

Question

Consider a quantum system made of one qubit. The system is in the initial state $|\psi\rangle = \frac{1}{\sqrt{N}} (a|0\rangle + b|1\rangle)$, where a and b are complex numbers and N is a normalization factor. A quantum gate Phase is applied on $|\psi\rangle$. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

The Phase gate, often denoted as "S" or "P", is a basic single-qubit gate in quantum computing. It introduces a phase shift of $\pi/2$ (or 90 degrees) to the state $|1\rangle$ while leaving the state $|0\rangle$ unchanged. In matrix form, the Phase gate is represented as:

$$\text{Phase} = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix},$$

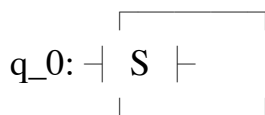
where i is the imaginary unit.

To describe the action of the Phase gate on the given initial state $|\psi\rangle = \frac{1}{\sqrt{N}} (a|0\rangle + b|1\rangle)$, we apply the gate to each component of the superposition:

$$\text{Phase} \left(\frac{1}{\sqrt{N}} (a|0\rangle + b|1\rangle) \right) = \frac{1}{\sqrt{N}} (a|0\rangle + ib|1\rangle),$$

where we have introduced the phase factor i to the $|1\rangle$ component.

For a graphical representation of the circuit, we can use the following quantum circuit diagram:



This circuit diagram shows a single qubit, labeled q_0 , with the Phase gate, denoted as "S," applied to it.

Finally, to determine the final state of the system, we substitute the transformed components back into the initial state expression:

$$|\psi\rangle = 1/\sqrt{N} (a|0\rangle + ib|1\rangle).$$

So, after the Phase gate is applied, the final state of the system is $|\psi\rangle = 1/\sqrt{N} (a|0\rangle + ib|1\rangle)$.

Answer 2

The Phase gate, denoted as $P(\theta)$, is a single-qubit gate that applies a phase shift of θ to the $|1\rangle$ state. The matrix representation of this gate in the computational basis is:

$$P(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$$

Thus, if we apply the Phase gate to the initial state $|\psi\rangle$, we get:

$$P(\theta) |\psi\rangle = 1/\sqrt{N} (a|0\rangle + e^{i\theta} b|1\rangle)$$

This means that the gate applies a phase shift of θ to the $|1\rangle$ state, while leaving the $|0\rangle$ state unchanged.

A graphical representation of the circuit for this gate on a single qubit is:

$$|\psi\rangle \xrightarrow{P(\theta)} P(\theta)|\psi\rangle$$

where the gate is represented by the symbol $[P(\theta)]$.

The final state of the system after applying the Phase gate is:

$$P(\theta) |\psi\rangle = 1/\sqrt{N} (a|0\rangle + e^{i\theta} b|1\rangle)$$

which is a superposition of the $|0\rangle$ and $|1\rangle$ states with a phase shift of θ on the $|1\rangle$ state.

Answer 3

The quantum gate you mentioned, the Phase gate (also known as the P gate or RZ gate), is a single-qubit gate commonly represented by the following matrix:

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$$

where θ is the phase angle. The gate acts on a qubit and applies a phase shift of θ to the state $|1\rangle$ while leaving the state $|0\rangle$ unchanged.

To describe the action of the Phase gate on the system in the initial state $|\psi\rangle = 1/\sqrt{N} (a|0\rangle + b|1\rangle)$, let's apply the gate to each basis state:

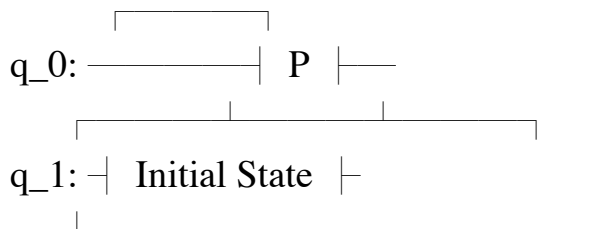
$\text{Phase}|0\rangle = |0\rangle$ (since there is no phase shift on $|0\rangle$)

$\text{Phase}|1\rangle = e^{i\theta}|1\rangle$

So, applying the Phase gate to the initial state $|\psi\rangle$, we have:

$$\text{Phase}(|\psi\rangle) = 1/\sqrt{N} (a|0\rangle + b(e^{i\theta})|1\rangle)$$

Now, to represent this circuit graphically, we can use the standard quantum circuit notation. The circuit for this scenario would look like this:



In the circuit diagram above, the box labeled "P" represents the Phase gate applied to the qubit q_0 . The qubit q_1 is prepared in the initial state $|\psi\rangle = 1/\sqrt{N} (a|0\rangle + b|1\rangle)$, which is represented by the "Initial State" box.

Finally, to determine the final state of the system after applying the Phase gate, we can rewrite the state as:

$$\text{Phase}(|\psi\rangle) = 1/\sqrt{N} (a|0\rangle + b(e^{i\theta})|1\rangle)$$

The final state can be obtained by normalizing the coefficients:

$$\text{Phase}(|\psi\rangle) = 1/\sqrt{N'} (a|0\rangle + b(e^{i\theta})|1\rangle)$$

where N' is the new normalization factor.