

Question

Consider a quantum system made of two qubits. The system is in the initial state $|\psi\rangle = |01\rangle$. A swap gate is applied on the system. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

Certainly! A swap gate is a unitary operation that exchanges the states of two qubits. In the case of a two-qubit system, the swap gate swaps the states of the two qubits.

The initial state of the system is $|\psi\rangle = |01\rangle$, which means the first qubit is in the state $|0\rangle$ and the second qubit is in the state $|1\rangle$.

The action of the swap gate can be represented by the following circuit:

```
--Swap--  
|  |  
Q0---Q1
```

The circuit diagram shows two qubits, Q0 and Q1, with the swap gate applied between them.

To understand the effect of the swap gate on the system, we can apply the gate to the initial state.

$$\begin{aligned}\text{Swap}(|\psi\rangle) &= \text{Swap}(|01\rangle) \\ &= |10\rangle\end{aligned}$$

After applying the swap gate, the final state of the system is $|10\rangle$, which means the first qubit is now in the state $|1\rangle$, and the second qubit is in the state $|0\rangle$.

So, the final state of the system after the swap gate is $|10\rangle$.

Answer 2

The swap gate exchanges the states of two qubits. In this case, the swap gate applied to the two-qubit system in the initial state $|01\rangle$ would exchange the states of the qubits, resulting in the final state $|10\rangle$.

The graphical representation of the circuit implementing this operation would look like:

$$\begin{array}{c} |0\rangle \text{---} [\text{SWAP}] \text{---} |1\rangle \\ |1\rangle \text{---} [\text{SWAP}] \text{---} |0\rangle \end{array}$$

where SWAP is the two-qubit swap gate.

To apply this gate to the initial state $|01\rangle$, we can represent the state as a tensor product of the states of each qubit:

$$|01\rangle = |0\rangle \otimes |1\rangle$$

Then, the swap gate applied to this state is given by:

$$\text{SWAP}(|0\rangle \otimes |1\rangle) = |1\rangle \otimes |0\rangle = |10\rangle$$

So the final state of the system after applying the swap gate is $|10\rangle$.

Answer 3

The swap gate, also known as the SWAP or CNOT gate, is a two-qubit gate that exchanges the states of two qubits. In the computational basis, the action of the swap gate is as follows:

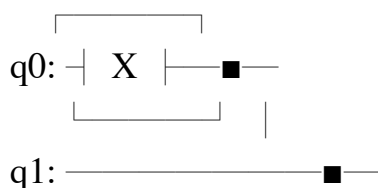
$$|00\rangle \rightarrow |00\rangle$$

$$|01\rangle \rightarrow |10\rangle$$

$$|10\rangle \rightarrow |01\rangle$$

$$|11\rangle \rightarrow |11\rangle$$

Given that the initial state of the system is $|\psi\rangle = |01\rangle$, we can represent the circuit diagrammatically as follows:



In this representation, q_0 and q_1 represent the two qubits. The X gate on q_0 represents the initial state $|\psi\rangle$, where the first qubit is in the state $|0\rangle$ and the second qubit is in the state $|1\rangle$. The swap gate is applied to the two qubits, and the final state of the system can be obtained by applying the corresponding matrix multiplication. In this case, the final state can be calculated as:

$$\text{SWAP}(|01\rangle) = \text{SWAP}(|0\rangle \otimes |1\rangle) = \text{SWAP}(|0\rangle) \otimes \text{SWAP}(|1\rangle) = |1\rangle \otimes |0\rangle = |10\rangle$$

Therefore, the final state of the system after applying the swap gate is $|10\rangle$.