Question
Consider a quantum system made of two qubits. The system is in the initial state $|\psi\rangle = |11\rangle$. A Control Z gate is applied on the system with the first qubit as target qubit and the second qubit as control qubit. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1
The Control Z gate, also known as the Controlled-Z gate or CZ gate, is a two-qubit gate that performs a phase flip on the target qubit (the qubit to which the gate is applied) if and only if the control qubit (the other qubit) is in the state $|1\rangle$.

In this case, we have the initial state $|\psi\rangle = |11\rangle$, where the first qubit is the target qubit and the second qubit is the control qubit. Applying the CZ gate to this system, we obtain the following action:

CZ gate:

- If the control qubit (second qubit) is in the state $|0\rangle$, there is no change to the target qubit (first qubit).
- If the control qubit (second qubit) is in the state $|1\rangle$, the target qubit (first qubit) undergoes a phase flip, resulting in the final state $|\psi\rangle = -|11\rangle$.

Now, let's represent the circuit graphically:

$|\psi\rangle$ ----[CZ]----

In this representation, $|\psi\rangle$ is the initial state and the CZ gate is denoted by the box with the label CZ. The first qubit is the target qubit, and the second qubit is the control qubit.

Therefore, the final state of the system after applying the CZ gate to the initial state $|\psi\rangle = |11\rangle$ is:

$|\psi\rangle = -|11\rangle$.

Note that the minus sign in front of the state indicates the phase flip applied to

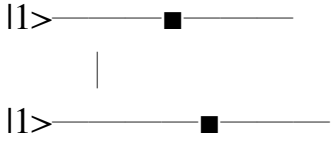
the target qubit.

Answer 2
The Control Z gate, also known as the Controlled Phase gate, is a two-qubit gate that performs a conditional phase shift on the target qubit based on the state of the control qubit. The gate acts as follows:

|00⟩ -> |00⟩
|01⟩ -> |01⟩
|10⟩ -> |10⟩
|11⟩ -> -|11⟩

So, if the control qubit is in the state |1⟩, the phase of the target qubit is flipped.

The graphical representation of the circuit is as follows:



<where the control qubit is the top qubit and the target qubit is the bottom qubit.

Starting with the initial state |11⟩, the action of the Control Z gate is to flip the phase of the target qubit, conditional on the state of the control qubit. Since the control qubit is also in the state |1⟩, the final state of the system is -|11⟩, or equivalently, the state with a negative phase. Thus, the final state of the system after applying the Control Z gate is:

|ψ⟩ = -|11⟩

Answer 3
In a quantum system made of two qubits, the Control Z gate (also known as the Controlled-Phase gate) is a two-qubit gate that applies a phase shift of -1 to the target qubit if and only if the control qubit is in the state |1⟩.

In this case, the initial state of the system is |ψ⟩ = |11⟩, where the first qubit is the target qubit and the second qubit is the control qubit. When the Control Z gate is applied, the action on the system can be described as follows:

If the control qubit (the second qubit) is in the state $|0\rangle$, there is no effect on the system. The phase of the target qubit remains unchanged.

If the control qubit is in the state $|1\rangle$, a phase shift of -1 (or a π radians phase shift) is applied to the target qubit. In this case, the resulting state of the system will be $|\psi'\rangle = -|11\rangle$.

Graphical representation of the circuit:

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----@----
    |
----Z----
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The circuit consists of two qubits, represented by vertical lines. The Control Z gate is applied with the second qubit as the control qubit (represented by the @ symbol) and the first qubit as the target qubit (represented by the Z gate). The lines are connected by a horizontal line, indicating that the gate is applied between the qubits.

The final state of the system, after the application of the Control Z gate on the initial state $|11\rangle$, is $|\psi'\rangle = -|11\rangle$, where the negative sign represents the phase shift of -1 applied to the target qubit due to the control qubit being in the state $|1\rangle$.