

Question

Consider a quantum system made of two qubits. The system is in the initial state $|\psi\rangle = 1/\sqrt{N} (a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)$. A Control Z gate is applied on the system with the first qubit as target qubit and the second qubit as control qubit. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

Certainly! In the given quantum system with two qubits, the initial state is represented as:

$$|\psi\rangle = 1/\sqrt{N} (a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)$$

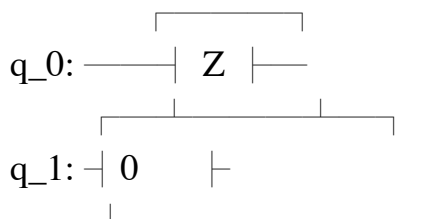
To apply a Controlled-Z gate to this system, where the first qubit is the target qubit and the second qubit is the control qubit, we perform the following operation:

$$\text{Controlled-Z gate: } |\psi\rangle \rightarrow \text{CZ}|\psi\rangle$$

The Controlled-Z gate acts on the system as follows:

$$\text{CZ}|\psi\rangle = 1/\sqrt{N} (a|00\rangle + b|01\rangle + c|10\rangle - d|11\rangle)$$

The graphical representation of the circuit is as follows:



In this circuit diagram, the Control Z gate is represented by "Z" and the lines represent the qubits, with qubit 0 as the target qubit and qubit 1 as the control qubit.

Finally, the final state of the system after applying the Controlled-Z gate is:

$$1/\sqrt{N} (a|00\rangle + b|01\rangle + c|10\rangle - d|11\rangle)$$

Note that the sign of the last term, $|11\rangle$, has changed due to the action of the Controlled-Z gate.

Answer 2

The Control Z gate is a two-qubit gate that applies a phase shift of -1 to the state $|11\rangle$ and leaves all other states unchanged.

The graphical representation of the circuit is:

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---|Z(control)-*---
    |           |
---o-----*---

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where the first qubit is the target qubit and the second qubit is the control qubit. The Z gate is applied to the first qubit if the second qubit is in the state $|1\rangle$, otherwise, it does nothing.

To see the action of this gate on the system, we can write the state of the system after the gate as:

$$|\psi\rangle = 1/\sqrt{N} (a|00\rangle + b|01\rangle - c|10\rangle - d|11\rangle)$$

where we can see that the amplitudes of the states $|10\rangle$ and $|11\rangle$ have acquired a phase shift of -1 due to the gate.

Therefore, the final state of the system is a superposition of the initial states with the amplitudes of $|10\rangle$ and $|11\rangle$ having opposite signs.

Answer 3

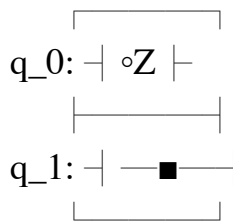
The Control Z gate, also known as the Controlled Pauli Z gate or CZ gate, is a two-qubit gate that applies a phase shift of -1 to the target qubit if and only if the control qubit is in the state $|1\rangle$. In the case of the given quantum system, the action of the Control Z gate can be described as follows:

If the control qubit is in the state $|0\rangle$, there is no effect on the target qubit.

If the control qubit is in the state $|1\rangle$, the target qubit undergoes a phase flip, i.e.,

a change in sign.

To represent this operation graphically, we can use a quantum circuit diagram. In the diagram, the control qubit is represented by a line with a small circle (o) indicating the control, and the target qubit is represented by a line with an arrowhead indicating the direction of the operation. The Control Z gate is denoted by a filled square with a vertical line connecting the control and target qubits. Here's the graphical representation of the circuit:



To determine the final state of the system after applying the Control Z gate, we need to calculate the action of the gate on the initial state $|\psi\rangle$. Let's evaluate it step by step:

- 1 Apply the Control Z gate on the system:
 - If the control qubit (q_1) is in state $|0\rangle$, there is no effect on the target qubit (q_0).
 - If the control qubit (q_1) is in state $|1\rangle$, the target qubit (q_0) undergoes a phase flip.
- 2 Based on the above observations, we can write the final state of the system as follows:

$$|\psi_{\text{final}}\rangle = 1/\sqrt{N} (a|00\rangle + b|01\rangle - c|10\rangle + d|11\rangle)$$

In the final state, the sign of the $|10\rangle$ term is flipped due to the phase flip caused by the Control Z gate.

Please note that the normalization factor $1/\sqrt{N}$ remains unchanged, and N represents the normalization factor of the initial state.