Question

Consider a quantum system made of two qubits. The system is in the initial state $|\psi\rangle = |10\rangle$. A Control Z gate is applied on the system with the first qubit as control qubit and the second qubit as target qubit. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

The Control Z gate, also known as the Controlled-Z gate or CZ gate, is a two-qubit gate that applies a phase shift of -1 to the target qubit if and only if the control qubit is in the state $|1\rangle$. In other words, it flips the sign of the target qubit's state when the control qubit is in the $|1\rangle$ state, and leaves it unchanged when the control qubit is in the $|0\rangle$ state.

Let's apply the Control Z gate to the initial state $|\psi\rangle = |10\rangle$, where the first qubit is the control qubit and the second qubit is the target qubit.

The graphical representation of the circuit would look like this:

```
      ┌───┐
q_0: ┤ X ├──■──
      └───┘  │
q_1: ───────■──h
```

In this circuit representation, the X gate is applied to the control qubit, representing the $|1\rangle$ state for the control qubit. The line connecting the control qubit and the target qubit represents the control relationship. The dot at the intersection indicates that the Control Z gate is applied.

Now, let's calculate the final state of the system after applying the Control Z gate to the initial state $|\psi\rangle = |10\rangle$.

The action of the Control Z gate on the initial state $|\psi\rangle$ can be represented as:

$CZ|10\rangle = CZ(X|0\rangle \otimes |1\rangle) = CZ(|0\rangle \otimes |1\rangle) = |0\rangle \otimes |1\rangle$

Therefore, the final state of the system is $|\psi\rangle = |01\rangle$.

In summary, the action of the Control Z gate on the system with the initial state $|10\rangle$ results in the final state $|01\rangle$.

Answer 2
The Control Z gate is a two-qubit gate that applies a phase shift of -1 to the target qubit if the control qubit is in the state $|1\rangle$. In the case of the initial state $|10\rangle$, the first qubit is in the state $|1\rangle$ and the second qubit is in the state $|0\rangle$. Therefore, applying the Control Z gate to this system will result in a phase shift of -1 to the second qubit, only if the first qubit is in the state $|1\rangle$.

The graphical representation of the circuit for the given scenario is as follows:

```
|1⟩ --@--      --     --
      |      |Z|   = |1⟩
|0⟩ --○--      --|0⟩--
```

In this circuit diagram, the first qubit is represented by the vertical line on the left, and the second qubit is represented by the vertical line on the right. The "@" symbol represents the Control Z gate, with the first qubit as the control qubit and the second qubit as the target qubit. The "○" symbol represents the initial state of the system, which is $|10\rangle$.

Since the first qubit is in the state $|1\rangle$, the Control Z gate will apply a phase shift of -1 to the second qubit. Therefore, the final state of the system will be:

$$|\psi\rangle = -|10\rangle$$

This means that the amplitude of the state $|10\rangle$ is flipped, and the overall state has acquired a global phase of -1.

Answer 3
The Control Z gate, also known as the Controlled-Phase gate, is a two-qubit gate that applies a phase shift of -1 to the target qubit if and only if the control qubit is in the state $|1\rangle$. In this case, the initial state of the system is $|\psi\rangle = |10\rangle$, where the first qubit is the control qubit and the second qubit is the target qubit.

The graphical representation of the circuit using quantum circuit notation is as follows:

```
     ┌───┐
q_0: ┤ X ├──■──
     └───┘  │
q_1: ───────■──
```

Here, the gate X on qubit 0 represents the Pauli-X gate, which flips the qubit from state $|0\rangle$ to $|1\rangle$, and vice versa. The control qubit is the top qubit, represented by qubit 0, and the target qubit is the bottom qubit, represented by qubit 1. The vertical line connecting the two qubits indicates that the Control Z gate is applied.

To determine the final state of the system, we need to apply the gate operation to the initial state $|10\rangle$. The Control Z gate will only affect the target qubit if the control qubit is in the state $|1\rangle$. In this case, the control qubit is in the state $|1\rangle$, and the target qubit is in the state $|0\rangle$.

Applying the Control Z gate, we obtain the following transformation:

$|10\rangle$ -> $-|10\rangle$

The minus sign (-) indicates a phase change of -1. Therefore, the final state of the system after applying the Control Z gate is:

$|\psi\rangle = -|10\rangle$