

Question

Consider a quantum system made of two qubits. The system is in the initial state $|\psi\rangle = |10\rangle$. A CNOT gate is applied on the system with the first qubit as control qubit and the second qubit as target qubit. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

In a quantum system consisting of two qubits, the CNOT (Controlled-NOT) gate is a two-qubit gate that acts on the control qubit and the target qubit. It flips the state of the target qubit if and only if the control qubit is in the state $|1\rangle$. The action of the CNOT gate can be described as follows:

If the control qubit is in the state $|0\rangle$, then the target qubit remains unchanged. If the control qubit is in the state $|1\rangle$, then the target qubit is flipped (changes from $|0\rangle$ to $|1\rangle$, or from $|1\rangle$ to $|0\rangle$).

In the given initial state $|\psi\rangle = |10\rangle$, the first qubit is in the state $|1\rangle$ and the second qubit is in the state $|0\rangle$.

The graphical representation of the circuit with the CNOT gate acting on the system would be:

```

|1>--[CNOT]--
      |
|0>-----
```

After applying the CNOT gate with the first qubit as the control qubit and the second qubit as the target qubit, the final state of the system can be calculated by applying the gate operation to the initial state.

Applying the CNOT gate with the control qubit (first qubit) in state $|1\rangle$ and the target qubit (second qubit) in state $|0\rangle$:

$$\text{CNOT}(|1\rangle \otimes |0\rangle) = \text{CNOT}(|10\rangle) = |11\rangle$$

```

|1>--[CNOT]--
```

$|$
 $|0\rangle$ -----

Therefore, the final state of the system after the CNOT gate is applied to the initial state $|10\rangle$ is $|11\rangle$.

Answer 2

Yes, I can certainly help you with that!

The CNOT gate is a two-qubit gate that acts on a control qubit and a target qubit. The action of the CNOT gate is to flip the state of the target qubit if the control qubit is in the state $|1\rangle$, and leave it unchanged if the control qubit is in the state $|0\rangle$.

In this case, the CNOT gate is applied with the first qubit ($|1\rangle$) as the control qubit and the second qubit ($|0\rangle$) as the target qubit. Since the control qubit is in the state $|1\rangle$, the target qubit will be flipped. Therefore, the action of the CNOT gate on the initial state $|10\rangle$ will be:

CNOT gate: $|10\rangle \rightarrow |11\rangle$

The graphical representation of this circuit is as follows:

$|1\rangle$ --@--
 $|$
 $|0\rangle$ --X--

Here, the qubit labeled with $|1\rangle$ is the control qubit, and the qubit labeled with $|0\rangle$ is the target qubit. The gate represented by "@--" is the CNOT gate.

The final state of the system after the CNOT gate is applied is $|11\rangle$, which means that the first qubit is in the state $|1\rangle$ and the second qubit is also in the state $|1\rangle$.

Answer 3

In a quantum system with two qubits, where the initial state is $|\psi\rangle = |10\rangle$, a CNOT gate with the first qubit as the control qubit and the second qubit as the target qubit can be described as follows:

The CNOT gate performs a NOT operation (or an X gate) on the target qubit if and only if the control qubit is in the state $|1\rangle$. If the control qubit is in the state $|0\rangle$, the target qubit remains unchanged.

Graphical representation of the circuit:

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-----●-----
-|CNOT|-
-----|-----

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In the above circuit, the qubit labeled 1 is the control qubit, and the qubit labeled 2 is the target qubit. The line connecting them represents the CNOT gate.

Now, let's apply the CNOT gate to the initial state $|10\rangle$:

Initially, the first qubit (control qubit) is in the state $|1\rangle$, and the second qubit (target qubit) is in the state $|0\rangle$.

Applying the CNOT gate with the control qubit as $|1\rangle$ and the target qubit as $|0\rangle$ results in flipping the state of the target qubit. Therefore, the final state of the system after applying the CNOT gate is:

$$|\psi\rangle = |11\rangle$$

So, the final state of the system is $|11\rangle$.