Question

Consider a quantum system made of one qubit. The system is in the initial state $|\psi\rangle = 1/\sqrt{N}$ ($a|0\rangle + b|1\rangle$), where a and b are complex numbers and N is a normalization factor. A quantum gate Pauli Z is applied on $|\psi\rangle$. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

The Pauli Z gate is a single-qubit gate that acts as a bit-flip operation. It leaves the state $|0\rangle$ unchanged and maps $|1\rangle$ to $-|1\rangle$. Therefore, the action of Pauli Z gate on the initial state $|\psi\rangle$ is:

$Z|\psi\rangle = 1/\sqrt{N} (a|0\rangle - b|1\rangle)$

The graphical representation of the circuit for applying Pauli Z gate on the qubit in state $|\psi\rangle$ is shown below:

$|\psi\rangle$ --Z---> $Z|\psi\rangle$

The final state of the system after applying the Pauli Z gate is $1/\sqrt{N} (a|0\rangle - b|1\rangle)$. This state can also be written as:

$|\psi'\rangle = 1/\sqrt{N} (a|0\rangle + (-b)|1\rangle)$

Thus, the Pauli Z gate flips the sign of the coefficient of the $|1\rangle$ state and leaves the coefficient of the $|0\rangle$ state unchanged.

Answer 2

The Pauli Z gate is a single-qubit gate in quantum computing that flips the sign of the $|1\rangle$ state while leaving the $|0\rangle$ state unchanged. It can be represented by the following matrix:

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

The action of the Pauli Z gate on the initial state $|\psi\rangle$ can be described as follows:

$Z |\psi\rangle = 1/\sqrt{N} (a|0\rangle - b|1\rangle)$

Graphical representation of the circuit:

$|\psi\rangle$ --- Z --- final state

The initial state $|\psi\rangle$ is represented by a qubit wire entering the circuit from the left, the Z gate is applied to the qubit, and the final state is represented by a qubit wire exiting the circuit from the right.

The final state of the system is $1/\sqrt{N} (a|0\rangle - b|1\rangle)$, which is the same as the initial state $|\psi\rangle$ but with the sign of the $|1\rangle$ state flipped. In other words, the Pauli Z gate has the effect of flipping the phase of the $|1\rangle$ state while leaving the $|0\rangle$ state unchanged.

Answer 3

The Pauli Z gate is a fundamental gate in quantum computing that operates on a single qubit. It is represented by the following matrix:

Z = [[1, 0],
 [0, -1]]

When the Pauli Z gate is applied to the initial state $|\psi\rangle = 1/\sqrt{N} (a|0\rangle + b|1\rangle)$, the action of the gate can be described as follows:

$Z|\psi\rangle = 1/\sqrt{N} (a|0\rangle - b|1\rangle)$

In other words, the Pauli Z gate flips the sign of the state $|1\rangle$ while leaving the state $|0\rangle$ unchanged.

As for the graphical representation of the circuit, it is represented by a single qubit and a Pauli Z gate acting on it. Here's an ASCII representation:

$|\psi\rangle$ ---[Z]---

The final state of the system after the Pauli Z gate is applied is given by:

$Z|\psi\rangle = 1/\sqrt{N} (a|0\rangle - b|1\rangle)$

So, the system will be in a superposition of the states $|0\rangle$ and $|1\rangle$, but with the sign of the $|1\rangle$ component flipped. The normalization factor N remains unchanged.