Question

Consider a quantum system made of one qubit. The system is in the initial state |ψ⟩ = |1⟩. The quantum gate Phase is applied on |ψ⟩. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

The Phase gate, also known as the S gate, is a fundamental quantum gate that operates on a single qubit. It introduces a phase shift of π/2 (or 90 degrees) to the state |1⟩ while leaving the state |0⟩ unchanged. Mathematically, it can be represented as:

Phase gate: S = [[1, 0], [0, i]]

where 'i' represents the imaginary unit.

To visualize the circuit representation, we can use a graphical notation called a quantum circuit diagram. In this case, the circuit consists of a single qubit and a Phase gate acting on it. Here is the graphical representation:

|1⟩ ----[S]----

The initial state |ψ⟩ = |1⟩ is represented by the vertical line labeled with '1', and the Phase gate 'S' is applied to it.

To determine the final state of the system, we need to apply the Phase gate to the initial state |ψ⟩ = |1⟩. Mathematically, the action of the Phase gate on |1⟩ can be computed as follows:

S|1⟩ = [[1, 0], [0, i]] * [[0], [1]] = [[0], [i]]

Therefore, the final state of the system after applying the Phase gate is |ψ⟩ = [[0], [i]]. In Dirac notation, this state can be written as:

|ψ⟩ = 0|0⟩ + i|1⟩

In other words, the final state has a coefficient of 0 for the |0⟩ state and a coefficient of 'i' (the imaginary unit) for the |1⟩ state.

Answer 2

The Phase gate, also known as the ϕ gate, is a single-qubit gate in quantum computing that introduces a relative phase between the basis states. In the computational basis, the Phase gate is represented by the following unitary matrix:

where ϕ is the phase angle that the gate introduces.

If the Phase gate is applied to the initial state $|1\rangle$, then the resulting state of the system is:

This means that the Phase gate introduces a relative phase of $e^{i\phi}$ to the $|1\rangle$ state, while leaving the $|0\rangle$ state unchanged.

A graphical representation of the circuit for the Phase gate acting on a single qubit is as follows:

|ψ⟩ --Ph(φ)--⟩

where Ph(φ) denotes the Phase gate with a phase angle of ϕ. The -- symbol represents a wire carrying a single qubit, and the ⟩ symbol represents the state of the qubit.

Therefore, if the Phase gate is applied to the initial state $|1\rangle$, the final state of the system is:

e^(i\phi) |1⟩

which means that the state vector acquires a global phase factor of $e^{i\phi}$, while the probability of measuring the $|1\rangle$ state is unchanged.

Answer 3

The Phase gate, also known as the S gate or the Z90 gate, is a fundamental single-qubit gate in quantum computing. It introduces a phase shift of $\pi/2$ (or 90 degrees) to the qubit state. Mathematically, the Phase gate can be represented as:

Phase Gate (S gate):
S = [[1, 0],
[0, i]],

where 'i' represents the imaginary unit.

When the Phase gate is applied to the initial state $|\psi\rangle = |1\rangle$, it transforms the state as follows:

$S|\psi\rangle = S|1\rangle$ = [[1, 0],
[0, i]] [[0],
[1]]
= [[0],
[i]],

So the final state of the system after applying the Phase gate is $|\psi\rangle$ = [0, i].