

### Question

Consider a quantum system made of one qubit. The system is in the initial state  $|\psi\rangle = |1\rangle$ . The quantum gate Pauli Z is applied on  $|\psi\rangle$ . Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

### Answer 1

The Pauli Z gate is a single-qubit gate that is represented by the following matrix:

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

When this gate is applied to the initial state  $|1\rangle$ , we obtain:

$$Z|1\rangle = |0\rangle$$

So, the Pauli Z gate changes the state  $|1\rangle$  to the state  $|0\rangle$ . Geometrically, this corresponds to a rotation of  $\pi$  around the Z-axis on the Bloch sphere.

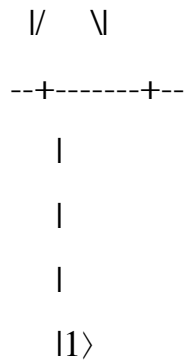
The circuit diagram for this operation is:

$$|1\rangle \xrightarrow{Z} |0\rangle$$

where the qubit starts in the state  $|1\rangle$  and then the Pauli Z gate is applied, resulting in the final state  $|0\rangle$ .

On the Bloch sphere, the initial state  $|1\rangle$  corresponds to the point on the +Z axis, and the final state  $|0\rangle$  corresponds to the point on the -Z axis, as shown below:

$$\begin{array}{c} |0\rangle \\ \uparrow \\ | \\ | \\ | \\ \hline \text{---+---*---+---} \\ | / \quad \backslash | \end{array}$$



Here, the \* represents the initial state  $|1\rangle$  and the + represents the final state  $|0\rangle$ . The dashed line represents the equator of the Bloch sphere, which corresponds to the states  $|+\rangle$  and  $|-\rangle$ . The rotation induced by the Pauli Z gate is a reflection across the equator, which takes  $|1\rangle$  to  $|0\rangle$ .

## Answer 2

The Pauli Z gate is a single-qubit gate that is represented by the matrix:

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

When applied to the qubit in the state  $|1\rangle$ , the Pauli Z gate flips the sign of the state. So the final state of the system after the application of the Pauli Z gate would be:

$$Z|1\rangle = -|1\rangle$$

Graphically, the circuit would be represented as:

$$|1\rangle \text{ ---- } Z \text{ ---- } (-|1\rangle)$$

Here, the state  $|1\rangle$  enters the Pauli Z gate, which flips the sign of the state, resulting in the final state of  $-|1\rangle$ .

In terms of the Bloch sphere representation, the initial state  $|1\rangle$  corresponds to the north pole of the sphere, while the final state of  $-|1\rangle$  corresponds to the south pole of the sphere. The Pauli Z gate is essentially a rotation of the qubit state around the Z-axis of the Bloch sphere by  $\pi$  radians.

## Answer 3

The Pauli Z gate, also known as the Z gate or the phase flip gate, is a fundamental quantum gate that operates on a single qubit. It is represented by the matrix:

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

When the Pauli Z gate is applied to the initial state  $|\psi\rangle = |1\rangle$ , it flips the phase of the qubit, leaving the probability amplitudes unchanged. In other words, it changes the sign of the state corresponding to the basis state  $|1\rangle$ .

Mathematically, the action of the Pauli Z gate on the initial state can be described as:

$$Z|\psi\rangle = Z|1\rangle = |0\rangle$$

Graphically, the circuit representation of the Pauli Z gate applied to a single qubit can be depicted as follows:

$$|\psi\rangle \text{ ---- } Z \text{ ---- } |0\rangle$$

This represents the qubit state  $|\psi\rangle$  as an input, followed by the application of the Pauli Z gate (Z), resulting in the final state  $|0\rangle$ .

Therefore, the final state of the system after the Pauli Z gate is applied to the initial state  $|\psi\rangle = |1\rangle$  is  $|0\rangle$ .