

### Question

Consider a quantum system made of one qubit. The system is in the initial state  $|\psi\rangle = \frac{1}{\sqrt{N}} (a|0\rangle + b|1\rangle)$ , where  $a$  and  $b$  are complex numbers and  $N$  is a normalization factor. A T gate is applied on  $|\psi\rangle$ . Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

### Answer 1

Certainly! The T gate is a single-qubit gate in quantum computing that performs a phase shift of  $\pi/4$  radians (or 45 degrees) on the  $|1\rangle$  state. It can be represented as the following matrix:

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix},$$

where  $i$  is the imaginary unit and  $e^{i\pi/4}$  represents a complex number with magnitude 1 and argument  $\pi/4$ .

To describe the action of the T gate on the system, we apply it to the initial state  $|\psi\rangle$ . The T gate only affects the  $|1\rangle$  state, so it leaves the  $|0\rangle$  state unchanged.

Therefore, the action of the T gate on  $|\psi\rangle$  is as follows:

$$T \text{ gate } (|\psi\rangle) = \frac{1}{\sqrt{N}} (a|0\rangle + b(e^{i\pi/4})|1\rangle).$$

Now, let's create a graphical representation of the circuit:



This circuit diagram represents a single qubit ( $q0$ ) on which the T gate is applied.

Finally, to determine the final state of the system, we can rewrite the state after applying the T gate:

$$T \text{ gate } (|\psi\rangle) = \frac{1}{\sqrt{N}} (a|0\rangle + b(e^{i\pi/4})|1\rangle).$$

Since the state needs to be normalized, we can multiply the factor  $e^{i\pi/4}$  with the normalization factor  $N$  to ensure that the final state remains normalized:

$$T \text{ gate } (|\psi\rangle) = \frac{1}{\sqrt{N}} (a|0\rangle + b(e^{i\pi/4})/\sqrt{N}|1\rangle).$$

So, the final state of the system after applying the T gate is:

$$|\psi'\rangle = 1/\sqrt{N} (a|0\rangle + b(e^{i\pi/4})/\sqrt{N}|1\rangle).$$

Please note that if you require a specific numerical representation of the final state, you'll need to provide the values of a, b, and N.

## Answer 2

The T gate is a single-qubit gate that is represented by the matrix:

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

When the T gate is applied to the initial state  $|\psi\rangle$ , we obtain:

$$T|\psi\rangle = 1/\sqrt{N} ( a|0\rangle + b e^{i\pi/4} |1\rangle )$$

This means that the T gate introduces a phase shift of  $e^{i\pi/4}$  to the  $|1\rangle$  state, while leaving the  $|0\rangle$  state unchanged.

Graphically, the circuit for applying the T gate to a single qubit is:

$$|\psi\rangle \xrightarrow{T} T|\psi\rangle$$

The final state of the system after the T gate is applied to  $|\psi\rangle$  is:

$$T|\psi\rangle = 1/\sqrt{N} ( a|0\rangle + b e^{i\pi/4} |1\rangle )$$

Note that the normalization factor N remains the same after the T gate is applied.

## Answer 3

Certainly! The T gate is a single-qubit gate in quantum computing that performs a rotation of  $\pi/4$  radians (45 degrees) around the Z-axis of the Bloch sphere. It can be represented by the following matrix:

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix},$$

where  $i$  is the imaginary unit and  $e^{i\pi/4}$  represents the complex phase factor.

Now, let's describe the action of the T gate on the initial state  $|\psi\rangle$ :

$$\begin{aligned} T|\psi\rangle &= T(1/\sqrt{N}) (a|0\rangle + b|1\rangle) \\ &= 1/\sqrt{N} (aT|0\rangle + bT|1\rangle) \\ &= 1/\sqrt{N} (a|0\rangle + be^{i\pi/4}|1\rangle). \end{aligned}$$

So, the final state of the system after applying the T gate is:

$$|\psi'\rangle = 1/\sqrt{N} (a|0\rangle + be^{i\pi/4}|1\rangle).$$

As for the graphical representation of the circuit, since we have a single qubit, it can be represented as follows:

$$|\psi\rangle \text{ ----[T]---- } |\psi'\rangle$$

Here, the initial state  $|\psi\rangle$  is acted upon by the T gate, resulting in the final state  $|\psi'\rangle$ .

It's important to note that the state  $|\psi'\rangle$  will have the same normalization factor  $N$  as the initial state  $|\psi\rangle$ , so the overall normalization of the state remains unchanged.