Question

Consider a quantum system made of one qubit. The system is in the initial state |ψ⟩ = |0⟩. The quantum gate Pauli Z is applied on |ψ⟩. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

The Pauli Z gate is a one-qubit gate that corresponds to a rotation of the qubit state around the z-axis of the Bloch sphere by π radians. Mathematically, the Pauli Z gate is represented by the following matrix:

Z = [1 0]

[0 -1]

Geometrically, the Pauli Z gate maps the qubit state |0⟩ to the state |0⟩ and the qubit state |1⟩ to the state -|1⟩ (where the minus sign indicates a global phase factor that does not affect any measurements).

The circuit diagram for applying the Pauli Z gate to a qubit in the state |0⟩ is shown below:

|0⟩ --Z---> |0⟩

The final state of the system after applying the Pauli Z gate is |1⟩, since the Pauli Z gate maps |0⟩ to |0⟩ and |1⟩ to -|1⟩. Therefore, applying the Pauli Z gate to the qubit in the initial state |0⟩ results in the final state |1⟩.

Answer 2

The Pauli Z gate is a quantum gate that acts on a single qubit. Its matrix representation is:

Z = [[1, 0],
[0, -1]]

When applied to the initial state |0⟩, the Pauli Z gate will transform the state as follows:

Z|0⟩ = [[1, 0],
[0, -1]]|0⟩

= [[1*0 + 0*0],
  [0*0 + (-1)*1]]
= [[0],
  [-1]]
= -|1⟩

So the final state of the system will be the state |1⟩, up to a global phase factor of -1.

In terms of a circuit diagram, the Pauli Z gate can be represented as a box with a "Z" inside, applied to a single qubit. Here is the circuit diagram for the gate applied to the initial state |0⟩:

|0⟩ ----Z---- (-1)|1⟩

Note that the final state has a phase factor of -1, which does not affect the measurement probabilities of any observable. Therefore, the measurement probabilities for measuring the qubit in the state |0⟩ or |1⟩ will be the same before and after the application of the Pauli Z gate.

Answer 3
The Pauli Z gate, also known as the Z gate, is a quantum gate that acts on a single qubit. It is defined by the following matrix:

Z = [1 0]
  [0 -1]

When the Pauli Z gate is applied to a qubit, it performs a phase flip, changing the sign of the |1⟩ state and leaving the |0⟩ state unchanged. Mathematically, the action of the Pauli Z gate on a qubit state |ψ⟩ is given by:

Z|ψ⟩ = [1 0] [a]
    [0 -1] [b]
  = [a]
    [-b]

So, if we apply the Pauli Z gate to the initial state |ψ⟩ = |0⟩, we have:

$$Z|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Graphically, we can represent the circuit as follows:

$$Z|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

The final state of the system, after applying the Pauli Z gate, is $|\psi\rangle = [1\ 0]$, which corresponds to the state $|0\rangle$. Thus, the Pauli Z gate does not change the initial state when applied to the qubit in the state $|0\rangle$.