Question
Consider a quantum system made of one qubit. The system is in the initial state $|\psi\rangle = |1\rangle$. A T gate is applied on $|\psi\rangle$. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1
The T gate, also known as the π/8 gate, is a single-qubit gate in quantum computing. It is represented by the following matrix:

```
T = [[1, 0],
     [0, e^(iπ/4)]]
```

When the T gate is applied to the initial state $|1\rangle$, the action of the gate can be described as follows:

```
T|1⟩ = [[1, 0],
        [0, e^(iπ/4)]] * [[0],
                          [1]]
     = [[0],
        [e^(iπ/4)]]
     = e^(iπ/4) * |1⟩
```

So, the final state of the system after applying the T gate to $|1\rangle$ is $e^{(i\pi/4)} * |1\rangle$. Note that $e^{(i\pi/4)}$ is a complex number representing a phase shift.

To represent this operation graphically, we can use a quantum circuit diagram. In this case, we have a single qubit, and the T gate is applied to it. Here's the circuit diagram:

```
|1⟩ -- T --
```

The initial state $|1\rangle$ is represented by a vertical line, and the T gate is denoted by a box labeled with T. The output of the gate is the final state of the system, which is $e^{(i\pi/4)} * |1\rangle$.

Answer 2
The T gate is a quantum gate that maps the quantum state $|\psi\rangle$ to the state obtained by applying a rotation of π/4 radians about the Z-axis of the Bloch

sphere. Mathematically, the T gate can be represented as:

T = [[1, 0], [0, e^(iπ/4)]]

where e^(iπ/4) is a complex number with magnitude 1 and argument π/4.

The circuit diagram for applying a T gate on a single qubit is shown below:

|ψ⟩ ——— T ————

To apply the T gate on the state |1⟩, we simply apply the T gate matrix to the column vector representing the state:

T|1⟩ = [[1, 0], [0, e^(iπ/4)]] |1⟩ = [0, e^(iπ/4)]^T

So the final state of the system is the quantum state represented by the column vector [0, e^(iπ/4)]^T. Geometrically, this state corresponds to a qubit with a probability amplitude of 1 at the |0⟩ state and a probability amplitude of e^(iπ/4) at the |1⟩ state on the Bloch sphere.

Answer 3
The T gate is one of the single-qubit gates in quantum computing, which is represented by the following matrix:

T = [[1, 0],
[0, e^(iπ/4)]],

where e^(iπ/4) represents a complex number with a phase of π/4.

When the T gate is applied to the initial state |ψ⟩ = |1⟩, the action can be represented by the following circuit:

|1⟩ ---- T ---- T|1⟩

In this circuit, the T gate is applied to the qubit in the initial state |1⟩, resulting in the final state:

T|1⟩ = [[1, 0],

[0, e^(iπ/4)]] |1⟩ = [[0],
[e^(iπ/4)]],

The final state can be written as:

|ψ'⟩ = [[0],
[e^(iπ/4)]].

So, the final state of the system after applying the T gate to the initial state |ψ⟩ = |1⟩ is |ψ'⟩ = [[0], [e^(iπ/4)]].