Question
Consider a quantum system made of two qubits. The system is in the initial state $|\psi\rangle = |00\rangle$. A Control Z gate is applied on the system with the first qubit as control qubit and the second qubit as target qubit. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1
The Control Z gate, also known as the Controlled-Z gate or CZ gate, is a two-qubit gate in quantum computing. It applies a phase flip of -1 to the target qubit if and only if the control qubit is in the state $|1\rangle$. In other words, it leaves the target qubit unchanged if the control qubit is in the state $|0\rangle$ and applies a phase flip to the target qubit if the control qubit is in the state $|1\rangle$.

In your case, the initial state of the system is $|\psi\rangle = |00\rangle$. The first qubit is the control qubit, and the second qubit is the target qubit. Applying the Control Z gate on this system means that if the first qubit is in the state $|1\rangle$, the second qubit will undergo a phase flip of -1.

Graphical representation of the circuit:

```
|0⟩ ---- CZ ----
          |
|0⟩ ------------
```

In the circuit diagram, the qubit wires are represented vertically. The gate CZ represents the Control Z gate. The top wire represents the control qubit (initialized as $|0\rangle$), and the bottom wire represents the target qubit (also initialized as $|0\rangle$).

Since the initial state is $|00\rangle$ and the control qubit is $|0\rangle$, the target qubit remains unchanged. Therefore, the final state of the system would still be $|00\rangle$.

Answer 2
The Control Z gate, also known as CZ gate, is a two-qubit gate in quantum computing that applies a phase shift to the target qubit based on the state of the control qubit. The gate is represented by the following unitary matrix:

$$CZ = \left|\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{matrix}\right|$$

In the computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, the CZ gate performs the following operation:

$CZ|00\rangle = |00\rangle$
$CZ|01\rangle = |01\rangle$
$CZ|10\rangle = |10\rangle$
$CZ|11\rangle = -|11\rangle$

Thus, the CZ gate acts as an identity gate on the first three basis states and applies a phase shift of -1 (i.e., a bit flip) to the last basis state.

The circuit diagram for the Control Z gate is shown below:

```
q0 ───■───
   |
q1 ───■───
```

In this diagram, the qubit on the left is the control qubit and the qubit on the right is the target qubit. The symbol "■" represents the CZ gate.

Applying the CZ gate on the initial state $|00\rangle$ yields:

$CZ(|00\rangle) = CZ(1|00\rangle) = 1|00\rangle = |00\rangle$

Therefore, the final state of the system remains $|00\rangle$.

Answer 3
The Control Z gate, also known as the Controlled-Z gate or CZ gate, is a two-qubit gate commonly used in quantum computing. It applies a phase shift of -1 to the target qubit if and only if the control qubit is in the state $|1\rangle$.

In the given quantum system with an initial state of $|\psi\rangle = |00\rangle$, the first qubit is in the state |0⟩ (control qubit) and the second qubit is also in the state |0⟩ (target qubit). Applying the Control Z gate to this system will have the following effect:

The Control Z gate does nothing to the target qubit if the control qubit is in the state |0⟩. Since the first qubit is |0⟩, the state of the target qubit remains unchanged. Therefore, the final state of the system is still |00⟩.

Here is the graphical representation of the circuit:

```
|0⟩--@--|0⟩
     |
     Z
```

In this representation, the vertical line represents a qubit, and the gate "Z" represents the Control Z gate. The control qubit is the top qubit, and the target qubit is the bottom qubit.

To summarize, the action of the Control Z gate on the given quantum system with the initial state |ψ⟩ = |00⟩ leaves the system unchanged, resulting in the final state |00⟩.