

### Question

Consider a quantum system made of one qubit. The system is in the initial state  $|\psi\rangle = |0\rangle$  and the quantum gate Identity is applied. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

### Answer 1

The Identity gate, denoted as  $I$ , is a quantum gate that has no effect on the state of a qubit. It essentially leaves the qubit in the same state as it was before the gate was applied. In mathematical notation, we can represent the action of the Identity gate as:

$$I|\psi\rangle = |\psi\rangle$$

where  $|\psi\rangle$  is the initial state of the qubit, in this case,  $|0\rangle$ .

The graphical representation of the circuit for this scenario would be a single qubit circuit with the Identity gate applied to it. This circuit would be represented by the following diagram:

$$|0\rangle \xrightarrow{I} |0\rangle$$

The final state of the system after the application of the Identity gate would be the same as the initial state, i.e., the qubit would still be in the state  $|0\rangle$ . Thus, the final state of the system would be:

$$|\psi\rangle = |0\rangle$$

### Answer 2

The Identity gate, denoted as  $I$ , is a quantum gate that does not change the state of the qubit it acts upon. In other words, if the qubit is in state  $|0\rangle$ , applying the Identity gate leaves it in the same state  $|0\rangle$ . Mathematically, we can represent the Identity gate as a  $2 \times 2$  unitary matrix:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The graphical representation of the circuit for this operation is as follows:

$$|0\rangle \xrightarrow{I} |0\rangle$$

Here, the state  $|0\rangle$  represents the initial state of the qubit,  $[I]$  represents the Identity gate, and the final state of the qubit is also  $|0\rangle$ .

Therefore, the final state of the system after applying the Identity gate on the initial state  $|\psi\rangle = |0\rangle$  is also  $|\psi\rangle = |0\rangle$ .

### Answer 3

The Identity gate, denoted as  $I$ , is a quantum gate that leaves the state of a qubit unchanged. It is often represented as a square matrix:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

When the Identity gate is applied to the initial state  $|0\rangle$ , the resulting state is still  $|0\rangle$ . Mathematically, we can represent this as:

$$I|0\rangle = |0\rangle$$

Graphically, the circuit representation of this operation is as follows:

$$|0\rangle \text{ ---- } [I] \text{ ---- }$$

The  $|0\rangle$  state is represented as a horizontal line, and the Identity gate ( $I$ ) is depicted as a box. The arrow indicates the direction of the qubit's flow through the circuit.

The final state of the system remains  $|0\rangle$ .