Question
Consider a quantum system made of one qubit. The system is in the initial state |ψ⟩ = |0⟩. The quantum gate Pauli Y is applied on |ψ⟩. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

The Pauli Y gate is a single-qubit gate that corresponds to a rotation around the y-axis of the Bloch sphere. Mathematically, it is represented by the following matrix:

Y = [[0, -i], [i, 0]]

Geometrically, it corresponds to a reflection of the qubit state about the y-axis of the Bloch sphere, as shown in the following image:

[broken image] PauliY

To represent this circuit diagrammatically, we can use the following notation:

|0⟩ -- Y -- |ψ⟩

where the vertical line represents a qubit, the Y represents the application of the Pauli Y gate, and the arrows represent the flow of quantum information.

Starting with the initial state |ψ⟩ = |0⟩, applying the Pauli Y gate gives us:

Y|0⟩ = [[0, -i], [i, 0]] [[1], [0]] = [[0], [i]]

Therefore, the final state of the system is |ψ⟩ = i|1⟩, where |1⟩ is the computational basis state representing the qubit being in the "up" position on the Bloch sphere.

Answer 2
The Pauli Y gate is a single-qubit quantum gate that flips the state of the qubit around the y-axis of the Bloch sphere. It can be represented by the following matrix:

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Y = [0 -i]
    [i  0]
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If we apply the Pauli Y gate on the initial state $|0\rangle$, we obtain:

$$Y|0\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i|1\rangle$$

So the action of the Pauli Y gate on the system takes the initial state $|0\rangle$ to the final state $|1\rangle$, up to a global phase factor of i. Geometrically, this corresponds to a 180-degree rotation of the state vector around the y-axis of the Bloch sphere.
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The circuit diagram for this operation is:

$|0\rangle$ --Y---> $i|1\rangle$

where the qubit starts in the state $|0\rangle$ and the Pauli Y gate is applied to it to obtain the final state $i|1\rangle$.

Note that the overall phase factor of i has no physical significance, as it only affects the global phase of the state vector and cannot be observed through any measurement. Therefore, we can also represent the final state as $|1\rangle$, which is equivalent to $i|1\rangle$ up to a global phase factor.

Answer 3

The Pauli Y gate is a single-qubit gate in quantum computing, which is represented by the following matrix:

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

When the Pauli Y gate is applied to the initial state $|\psi\rangle = |0\rangle$, we can calculate the final state by multiplying the Pauli Y matrix with the initial state vector:

[0 -i] [1]
Y * [i 0] [0]

Performing the matrix multiplication, we get:

[0 -i] [1] = [0]
[i 0] [0] [-i]

So, the final state of the system after applying the Pauli Y gate on |ψ⟩ = |0⟩ is:

|ψ'⟩ = 0|0⟩ - i|-⟩
= -i|-⟩

In terms of the Bloch sphere representation, the state |-⟩ corresponds to the state vector pointing in the negative y-direction on the Bloch sphere. Therefore, the action of the Pauli Y gate on the system can be visualized as a rotation of 180 degrees around the y-axis.

Regarding the graphical representation of the circuit, it would consist of a single-qubit quantum gate labeled as Y, with the input state |0⟩ and the output state |-⟩ connected to it. The circuit diagram would look like this:

|0⟩ ---- Y ---- |-⟩

Note: The notation |-⟩ represents the state vector pointing in the negative y-direction, which is often denoted as |1⟩ in other representations.