

### Question

Consider a quantum system made of two qubits. The system is in the initial state  $|\psi\rangle = |00\rangle$ . A CNOT gate is applied on the system with the first qubit as control qubit and the second qubit as target qubit. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

### Answer 1

Certainly! In the given scenario, we have a quantum system composed of two qubits. The initial state of the system is  $|\psi\rangle = |00\rangle$ , where the first qubit is in state  $|0\rangle$  and the second qubit is also in state  $|0\rangle$ .

The CNOT (Controlled-NOT) gate is a two-qubit gate that performs a NOT operation on the target qubit (second qubit) only if the control qubit (first qubit) is in state  $|1\rangle$ . Otherwise, it leaves the target qubit unchanged.

The action of the CNOT gate on the system can be described as follows:

$$\text{CNOT gate: } |c\rangle \otimes |t\rangle \rightarrow |c\rangle \otimes (X|t \oplus c\rangle)$$

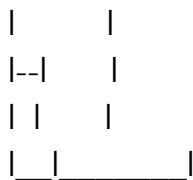
where  $|c\rangle$  represents the control qubit,  $|t\rangle$  represents the target qubit,  $X$  denotes the NOT operation, and  $\oplus$  represents the XOR (exclusive OR) operation.

In this case, the first qubit (control qubit) is  $|0\rangle$  and the second qubit (target qubit) is  $|0\rangle$ . Applying the CNOT gate to the system results in:

$$|0\rangle \otimes |0\rangle \rightarrow |0\rangle \otimes (X|0 \oplus 0\rangle) \rightarrow |0\rangle \otimes |0\rangle$$

The graphical representation of this circuit is as follows:

----CNOT----



In this representation, the control qubit is represented by the line with the control symbol (a circle or dot), and the target qubit is represented by the line with the target symbol (an arrow or line).

As a result of applying the CNOT gate to the initial state  $|\psi\rangle = |00\rangle$ , the final state of the system remains unchanged:  $|\psi\rangle = |00\rangle$ . The CNOT gate, in this case, has no effect since the control qubit is in state  $|0\rangle$ , and the target qubit is left unchanged.

## Answer 2

Sure, I'd be happy to help!

The CNOT gate is a two-qubit gate that is commonly used in quantum computing. It is a conditional gate that flips the state of the target qubit (the second qubit) if and only if the control qubit (the first qubit) is in the state  $|1\rangle$ . Here is the truth table for the CNOT gate:

Control (qubit 1) | Target (qubit 2) | Result

$ 0\rangle$	$ 0\rangle$	$ 0\rangle$
$ 0\rangle$	$ 1\rangle$	$ 1\rangle$
$ 1\rangle$	$ 0\rangle$	$ 1\rangle$
$ 1\rangle$	$ 1\rangle$	$ 0\rangle$

So if the first qubit is  $|0\rangle$ , the state of the second qubit is not changed, whereas if the first qubit is  $|1\rangle$ , the second qubit is flipped.

The CNOT gate can be represented by the following circuit:

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----@----
   |
----X----

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where the control qubit is the line coming in from the left, and the target qubit is the line coming in from the bottom. The "@" symbol represents the CNOT gate.

In your case, the initial state of the two-qubit system is  $|00\rangle$ , which means that both qubits are in the state  $|0\rangle$ . Applying the CNOT gate with the first qubit as the control qubit and the second qubit as the target qubit leads to the following calculation:

$$\text{CNOT}|00\rangle = \text{CNOT}(|0\rangle \otimes |0\rangle) = |0\rangle \otimes |0\rangle = |00\rangle$$

So the final state of the system is  $|00\rangle$ , which is the same as the initial state. Therefore, applying the CNOT gate with the given initial state does not change the state of the system.

### Answer 3

The CNOT gate, also known as the controlled-NOT gate or the controlled-X gate, is a two-qubit gate that performs an operation based on the state of the control qubit. In this case, the control qubit is the first qubit, and the target qubit is the second qubit.

The CNOT gate applies the Pauli-X gate (also known as the bit-flip gate or NOT gate) to the target qubit if and only if the control qubit is in the state  $|1\rangle$ . If the control qubit is in the state  $|0\rangle$ , no operation is performed on the target qubit.

The graphical representation of the circuit for this scenario is as follows:

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----CNOT----
|      |
|      |
--|0|--|1|--
  |  |
--○---○--

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The initial state of the system is  $|00\rangle$ . Applying the CNOT gate with the first qubit as the control qubit and the second qubit as the target qubit, we determine the action based on the state of the control qubit.

In this case, the control qubit is  $|0\rangle$ , and the target qubit is  $|0\rangle$ . Since the control qubit is in the state  $|0\rangle$ , no operation is performed on the target qubit. Therefore, the final state of the system remains  $|00\rangle$ .

Please note that in this specific scenario, since the control qubit is always  $|0\rangle$ , the target qubit remains unchanged, resulting in the same initial state.