

Question

Consider a quantum system made of one qubit. The system is in the initial state $|\psi\rangle = 1/\sqrt{N} (a|0\rangle + b|1\rangle)$, where a and b are complex numbers and N is a normalization factor. A quantum gate Hadamard is applied on $|\psi\rangle$. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

Sure! The Hadamard gate, denoted as H , is a fundamental quantum gate that acts on a single qubit. The action of H on a qubit is described by the following matrix:

$$H = 1/\sqrt{2} * \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

When this gate is applied to the initial state $|\psi\rangle = 1/\sqrt{N} (a|0\rangle + b|1\rangle)$, we get the following:

$$\begin{aligned} H|\psi\rangle &= 1/\sqrt{2} * \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} * \begin{bmatrix} 1/\sqrt{N} * a \\ 1/\sqrt{N} * b \end{bmatrix} \\ &= 1/\sqrt{2} * \begin{bmatrix} 1/\sqrt{N} * a + 1/\sqrt{N} * b \\ 1/\sqrt{N} * a - 1/\sqrt{N} * b \end{bmatrix} \\ &= 1/\sqrt{N} * \begin{bmatrix} (a + b) / \sqrt{2} \\ (a - b) / \sqrt{2} \end{bmatrix} \end{aligned}$$

So, the final state of the system after applying the Hadamard gate is:

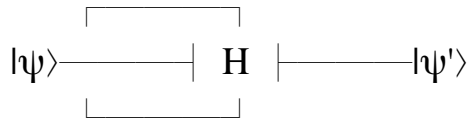
$$|\psi'\rangle = 1/\sqrt{N} * \begin{bmatrix} (a + b) / \sqrt{2} \\ (a - b) / \sqrt{2} \end{bmatrix} |0\rangle + 1/\sqrt{N} * \begin{bmatrix} (a - b) / \sqrt{2} \\ (a + b) / \sqrt{2} \end{bmatrix} |1\rangle$$

The circuit diagram of this operation is as follows:

$$|\psi\rangle \text{ ----H----> } |\psi'\rangle$$

where the qubit is initialized in the state $|\psi\rangle$, the Hadamard gate H is applied, and the resulting state is $|\psi'\rangle$.

Graphically, the circuit can be represented as follows:



I hope this helps!

Answer 2

Sure! The Hadamard gate is a fundamental gate in quantum computing, and it acts on a single qubit. The action of the Hadamard gate on the qubit state $|\psi\rangle$ is described by the following matrix:

$$H = \frac{1}{\sqrt{2}} * \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

If we apply the Hadamard gate to the initial state $|\psi\rangle$, we get:

$$H|\psi\rangle = \frac{1}{\sqrt{2}} * \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} * \frac{1}{\sqrt{N}} * \begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{\sqrt{2N}} * \begin{bmatrix} a+b \\ a-b \end{bmatrix}$$

So the final state of the system after applying the Hadamard gate is:

$$|\psi'\rangle = H|\psi\rangle = \frac{1}{\sqrt{2N}} * (a|0\rangle + b|1\rangle + a|1\rangle - b|0\rangle)$$

We can represent the circuit of this quantum operation graphically using the following diagram:

$$|\psi\rangle \text{ ---H--- } |\psi'\rangle$$

In this diagram, the qubit state $|\psi\rangle$ is input to the Hadamard gate, which produces the output state $|\psi'\rangle$.

As we can see from the final state expression, the Hadamard gate puts the qubit in a superposition of the $|0\rangle$ and $|1\rangle$ states, with equal probability amplitudes. In other words, it creates a state that is halfway between $|0\rangle$ and $|1\rangle$. This state is often denoted as $|+\rangle$, and it can be written as:

$$|+\rangle = \frac{1}{\sqrt{2}} * (|0\rangle + |1\rangle)$$

Therefore, the final state of the system after applying the Hadamard gate can be written as:

$$|\psi'\rangle = 1/\sqrt{2N} * (a|0\rangle + b|1\rangle + a|1\rangle - b|0\rangle) = 1/\sqrt{2} * (a+b)|+\rangle + 1/\sqrt{2} * (a-b)|-\rangle$$

where $|+\rangle$ and $|-\rangle$ are the eigenstates of the Hadamard gate.

Answer 3

The Hadamard gate is a fundamental quantum gate that acts on a single qubit. It transforms the computational basis states $|0\rangle$ and $|1\rangle$ into superposition states.

The action of the Hadamard gate on the initial state $|\psi\rangle = 1/\sqrt{N} (a|0\rangle + b|1\rangle)$ can be described as follows:

$$H(|\psi\rangle) = 1/\sqrt{2} (a|0\rangle + b|1\rangle) + 1/\sqrt{2} (a|1\rangle - b|0\rangle)$$

In other words, the Hadamard gate applies a rotation to the qubit's state, creating a superposition of both $|0\rangle$ and $|1\rangle$ with specific amplitudes.

A graphical representation of the circuit would look like this:

$|\psi\rangle$ ---[H]---

Here, the qubit state $|\psi\rangle$ enters the Hadamard gate [H] and undergoes the transformation described above.

The final state of the system after applying the Hadamard gate is given by:

$$1/\sqrt{2} (a|0\rangle + b|1\rangle) + 1/\sqrt{2} (a|1\rangle - b|0\rangle)$$

This can be further simplified as:

$$1/\sqrt{2} [(a + b)|0\rangle + (a - b)|1\rangle]$$

So, the final state of the system is a superposition of $|0\rangle$ and $|1\rangle$ with specific amplitudes, $(a + b)/\sqrt{2}$ and $(a - b)/\sqrt{2}$, respectively.