Question

Consider a quantum system made of two qubits. The system is in the initial state |ψ⟩ = |01⟩. A CNOT gate is applied on the system with the first qubit as control qubit and the second qubit as target qubit. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

In the given quantum system, we have two qubits initially in the state |ψ⟩ = |01⟩. The state |01⟩ represents the first qubit being in the state |0⟩ and the second qubit being in the state |1⟩.

The CNOT gate (Controlled-NOT gate) is a two-qubit gate that acts on two qubits, one acting as the control qubit and the other as the target qubit. It applies a NOT gate (bit flip) operation on the target qubit if and only if the control qubit is in the state |1⟩. Otherwise, it leaves the target qubit unchanged.

In our case, the CNOT gate is applied with the first qubit (qubit 1) as the control qubit and the second qubit (qubit 2) as the target qubit. Since the control qubit is in the state |0⟩ and the target qubit is in the state |1⟩, the action of the CNOT gate is as follows:

If the control qubit (qubit 1) is |0⟩, the target qubit (qubit 2) remains unchanged.
If the control qubit (qubit 1) is |1⟩, the target qubit (qubit 2) undergoes a bit flip, changing from |1⟩ to |0⟩.

The graphical representation of this circuit can be depicted as follows:

```
|ψ⟩ --[CNOT]--
   |
   |
   v
|ψ'⟩
```

where the CNOT gate is represented by the label [CNOT].

Applying the CNOT gate to the initial state |ψ⟩ = |01⟩, we have the following transformation:

|01⟩ --[CNOT]--> |01⟩

Since the control qubit is |0⟩, the target qubit remains unchanged. Therefore, the final state of the system is |01⟩, which is the same as the initial state.

Answer 2
The CNOT (Controlled-NOT) gate is a two-qubit gate that acts on a control qubit and a target qubit. If the control qubit is in the state |1⟩, the CNOT gate applies a NOT gate to the target qubit; otherwise, it leaves the target qubit unchanged. The CNOT gate can be represented as follows:

|00⟩ → |00⟩
|01⟩ → |01⟩
|10⟩ → |11⟩
|11⟩ → |10⟩

In this case, the first qubit is the control qubit and the second qubit is the target qubit. The initial state of the system is |ψ⟩ = |01⟩, which means that the first qubit is in the state |0⟩ and the second qubit is in the state |1⟩. Applying the CNOT gate to this system results in the following:

|01⟩ → |11⟩

So the final state of the system is |11⟩, which means that the first qubit is still in the state |0⟩ and the second qubit has been flipped from |1⟩ to |0⟩. We can represent this circuit using the following graphical notation:

Copy code

```
  ┌─────────┐
──┤ 0 ├──
  │   │
──┤ 1 ├──
  └─────────┘
```

This represents the two qubits in the circuit, with the control qubit labeled as qubit 0 and the target qubit labeled as qubit 1. The CNOT gate is applied between these two qubits, with the control qubit as the control and the target qubit as the target.

Answer 3
The CNOT gate, also known as the controlled-NOT gate, is a two-qubit gate

commonly used in quantum computing. It performs an operation on the target qubit conditioned on the state of the control qubit. In this case, the CNOT gate is applied with the first qubit as the control qubit and the second qubit as the target qubit.

The initial state of the system is $|\psi\rangle = |01\rangle$. Let's break down the state notation: the first qubit is in the state $|0\rangle$ (0 eigenstate) and the second qubit is in the state $|1\rangle$ (1 eigenstate).

The CNOT gate applies the NOT operation (X gate) to the target qubit if and only if the control qubit is in the state $|1\rangle$. The action of the CNOT gate can be described as follows:

- If the control qubit is in the state $|0\rangle$, there is no change to the system. The target qubit remains unchanged.
- If the control qubit is in the state $|1\rangle$, the target qubit undergoes a NOT operation (bit-flip) and changes its state.

Now, let's represent the circuit graphically:

```
---•---
   |
---X---
```

In the above circuit diagram, the first qubit (control qubit) is represented by a line with a dot, and the second qubit (target qubit) is represented by a line with an X. The CNOT gate is represented by a cross ('X') between the two lines.

In our case, the control qubit is in the state $|0\rangle$ and the target qubit is in the state $|1\rangle$. Applying the CNOT gate, since the control qubit is $|0\rangle$, there is no change to the system. Therefore, the final state of the system remains $|01\rangle$.