Question

Consider a quantum system made of two qubits. The system is in the initial state $|\psi\rangle = 1/\sqrt{N}\,(a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)$. A Control Z gate is applied on the system with the first qubit as control qubit and the second qubit as target qubit. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

The Control Z gate, also known as the Controlled-Phase gate or CPHASE gate, is a two-qubit gate that applies a phase factor of -1 to the target qubit if and only if the control qubit is in the state $|1\rangle$. In this case, the first qubit is the control qubit and the second qubit is the target qubit.

The action of the Control Z gate on the system can be described as follows:

- If the control qubit (the first qubit) is in the state $|0\rangle$, the gate has no effect on the target qubit (the second qubit).
- If the control qubit is in the state $|1\rangle$, the gate applies a phase factor of -1 to the target qubit.

To represent this gate in a circuit diagram, we can use the following graphical notation:

```
----CZ----
--|     |--
-|  Z  |-
--|     |--
----| |----
```

Here, the line crossing between the two qubits represents the Control Z gate. The qubit with the control operation (first qubit) is on the top, and the qubit being controlled (second qubit) is at the bottom.

Now, let's calculate the final state of the system after the Control Z gate is applied to the initial state $|\psi\rangle$:

$1/\sqrt{N}\,(a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)$

Since the Control Z gate applies a phase factor of -1 to the target qubit when the control qubit is in the state $|1\rangle$, the final state will be:

$1/\sqrt{N}\,(a|00\rangle + b|01\rangle - c|10\rangle - d|11\rangle)$

The signs of the states $|10\rangle$ and $|11\rangle$ have been flipped due to the action of the Control Z gate when the control qubit is $|1\rangle$.

So, the final state of the system is $1/\sqrt{N}\,(a|00\rangle + b|01\rangle - c|10\rangle - d|11\rangle)$.

Answer 2

The Control Z gate is a two-qubit gate that applies a phase shift of -1 to the target qubit when the control qubit is in the state $|1\rangle$. The gate leaves the state $|00\rangle$ invariant.

The action of the Control Z gate on the initial state $|\psi\rangle$ can be represented by the following matrix:

$|00\rangle\langle 00| + a|01\rangle\langle 01| + b|10\rangle\langle 10| - c|11\rangle\langle 11|$

where the first term corresponds to the case where the control qubit is in state $|0\rangle$, and the last term corresponds to the case where the control qubit is in state $|1\rangle$.

To obtain a graphical representation of the circuit, we can use the following convention: the control qubit is represented by a solid line with a control dot, and the target qubit is represented by a solid line without a control dot. The Control Z gate is represented by a box with a Z in the middle, as shown below:

```
   ┌─────────┐
───┤   CZ   ├───
   └─────────┘
```

Applying the Control Z gate to the initial state $|\psi\rangle$ gives:

$1/\sqrt{N}\,(a|00\rangle + b|01\rangle + c|10\rangle - d|11\rangle)$

Thus, the final state of the system is obtained by applying a phase shift of -1 to the state $|11\rangle$ when the control qubit is in the state $|1\rangle$.

Answer 3
The Control Z gate, also known as the Controlled-Z gate or CNOT gate, is a two-qubit gate commonly used in quantum computing. It acts on two qubits, with the first qubit acting as the control qubit and the second qubit as the target qubit. The gate applies a phase flip (Z gate) to the target qubit only if the control qubit is in the state $|1\rangle$. If the control qubit is in the state $|0\rangle$, no operation is applied to the target qubit.

Let's describe the action of the Control Z gate on the initial state $|\psi\rangle$:

- If the control qubit is in the state $|0\rangle$, no operation is applied to the target qubit. The state remains unchanged.

- If the control qubit is in the state $|1\rangle$, a phase flip (Z gate) is applied to the target qubit. The Z gate negates the phase of the state $|1\rangle$, while leaving the state $|0\rangle$ unchanged.

Using this information, let's calculate the final state of the system.

Applying the Control Z gate to $|\psi\rangle$:

$|\psi'\rangle = 1/\sqrt{N}\ (a|00\rangle + b|01\rangle + c|10\rangle - d|11\rangle)$

The resulting state $|\psi'\rangle$ is the final state of the system after applying the Control Z gate.

Now, let's provide a graphical representation of the circuit:

```
          ┌────────┐
q_0: ─────┤ ○────── ├──
       ┌─┴─────────┴─┐
q_1: ┤ Z  ├──
       └─────────────┘
```

In the above circuit diagram, q_0 represents the control qubit (first qubit) and q_1 represents the target qubit (second qubit). The gate represented by the '○' symbol is the Control Z gate, which is applied between q_0 and q_1.

Finally, the final state of the system after applying the Control Z gate is $|\psi'\rangle = 1/\sqrt{N} (a|00\rangle + b|01\rangle + c|10\rangle - d|11\rangle)$.