

### Question

Consider a quantum system made of one qubit. The system is in the initial state  $|\psi\rangle = |1\rangle$ . The quantum gate Pauli Y is applied on  $|\psi\rangle$ . Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

### Answer 1

The Pauli Y gate is represented by the following matrix:

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

When applied to a single-qubit state, this gate performs a rotation around the y-axis of the Bloch sphere, mapping the initial state  $|1\rangle$  to the state  $|0\rangle$  and vice versa. Mathematically, we can represent this as follows:

$$Y|1\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}|1\rangle = [i, 0] = i|0\rangle$$

Thus, the action of the Pauli Y gate on the initial state  $|1\rangle$  is to rotate it to the state  $|0\rangle$ , up to a global phase of  $i$ . The final state of the system is therefore:

$$|\psi\rangle = Y|1\rangle = i|0\rangle$$

A graphical representation of this circuit can be given by the following diagram:

$$|1\rangle \text{ --- } Y \text{ --- } i|0\rangle$$

Here, the state  $|1\rangle$  is input into the Y gate, which outputs the state  $i|0\rangle$ . Note that the global phase of  $i$  does not affect any observable quantities, so this final state is equivalent to the state  $|0\rangle$  up to an overall phase factor.

In terms of the Bloch sphere representation, the Pauli Y gate rotates the qubit state by 180 degrees around the y-axis, reflecting it across the x-z plane.

### Answer 2

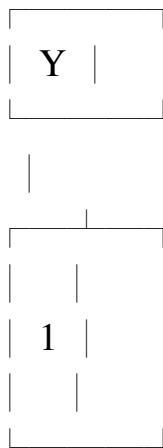
The Pauli Y gate is a single-qubit gate that corresponds to a rotation around the Y-axis of the Bloch sphere by 180 degrees. The matrix representation of the Pauli Y gate is:

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

Applying the Pauli Y gate to the initial state  $|1\rangle$  gives:

$$\begin{aligned}
 Y|1\rangle &= \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} |1\rangle \\
 &= \begin{bmatrix} -i \\ 0 \end{bmatrix} \\
 &= -i|0\rangle
 \end{aligned}$$

Therefore, the final state of the system is  $-i|0\rangle$ . Geometrically, the Pauli Y gate corresponds to a reflection of the state vector about the Y-axis of the Bloch sphere. The circuit representation of applying the Pauli Y gate to the initial state  $|1\rangle$  is:



Here, the gate Y is applied to the qubit in the initial state  $|1\rangle$ . The final state of the system is the output of this gate, which is  $-i|0\rangle$ .

### Answer 3

The Pauli Y gate is one of the basic quantum gates, which acts on a single qubit. It is represented by the following matrix:

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

where 'i' represents the imaginary unit.

The action of the Pauli Y gate on the initial state  $|\psi\rangle = |1\rangle$  can be calculated by multiplying the matrix representation of the gate with the vector representation of the state. In this case:

$$Y * |1\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} * \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix}$$

So, the final state of the system after applying the Pauli Y gate is  $| -i \rangle$ , which can also be expressed as  $-i|0\rangle$ .

Graphical representation of the circuit:

$$|1\rangle \text{ ----- } Y \text{ ----- } | -i \rangle$$

In the circuit diagram, the qubit starts in the state  $|1\rangle$ , then the Pauli Y gate is applied, and the final state is  $| -i \rangle$ .

Note: In quantum mechanics, we often represent the states as unit vectors, but for simplicity, I have used the Dirac notation to represent the states as kets (e.g.,  $|1\rangle$ ,  $| -i \rangle$ ).