Question
Consider a quantum system made of two qubits. The system is in the initial state $|\psi\rangle = |11\rangle$. A Control Z gate is applied on the system with the first qubit as control qubit and the second qubit as target qubit. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1
The Control Z gate, also known as the Controlled-Phase (CZ) gate, is a two-qubit gate commonly used in quantum computing. It applies a phase shift of -1 to the target qubit if and only if the control qubit is in the state $|1\rangle$. In other words, if the control qubit is in the state $|0\rangle$, the target qubit remains unchanged, and if the control qubit is in the state $|1\rangle$, the target qubit acquires a phase shift of -1.

In the given scenario, the initial state of the two-qubit system is $|\psi\rangle = |11\rangle$, where the first qubit is the control qubit and the second qubit is the target qubit. Applying the Control Z gate means that if the control qubit (the first qubit) is in the state $|1\rangle$, the target qubit (the second qubit) will acquire a phase shift of -1.

To represent this circuit graphically, we can use the following notation:

```
----CZ----
|      |
Qubit 1  Qubit 2
|      |
----------
```

The "CZ" gate represents the Control Z gate. The wires represent the qubits, with qubit 1 being the control qubit and qubit 2 being the target qubit.

Applying the Control Z gate to the initial state $|11\rangle$, we have:

$|11\rangle$ --> CZ --> $-|11\rangle$

The final state of the system after the application of the Control Z gate is $-|11\rangle$. The minus sign indicates the phase shift of -1 acquired by the target qubit.

Answer 2

The Control Z gate is a two-qubit gate that applies a phase shift of -1 to the target qubit if the control qubit is in the state |1⟩. It can be represented by the following matrix:

```
CZ = |1 0 0 0|
     |0 1 0 0|
     |0 0 1 0|
     |0 0 0 -1|
```

The graphical representation of the circuit is shown below:

```
|ψ⟩ --●--      --●--
       CZ      |
          --|--
       |      |
       |      CZ
      \|/    /|\
|00⟩ --●-- --●-- --●-- |11⟩
```

Here, the first qubit is the control qubit and the second qubit is the target qubit. The circuit starts with the initial state |11⟩, represented by the two qubits in the state |1⟩. The Control Z gate is applied with the first qubit as the control qubit and the second qubit as the target qubit. The gate acts on the state |11⟩ as follows:

CZ |11⟩ = CZ (|1⟩ ⊗ |1⟩) = |1⟩ ⊗ (-|1⟩) = -|11⟩

Therefore, the final state of the system is -|11⟩.

Answer 3
The Control Z gate, also known as the Controlled-Z gate or CZ gate, is a two-qubit gate in quantum computing. It applies a phase flip to the target qubit if and only if the control qubit is in the state |1⟩. In other words, if the control qubit is |1⟩, the phase of the target qubit is flipped (multiplied by -1), and if the control qubit is |0⟩, no change occurs to the target qubit.

In the given scenario, the initial state of the system is |ψ⟩ = |11⟩, where the first qubit (control qubit) is in state |1⟩ and the second qubit (target qubit) is also in state |1⟩. When the Control Z gate is applied, the action can be described as follows:

If the control qubit is |1⟩, the phase of the target qubit is flipped (multiplied by -1). Since the control qubit is |1⟩ in the initial state, the phase of the target qubit will be flipped.

The graphical representation of this circuit is as follows:

```
--H--
-|  |-
-⊕  ⊕-
--|  |--
---Z---
--|  |--
-⊕  ⊕-
-|  |-
--H--
```

Here, the "H" gate represents a Hadamard gate applied to each qubit before and after the Control Z gate. The "⊕" symbol represents a controlled-X gate (also known as a Controlled-NOT or CNOT gate), which can be used to create the controlled-Z gate by applying a Hadamard gate before and after it.

Now, let's calculate the final state of the system after applying the Control Z gate to the initial state $|\psi\rangle = |11\rangle$:

Applying the Hadamard gate to each qubit:

$H \otimes H |11\rangle = (1/\sqrt{2})(|0\rangle + |1\rangle) \otimes (1/\sqrt{2})(|0\rangle + |1\rangle) = (1/2)(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$

Applying the CNOT gate with the first qubit as control and the second qubit as the target:

$(1/2)(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \rightarrow (1/2)(|00\rangle + |01\rangle + |11\rangle + |10\rangle)$

Applying the Hadamard gate to each qubit again:

$H \otimes H$ (1/2)(|00⟩ + |01⟩ + |11⟩ + |10⟩) = (1/4)(|00⟩ + |01⟩ + |11⟩ + |10⟩ + |10⟩ - |11⟩ - |01⟩ - |00⟩)
= (1/4)(2|00⟩ - 2|01⟩ + |11⟩ + |10⟩)

Simplifying the expression, we get:

(1/2)(|00⟩ - |01⟩ + |11⟩ + |10⟩)

Therefore, the final state of the system after applying the Control Z gate to the initial state |ψ⟩ = |11⟩ is (1/2)(|00⟩ - |01⟩ + |11⟩ + |10⟩).