**Question**

Consider a quantum system made of one qubit. The system is in the initial state $|\psi\rangle = 1/\sqrt{N}\ ( a|0\rangle + b|1\rangle )$, where a and b are complex numbers and N is a normalization factor. A quantum gate Pauli X is applied on $|\psi\rangle$. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

**Answer 1**

The Pauli X gate is a quantum gate that flips the state of a qubit, also known as a NOT gate. It maps the state $|0\rangle$ to $|1\rangle$ and the state $|1\rangle$ to $|0\rangle$. The matrix representation of the Pauli X gate is:

| 0 1 |

X = X =| 1 0 |

The action of the Pauli X gate on the initial state $|\psi\rangle$ is given by:

$X|\psi\rangle = X(1/\sqrt{N}\ (a|0\rangle + b|1\rangle)) = 1/\sqrt{N}\ (b|0\rangle + a|1\rangle)$

In other words, the Pauli X gate swaps the amplitudes of the states $|0\rangle$ and $|1\rangle$.

The graphical representation of the circuit is:

$|\psi\rangle$ -- X -- final state

The initial state $|\psi\rangle$ is fed into the Pauli X gate, and the output is the final state of the system.

The final state of the system after applying the Pauli X gate is:

$X|\psi\rangle = 1/\sqrt{N}\ (b|0\rangle + a|1\rangle)$

The amplitudes of the states $|0\rangle$ and $|1\rangle$ have been swapped, so the system is now in a superposition of $|0\rangle$ and $|1\rangle$ with different coefficients.

**Answer 2**

The Pauli X gate is a basic quantum gate that acts on a single qubit and is also called a NOT gate. It is represented by the matrix

X = [0 1]

[1 0]
When the Pauli X gate is applied to the initial state $|\psi\rangle$, the resulting state is given by:

$X|\psi\rangle = 1/\sqrt{N} \, ( a|1\rangle + b|0\rangle )$

This operation flips the state of the qubit from $|0\rangle$ to $|1\rangle$ and vice versa. Geometrically, the Pauli X gate corresponds to a rotation of the qubit state vector around the X-axis of the Bloch sphere by $\pi$ radians.

The circuit diagram for applying the Pauli X gate to the initial state $|\psi\rangle$ is shown below:

$|\psi\rangle$ --X---> $X|\psi\rangle$

The final state of the system after the Pauli X gate is applied is:

$X|\psi\rangle = 1/\sqrt{N} \, ( a|1\rangle + b|0\rangle )$

This state is obtained by swapping the amplitudes of the $|0\rangle$ and $|1\rangle$ basis states in the initial state.

**Answer 3**

The Pauli X gate, also known as the "bit-flip" gate, is a single-qubit gate that acts as a NOT gate. It flips the state of a qubit, transforming $|0\rangle$ to $|1\rangle$ and vice versa, while preserving the coefficients. In the case of the initial state $|\psi\rangle = 1/\sqrt{N} (a|0\rangle + b|1\rangle)$, applying the Pauli X gate will yield the following result:

Pauli $X|\psi\rangle = 1/\sqrt{N} (a|1\rangle + b|0\rangle)$

In other words, the Pauli X gate interchanges the coefficients a and b, while flipping the state of the qubit. Geometrically, you can represent this operation as a rotation of the Bloch sphere by $\pi$ radians around the x-axis.

To depict the circuit diagram, we can represent the initial state $|\psi\rangle$ as an input qubit, apply the Pauli X gate, and show the final state as the output qubit. The circuit diagram looks like this:

$|\psi\rangle$ ----[X]---- Pauli X ---- $|\psi'\rangle$

Where $|\psi\rangle$ represents the initial state, [X] represents the Pauli X gate, and $|\psi'\rangle$ represents the final state after the gate is applied.

As for the final state of the system, after applying the Pauli X gate, the state becomes $|\psi'\rangle = 1/\sqrt{N} (a|1\rangle + b|0\rangle)$. The coefficients a and b are swapped compared to the initial state, and the qubit is flipped.