

Question

Consider a quantum system made of two qubits. The system is in the initial state $|\psi\rangle = 1/\sqrt{N} (a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)$. A Swap gate is applied on the system. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

The Swap gate exchanges the states of two qubits. In the case of a two-qubit system, the Swap gate interchanges the states of the two qubits.

Graphically, the Swap gate can be represented using the following circuit:



Here, the vertical lines represent the qubits, and the circle with the "X" inside represents the Swap gate.

To determine the final state of the system after applying the Swap gate, let's consider the initial state $|\psi\rangle$:

$$|\psi\rangle = 1/\sqrt{N} (a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)$$

When the Swap gate is applied, the states of the two qubits are exchanged. Therefore, the final state of the system, $|\psi'\rangle$, would be:

$$|\psi'\rangle = 1/\sqrt{N} (a|00\rangle + c|01\rangle + b|10\rangle + d|11\rangle)$$

In the final state, the amplitudes of the states $|01\rangle$ and $|10\rangle$ have been swapped compared to the initial state. The amplitudes of the states $|00\rangle$ and $|11\rangle$ remain the same.

Answer 2

The Swap gate, also known as the SWAP gate or the CNOT gate with the control and target qubits exchanged, is a quantum gate that exchanges the states of two qubits. The action of the Swap gate on a two-qubit system can be

described as follows:

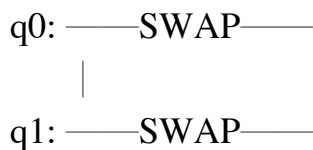
$$\text{SWAP } |00\rangle = |00\rangle$$

$$\text{SWAP } |01\rangle = |10\rangle$$

$$\text{SWAP } |10\rangle = |01\rangle$$

$$\text{SWAP } |11\rangle = |11\rangle$$

To represent this operation graphically, we can draw the following circuit:



Here, q_0 and q_1 represent the two qubits in the system, and the Swap gate is applied to each qubit in succession.

To apply the Swap gate to the initial state $|\psi\rangle$, we can simply apply the gate to each qubit in the state:

$$\text{SWAP } (a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle) = a|00\rangle + c|01\rangle + b|10\rangle + d|11\rangle$$

Therefore, the final state of the system is:

$$|\psi'\rangle = a|00\rangle + c|01\rangle + b|10\rangle + d|11\rangle$$

This is a state where the amplitudes of the states $|01\rangle$ and $|10\rangle$ have been swapped compared to the initial state $|\psi\rangle$.

Answer 3

The Swap gate is a quantum gate that exchanges the states of two qubits. In the case of a two-qubit system, the Swap gate swaps the amplitudes of the states $|01\rangle$ and $|10\rangle$ while leaving the other states unchanged.

The graphical representation of the circuit with a Swap gate applied to the two-qubit system is as follows:



q_1: _____

The final state of the system after applying the Swap gate can be obtained by evaluating the action of the gate on the initial state $|\psi\rangle$. Let's calculate it step by step:

Applying the Swap gate to the initial state $|\psi\rangle$:

$$\text{SWAP}|\psi\rangle = 1/\sqrt{N} (a|00\rangle + c|01\rangle + b|10\rangle + d|11\rangle)$$

Therefore, the final state of the system after the Swap gate is:

$$|\psi'\rangle = 1/\sqrt{N} (a|00\rangle + c|01\rangle + b|10\rangle + d|11\rangle)$$

As you can see, the Swap gate simply swaps the amplitudes of the states $|01\rangle$ and $|10\rangle$, leaving the other states unchanged.