Question

Consider a quantum system made of one qubit. The system is in the initial state $|\psi\rangle = |1\rangle$. The quantum gate Pauli Z is applied on $|\psi\rangle$. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

The Pauli Z gate is a single-qubit gate that is represented by the following matrix:

```
Z = |1 0|
    |0 -1|
```

When this gate is applied to the initial state $|1\rangle$, we obtain:

$Z|1\rangle = |0\rangle$

So, the Pauli Z gate changes the state $|1\rangle$ to the state $|0\rangle$. Geometrically, this corresponds to a rotation of π around the Z-axis on the Bloch sphere.

The circuit diagram for this operation is:

```
|1⟩ --Z-- |0⟩
```

where the qubit starts in the state $|1\rangle$ and then the Pauli Z gate is applied, resulting in the final state $|0\rangle$.

On the Bloch sphere, the initial state $|1\rangle$ corresponds to the point on the +Z axis, and the final state $|0\rangle$ corresponds to the point on the -Z axis, as shown below:

```
     |0⟩
      ↑
      |
      |
      |
--+--*--+--
  | /  \ |
```

```
  |/    \|
--+-------+--
    |
    |
    |
    |1⟩
```

Here, the * represents the initial state |1⟩ and the + represents the final state |0⟩. The dashed line represents the equator of the Bloch sphere, which corresponds to the states |+⟩ and |-⟩. The rotation induced by the Pauli Z gate is a reflection across the equator, which takes |1⟩ to |0⟩.

Answer 2

The Pauli Z gate is a single-qubit gate that is represented by the matrix:

```
Z = [1  0]
    [0 -1]
```

When applied to the qubit in the state |1⟩, the Pauli Z gate flips the sign of the state. So the final state of the system after the application of the Pauli Z gate would be:

Z|1⟩ = -|1⟩

Graphically, the circuit would be represented as:

|1⟩ ----Z---- (-|1⟩)

Here, the state |1⟩ enters the Pauli Z gate, which flips the sign of the state, resulting in the final state of -|1⟩.

In terms of the Bloch sphere representation, the initial state |1⟩ corresponds to the north pole of the sphere, while the final state of -|1⟩ corresponds to the south pole of the sphere. The Pauli Z gate is essentially a rotation of the qubit state around the Z-axis of the Bloch sphere by π radians.

Answer 3

The Pauli Z gate, also known as the Z gate or the phase flip gate, is a fundamental quantum gate that operates on a single qubit. It is represented by the matrix:

Z = |1 0|
|0 -1|

When the Pauli Z gate is applied to the initial state $|\psi\rangle = |1\rangle$, it flips the phase of the qubit, leaving the probability amplitudes unchanged. In other words, it changes the sign of the state corresponding to the basis state $|1\rangle$. Mathematically, the action of the Pauli Z gate on the initial state can be described as:

$Z|\psi\rangle = Z|1\rangle = |0\rangle$

Graphically, the circuit representation of the Pauli Z gate applied to a single qubit can be depicted as follows:

$|\psi\rangle$ ---- Z ---- $|0\rangle$

This represents the qubit state $|\psi\rangle$ as an input, followed by the application of the Pauli Z gate (Z), resulting in the final state $|0\rangle$.

Therefore, the final state of the system after the Pauli Z gate is applied to the initial state $|\psi\rangle = |1\rangle$ is $|0\rangle$.