

Question

Consider a quantum system made of two qubits. The system is in the initial state $|\psi\rangle = |11\rangle$. A CNOT gate is applied on the system with the first qubit as control qubit and the second qubit as target qubit. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

The CNOT gate, also known as the Controlled-NOT gate, is a fundamental gate in quantum computing. It operates on two qubits: a control qubit and a target qubit. The gate flips the state of the target qubit if and only if the control qubit is in the state $|1\rangle$.

In this case, the initial state of the system is $|\psi\rangle = |11\rangle$, where the first qubit is the control qubit and the second qubit is the target qubit. Applying the CNOT gate with the first qubit as the control and the second qubit as the target, the action of the gate can be described as follows:

If the control qubit is in the state $|0\rangle$, there is no change in the target qubit. If the control qubit is in the state $|1\rangle$, the target qubit undergoes a bit flip, changing from $|0\rangle$ to $|1\rangle$ or from $|1\rangle$ to $|0\rangle$.

Graphical representation of the circuit:

--CNOT--
| |
[Q₀]-[Q₁]

Where [Q₀] represents the first qubit (control) and [Q₁] represents the second qubit (target). The line representing the CNOT gate connects the control qubit to the target qubit.

To determine the final state of the system, let's apply the CNOT gate to the initial state $|\psi\rangle = |11\rangle$:

- 1 The control qubit is $|1\rangle$ and the target qubit is $|1\rangle$.
- 2 Since the control qubit is in the state $|1\rangle$, the target qubit undergoes a bit flip.
- 3 The final state of the system after applying the CNOT gate is $|\psi\rangle = |10\rangle$.

Therefore, the final state of the system is $|10\rangle$, where the first qubit is $|1\rangle$ (control) and the second qubit is $|0\rangle$ (target).

Answer 2

Yes, I can describe the action of the CNOT gate on this two-qubit system and provide a graphical representation of the circuit.

The CNOT (Controlled-NOT) gate is a two-qubit gate where one qubit, called the control qubit, controls the operation of the gate on the other qubit, called the target qubit. The gate applies a NOT gate (also known as a Pauli-X gate) to the target qubit only when the control qubit is in the state $|1\rangle$, and leaves the target qubit unchanged when the control qubit is in the state $|0\rangle$.

In this case, the initial state of the system is $|\psi\rangle = |11\rangle$, where the first qubit is the control qubit and the second qubit is the target qubit. Applying the CNOT gate with the first qubit as the control qubit and the second qubit as the target qubit results in the following transformation:

$$\text{CNOT gate: } |c\rangle|t\rangle \rightarrow |c\rangle|c \oplus t\rangle$$

where \oplus denotes the modulo-2 sum (XOR) of the two qubits. In this case, the transformation can be written as:

$$|11\rangle \rightarrow |11 \oplus 1\rangle = |10\rangle$$

Therefore, the final state of the system after applying the CNOT gate is $|10\rangle$.

Here is a graphical representation of the circuit:

$$\begin{array}{c} |1\rangle \text{--} @ \text{--} | \\ | \\ |1\rangle \text{--} X \text{--} | \end{array} = |1\rangle \otimes (X) |1\rangle \text{--} |1\rangle \otimes (X) |0\rangle \text{--} |1\rangle \otimes |0\rangle = |10\rangle$$

In this representation, the qubit on the left is the control qubit, and the qubit on the right is the target qubit. The symbol "@" represents the CNOT gate. The vertical line indicates that the qubits are in an entangled state after the application of the CNOT gate.

Answer 3

Certainly! A CNOT gate, or Controlled-NOT gate, is a two-qubit gate commonly used in quantum computing. It applies a Pauli-X gate (also known as a bit-flip gate) on the target qubit if and only if the control qubit is in the state $|1\rangle$. Otherwise, it leaves the target qubit unchanged.

In your case, the initial state of the system is $|\psi\rangle = |11\rangle$, where the first qubit is the control qubit (C) and the second qubit is the target qubit (T). Applying the CNOT gate to this system will result in the following transformation:

CNOT gate (C as control, T as target):

$$|11\rangle \rightarrow |11 \oplus 1\rangle = |10\rangle,$$

where the symbol \oplus denotes the XOR operation. The control qubit remains unchanged, and the target qubit is flipped because the control qubit was in the state $|1\rangle$.

Here is a graphical representation of the circuit:

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--C--  
  |  
--T--
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The "C" above the top wire represents the control qubit, and the "T" below the bottom wire represents the target qubit. The line connecting them represents the CNOT gate.

So, after applying the CNOT gate to the initial state $|11\rangle$, the final state of the system is $|10\rangle$.