Question

Consider a quantum system made of one qubit. The system is in the initial state $|\psi\rangle = |1\rangle$ and the quantum gate Identity is applied. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

The Identity gate, also known as the Identity matrix, is a quantum gate that leaves the state of the qubit unchanged. Mathematically, the Identity matrix is defined as:

$I = |0\rangle\langle 0| + |1\rangle\langle 1|$

where $|0\rangle$ and $|1\rangle$ are the computational basis states of the qubit.

When the Identity gate is applied to the state $|1\rangle$, the resulting state is:

$I\,|1\rangle = (|0\rangle\langle 0| + |1\rangle\langle 1|)\,|1\rangle = |0\rangle\langle 1|\,|1\rangle + |1\rangle\langle 1|\,|1\rangle = |0\rangle\,(\langle 1|1\rangle) + |1\rangle\,(\langle 1|1\rangle) = |1\rangle$

In other words, the Identity gate leaves the state $|1\rangle$ unchanged.

Graphically, the circuit for this operation can be represented as:

$|1\rangle$ --I-- $|1\rangle$

where the qubit is initialized in the state $|1\rangle$, the Identity gate I is applied, and the final state is also $|1\rangle$.

So, the final state of the system is still $|1\rangle$.

Answer 2

The Identity gate in quantum computing is a gate that leaves the state of the qubit unchanged. In other words, when the Identity gate is applied to a qubit, it does nothing to the qubit. Mathematically, the Identity gate is represented by the 2x2 identity matrix:

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I = [[1, 0],
     [0, 1]]
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Graphically, the circuit for the Identity gate is simply a wire that connects the input qubit to the output qubit without any additional gates or operations.

|1⟩ --[I]--|1⟩

Since the Identity gate does not change the state of the qubit, the final state of the system will be the same as the initial state:

|ψ⟩ = |1⟩

Answer 3

The Identity gate, often denoted as I or sometimes as the "do nothing" gate, is a quantum gate that leaves the state of a qubit unchanged. When applied to a qubit, it has no effect on the qubit's state. In other words, if the system is initially in the state |1⟩ and the Identity gate is applied, the final state remains |1⟩.

Graphical representation of the circuit:

|1⟩ -- I -- |1⟩

In this circuit representation, the initial state |1⟩ is input to the Identity gate (represented by the symbol I), and the resulting state is also |1⟩.

To summarize:

Initial state: |1⟩
Quantum gate applied: Identity (I)
Final state: |1⟩