

Questions

Consider a quantum system made of two qubits. The system is in the initial state $|\psi\rangle = |01\rangle$. A CNOT gate is applied on the system with the first qubit as target qubit and the second qubit as control qubit. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

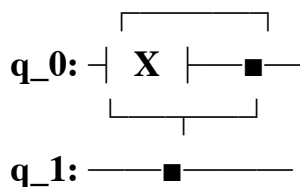
The CNOT (Controlled-NOT) gate is a two-qubit gate in quantum computing. It performs a NOT (or X) operation on the target qubit if and only if the control qubit is in the state $|1\rangle$. If the control qubit is in the state $|0\rangle$, the target qubit remains unchanged.

In your case, the initial state of the system is $|01\rangle$, where the first qubit is the target qubit and the second qubit is the control qubit. Applying the CNOT gate with the first qubit as the target and the second qubit as the control leads to the following action:

If the control qubit (second qubit) is in the state $|0\rangle$, then the target qubit (first qubit) remains unchanged. Therefore, the resulting state is still $|01\rangle$.

If the control qubit is in the state $|1\rangle$, then the target qubit undergoes a NOT operation. Thus, the resulting state is $|11\rangle$.

Graphical representation of the circuit:



In this circuit diagram, the "X" gate represents the NOT operation. The control qubit q₁ is represented by the line coming from the left, and the target qubit q₀ is represented by the line coming from the right. The line connecting the two qubits represents the CNOT gate.

Therefore, in this case, the final state of the system is $|11\rangle$.

Answer 2

The CNOT gate is a two-qubit gate that acts on a target qubit (usually the first qubit) depending on the state of a control qubit (usually the second qubit). It is represented by the following matrix:

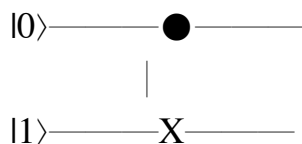
$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

If we apply this gate to the initial state $|01\rangle$, the second qubit is the control qubit, and the first qubit is the target qubit. The action of the gate is then to leave the state of the second qubit unchanged and to apply a NOT gate to the first qubit if the second qubit is in state $|1\rangle$. So, we have:

$$\text{CNOT}(|01\rangle) = \text{CNOT}(|0\rangle \otimes |1\rangle) = |0\rangle \otimes \text{CNOT}(|1\rangle) = |0\rangle \otimes |1 \oplus 1\rangle = |00\rangle$$

where \otimes denotes the tensor product and \oplus denotes addition modulo 2.

So, the final state of the system after the CNOT gate is applied is $|00\rangle$. Here's a graphical representation of the circuit:



where the circle represents the CNOT gate, the vertical line on the left is the first qubit (target qubit) in state $|0\rangle$, and the vertical line on the right is the second qubit (control qubit) in state $|1\rangle$. The output is a vertical line on the left in state $|0\rangle$ and a vertical line on the right in state $|0\rangle$, representing the final state $|00\rangle$.

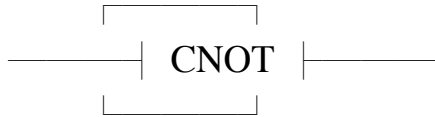
Answer 3

Certainly! In a quantum system with two qubits, the CNOT gate, also known as the Controlled-NOT gate, is a two-qubit gate that performs an operation on the target qubit based on the state of the control qubit.

In this case, the CNOT gate is applied with the first qubit as the target qubit

(denoted as T) and the second qubit as the control qubit (denoted as C). The initial state of the system is $|\psi\rangle = |01\rangle$, which means the first qubit is in the state $|0\rangle$ and the second qubit is in the state $|1\rangle$.

The action of the CNOT gate can be represented using a quantum circuit diagram. Here's the graphical representation of the circuit:



In this diagram, the control qubit (C) is represented by the line on top, and the target qubit (T) is represented by the line at the bottom. The CNOT gate is applied on these qubits.

Now, let's analyze the action of the CNOT gate on the initial state $|\psi\rangle = |01\rangle$:

The control qubit (C) is $|0\rangle$, and the target qubit (T) is $|1\rangle$.

- 1 The CNOT gate checks the state of the control qubit. If it is $|0\rangle$, it leaves the target qubit unchanged. If it is $|1\rangle$, it flips the state of the target qubit.
- 2 In this case, since the control qubit (C) is $|0\rangle$ and the target qubit (T) is $|1\rangle$, the CNOT gate does not flip the state of the target qubit.
- 3 Therefore, the final state of the system remains the same: $|\psi\rangle = |01\rangle$.

So, after applying the CNOT gate, the final state of the system remains $|01\rangle$.