

Question

Consider a quantum system made of two qubits. The system is in the initial state $|\psi\rangle = |10\rangle$. A Swap gate is applied on the system. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

Certainly! In a quantum system composed of two qubits, applying a Swap gate exchanges the states of the two qubits. Let's analyze the action of the Swap gate on the initial state $|\psi\rangle = |10\rangle$.

A graphical representation of the circuit would look like this:

$$|1\rangle \text{ --[Swap]-- } |0\rangle$$

Before we apply the Swap gate, let's express the initial state $|\psi\rangle = |10\rangle$ in the computational basis, where the first qubit represents the state of the leftmost qubit and the second qubit represents the state of the rightmost qubit. The computational basis is given by $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$.

In this case, the initial state $|\psi\rangle = |10\rangle$ can be expressed as:

$$|10\rangle = 0|00\rangle + 0|01\rangle + 1|10\rangle + 0|11\rangle.$$

Applying the Swap gate exchanges the states of the two qubits. After applying the Swap gate, the final state of the system would be:

$$|\psi'\rangle = \text{Swap}(|10\rangle) = |01\rangle.$$

So, the final state of the system is $|01\rangle$, indicating that the first qubit is now in the state $|0\rangle$, and the second qubit is in the state $|1\rangle$.

Answer 2

Sure, I can help you with that!

A Swap gate exchanges the states of two qubits. In this case, the Swap gate would exchange the states of the first qubit ($|1\rangle$) and the second qubit ($|0\rangle$). The graphical representation of this circuit would look like:



where the symbol \oplus represents the Swap gate.

So, the action of the Swap gate on the initial state $|10\rangle$ would be to exchange the states of the two qubits, resulting in the final state:

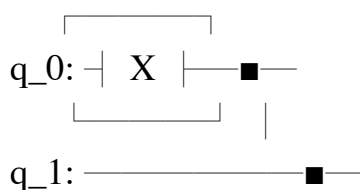
$|01\rangle$

So, the final state of the system would be $|01\rangle$, which means that the first qubit is in state $|0\rangle$ and the second qubit is in state $|1\rangle$.

Answer 3

The Swap gate, denoted as SWAP, exchanges the states of two qubits. In this case, we have a quantum system composed of two qubits, initially in the state $|\psi\rangle = |10\rangle$.

The graphical representation of the circuit for the Swap gate acting on this system is as follows:



Here, the X gate is applied to the first qubit (q_0) and acts as a NOT gate, flipping its state from $|1\rangle$ to $|0\rangle$. The control qubit (q_0) controls the SWAP operation, while the target qubit (q_1) is swapped with the state of q_0 .

The action of the Swap gate on the initial state $|10\rangle$ can be described as follows:

The X gate flips the state of the first qubit, resulting in the state $|\psi_1\rangle = |00\rangle$.

The Swap gate exchanges the states of the two qubits, resulting in the final state

$$|\psi_f\rangle = |01\rangle.$$

Therefore, the final state of the system after applying the Swap gate is $|\psi_f\rangle = |01\rangle$, where the first qubit is in the state $|0\rangle$ and the second qubit is in the state $|1\rangle$.