

### Question

Consider a quantum system made of one qubit. The system is in the initial state  $|\psi\rangle = |0\rangle$ . The quantum gate Hadamard is applied on  $|\psi\rangle$ . Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

### Answer 1

Yes, I can describe the action of the Hadamard gate on a qubit in the initial state  $|0\rangle$ . The Hadamard gate is represented by the matrix:

$$H = \frac{1}{\sqrt{2}} * \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

When this gate is applied to the initial state  $|0\rangle$ , we obtain:

$$\begin{aligned} H|0\rangle &= \frac{1}{\sqrt{2}} * \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} * \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &= \frac{1}{\sqrt{2}} * \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{aligned}$$

So the final state of the system is  $|\psi\rangle = \frac{1}{\sqrt{2}} * |0\rangle + \frac{1}{\sqrt{2}} * |1\rangle$ , which is a superposition of the  $|0\rangle$  and  $|1\rangle$  states.

The graphical representation of the circuit for applying the Hadamard gate to a qubit in the initial state  $|0\rangle$  is:

$$|0\rangle \xrightarrow{H} \frac{1}{\sqrt{2}} * (|0\rangle + |1\rangle)$$

This represents the qubit in the initial state  $|0\rangle$  being input to the Hadamard gate, resulting in the final state  $\frac{1}{\sqrt{2}} * (|0\rangle + |1\rangle)$ .

### Answer 2

Yes, I can certainly describe the action of the Hadamard gate on a single qubit quantum system.

The Hadamard gate is a quantum gate that operates on a single qubit. It transforms the qubit from the computational basis state to a superposition state that is equally weighted between the  $|0\rangle$  and  $|1\rangle$  basis states.

Mathematically, the Hadamard gate is represented by the following unitary matrix:

$$H = 1/\sqrt{2} * \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Geometrically, the Hadamard gate can be represented as a rotation of the Bloch sphere by 180 degrees around the X-axis followed by a rotation of the Bloch sphere by 90 degrees around the Y-axis.

The circuit diagram for applying the Hadamard gate to a qubit is shown below:

$$|0\rangle \xrightarrow{H} 1/\sqrt{2} * (|0\rangle + |1\rangle)$$

So, when the Hadamard gate is applied to the initial state  $|\psi\rangle = |0\rangle$ , the resulting state of the system is:

$$H|0\rangle = 1/\sqrt{2} * (|0\rangle + |1\rangle)$$

Therefore, the final state of the system is a superposition state that is equally weighted between the  $|0\rangle$  and  $|1\rangle$  basis states.

### Answer 3

The Hadamard gate, often denoted as H, is a fundamental quantum gate that acts on a single qubit. It transforms the basis states  $|0\rangle$  and  $|1\rangle$  into superposition states.

When the Hadamard gate is applied to the initial state  $|\psi\rangle = |0\rangle$ , the action of the gate can be represented as follows:

$$H(|\psi\rangle) = H(|0\rangle)$$

The Hadamard gate can be represented by the following matrix:

$$H = 1/\sqrt{2} * \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Applying the Hadamard gate to the  $|0\rangle$  state, we get:

$$H(|0\rangle) = \frac{1}{\sqrt{2}} * \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} * \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} * \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

The final state of the system after applying the Hadamard gate is:

$$|\psi\rangle = \frac{1}{\sqrt{2}} * (|0\rangle + |1\rangle)$$

Graphical representation of the circuit:

$$|0\rangle \text{ ---- } H \text{ ---- } |\psi\rangle$$

In this representation, the input qubit  $|0\rangle$  is transformed by the Hadamard gate (H), resulting in the final state  $|\psi\rangle$ .

The final state of the system, after applying the Hadamard gate, is given by:

$$|\psi\rangle = \frac{1}{\sqrt{2}} * (|0\rangle + |1\rangle)$$

This represents a superposition of the basis states  $|0\rangle$  and  $|1\rangle$  with equal amplitudes, which corresponds to a qubit in an equal probability distribution between the two states.