Question

Consider a quantum system made of two qubits. The system is in the initial state $|\psi\rangle = |11\rangle$. A CNOT gate is applied on the system with the first qubit as target qubit and the second qubit as control qubit. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

In a quantum system composed of two qubits, the CNOT gate is a two-qubit gate that performs a controlled-NOT operation. It flips the target qubit (the qubit to which the gate is applied) if and only if the control qubit (the other qubit involved in the gate) is in the state $|1\rangle$.

In your case, the initial state of the system is $|\psi\rangle = |11\rangle$, where the first qubit is in the state $|1\rangle$ and the second qubit is also in the state $|1\rangle$.

The CNOT gate, with the first qubit as the target and the second qubit as the control, will perform the following operation:

- If the control qubit is in the state $|0\rangle$, there is no change to the target qubit.
- If the control qubit is in the state $|1\rangle$, the target qubit will be flipped.

Let's represent this operation in a circuit diagram:

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          ┌────┐
q_1: ─────┤ CX ├──
          └─┬──┘
q_2: ───────■──────
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In this circuit diagram, the qubits are represented as horizontal lines, and the CNOT gate is denoted by the "CX" symbol. The control qubit is qubit q_2, and the target qubit is qubit q_1.

Since the initial state of the system is $|11\rangle$, the control qubit (q_2) is in the state $|1\rangle$. Therefore, the target qubit (q_1) will be flipped, resulting in the final state of the system as $|\psi\rangle = |10\rangle$.

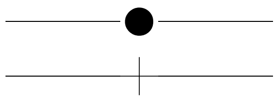
Answer 2

The CNOT (Controlled-NOT) gate is a two-qubit gate commonly used in quantum computing. It acts on two qubits, a control qubit and a target qubit. If the control qubit is in the state $|0\rangle$, then the target qubit remains unchanged. However, if the control qubit is in the state $|1\rangle$, then the target qubit is flipped (i.e., changes from $|0\rangle$ to $|1\rangle$ or vice versa).

In the given scenario, the quantum system is initially in the state $|\psi\rangle = |11\rangle$. This means that both qubits are in the state $|1\rangle$.

The CNOT gate is applied with the first qubit (leftmost qubit) as the target qubit and the second qubit (rightmost qubit) as the control qubit. This means that the second qubit acts as a switch that controls whether the first qubit is flipped or not.

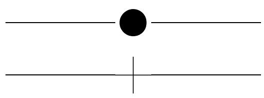
Graphically, the CNOT gate is represented as:



where the control qubit is the bottom qubit and the target qubit is the top qubit.

Applying the CNOT gate on the initial state $|11\rangle$, we get:

CNOT gate:



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Final state:
$|\psi\rangle = |10\rangle$

So, the final state of the system is $|10\rangle$, which means that the first qubit (target qubit) is flipped because the second qubit (control qubit) is in the state $|1\rangle$.

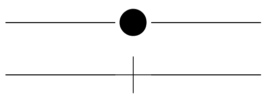
Answer 3

The CNOT gate, also known as the Controlled-NOT gate or CX gate, is a fundamental gate in quantum computing. It operates on two qubits, with one acting as the control qubit and the other as the target qubit. The gate flips the state of the target qubit (changes its value from 0 to 1 or from 1 to 0) only if the control qubit is in the state $|1\rangle$.

In your case, the initial state of the two qubits is $|11\rangle$, where the first qubit is the target qubit, and the second qubit is the control qubit. Applying a CNOT gate with the first qubit as the target and the second qubit as the control will result in the following action:

CNOT gate($|11\rangle$) = $|10\rangle$

The graphical representation of this circuit is as follows:



Here, the control qubit is represented by the line on top, and the target qubit is represented by the line on the bottom. The circle with an X denotes the CNOT gate, where the control qubit is connected to the gate, and the target qubit is the output of the gate.

After applying the CNOT gate, the final state of the system is $|10\rangle$, where the first qubit is 1 (flipped), and the second qubit remains 0.