

Question

Consider a quantum system made of two qubits. The system is in the initial state $|\psi\rangle = |10\rangle$. A Control Z gate is applied on the system with the first qubit as target qubit and the second qubit as control qubit. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

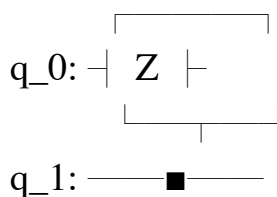
The Control Z gate, also known as the Controlled-Z gate or CZ gate, is a two-qubit gate that performs a phase flip on the target qubit if and only if the control qubit is in the state $|1\rangle$. In other words, it applies a conditional phase shift of π (180 degrees) to the target qubit if the control qubit is in the state $|1\rangle$, and leaves the target qubit unchanged if the control qubit is in the state $|0\rangle$.

In the given system, the initial state is $|\psi\rangle = |10\rangle$, where the first qubit is in state $|1\rangle$ and the second qubit is in state $|0\rangle$. When the Control Z gate is applied with the first qubit as the target qubit and the second qubit as the control qubit, the gate acts as follows:

If the control qubit (second qubit) is in the state $|0\rangle$, the target qubit (first qubit) remains unchanged.

If the control qubit (second qubit) is in the state $|1\rangle$, the target qubit (first qubit) undergoes a phase flip of π radians.

Now let's represent the circuit graphically:



In this circuit representation, q_0 represents the first qubit (target qubit) and q_1 represents the second qubit (control qubit). The Z gate is applied to the target qubit, controlled by the control qubit.

Based on the initial state $|\psi\rangle = |10\rangle$ and the action of the Control Z gate, we can determine the final state of the system.

Since the control qubit is initially in state $|0\rangle$, it does not affect the target qubit. Therefore, the final state of the system remains the same as the initial state: $|\psi\rangle = |10\rangle$.

Hence, the final state of the system after applying the Control Z gate is still $|10\rangle$.

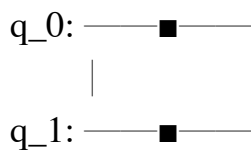
Answer 2

A Control Z gate is a two-qubit gate that applies a phase shift to the target qubit (the qubit that the gate acts on) if and only if the control qubit (the other qubit) is in the state $|1\rangle$. Specifically, the Control Z gate acts on the computational basis states as follows:

$|00\rangle \rightarrow |00\rangle$
 $|01\rangle \rightarrow |01\rangle$
 $|10\rangle \rightarrow |10\rangle$
 $|11\rangle \rightarrow -|11\rangle$

Here, the minus sign in the last row indicates a phase shift of π radians (i.e., a 180-degree phase shift). Note that the Control Z gate leaves the states $|00\rangle$, $|01\rangle$, and $|10\rangle$ unchanged, but it introduces a relative phase of -1 between the basis states $|11\rangle$ and $|10\rangle$.

A graphical representation of the circuit for the Control Z gate is shown below:



In this diagram, the horizontal lines represent the qubits, and the vertical line with a circle at the bottom represents the Control Z gate. The control qubit is represented by the qubit line on the left, and the target qubit is represented by the qubit line on the right. The arrows on the control and target qubit lines indicate the direction of information flow (from left to right). The small circles at the ends of the qubit lines represent the initial state of the qubits ($|1\rangle$ for the control qubit and $|0\rangle$ for the target qubit).

To apply the Control Z gate to the initial state $|10\rangle$, we represent this state as the tensor product of the individual qubit states:

$$|10\rangle = |1\rangle \otimes |0\rangle$$

Then, we apply the gate to this state:

$$\text{Control Z gate: } (|1\rangle\langle 1| \otimes I) \otimes Z$$

Here, I is the identity matrix, and Z is the Pauli Z matrix (which acts on a single qubit). The tensor product symbol (\otimes) indicates that we apply the Control Z gate to the second qubit (the target qubit) and apply the identity matrix to the first qubit (the control qubit).

Applying this gate to the initial state $|10\rangle$, we obtain:

$$(\text{Control Z gate}) |10\rangle = (|1\rangle\langle 1| \otimes I) \otimes Z |1\rangle \otimes |0\rangle$$

$$= |1\rangle \otimes Z|0\rangle$$

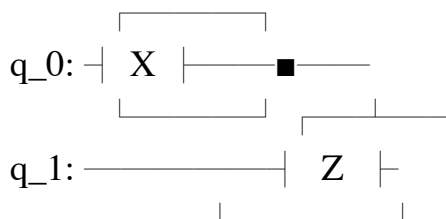
$$= |1\rangle \otimes |0\rangle$$

Thus, the final state of the system is $|10\rangle$, which is the same as the initial state. The Control Z gate did not introduce any phase shift or change the state of the qubits.

Answer 3

The Control Z gate, also known as the Controlled-Phase (CZ) gate, is a two-qubit gate that applies a phase flip to the target qubit if and only if the control qubit is in the state $|1\rangle$. In the given scenario, the first qubit is the target qubit and the second qubit is the control qubit.

The graphical representation of the circuit for the given scenario would look as follows:



In this circuit diagram, the gate represented by "X" is the Pauli-X gate, which is applied to the target qubit to flip it from the state $|0\rangle$ to $|1\rangle$. The gate represented by "Z" is the Pauli-Z gate, which applies a phase flip to the target qubit.

Let's analyze the action of the Control Z gate on the initial state $|\psi\rangle = |10\rangle$:

- 1 Start with the initial state $|10\rangle$.
- 2 Apply the Pauli-X gate (represented as "X") on the target qubit (qubit 0) to flip it from $|0\rangle$ to $|1\rangle$. Now the state becomes $|11\rangle$.
- 3 Apply the CZ gate (represented as "Z") with qubit 0 as the target and qubit 1 as the control. The CZ gate applies a phase flip to the target qubit (qubit 0) if the control qubit (qubit 1) is in the state $|1\rangle$. Since qubit 1 is in the state $|1\rangle$, the phase of qubit 0 gets flipped. Therefore, the final state of the system is $|-11\rangle$.

Note that the minus sign in front of the state $|11\rangle$ indicates the phase flip that was applied.