the Phase gate has no effect on the initial state $|0\rangle$.

Now, let's represent the circuit diagram for this system:

$|\psi\rangle$ ——S————

In this circuit diagram, the initial state $|\psi\rangle$ is passed through the Phase gate S.

The final state of the system remains $|\psi\rangle = |0\rangle$ since the Phase gate does not alter the state when applied to $|0\rangle$.