Question
Consider a quantum system made of two qubits. The system is in the initial state $|\psi\rangle = |00\rangle$. A CNOT gate is applied on the system with the first qubit as target qubit and the second qubit as control qubit. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1
In a quantum system composed of two qubits, the CNOT (Controlled-NOT) gate is a fundamental two-qubit gate that operates on both qubits. The CNOT gate has the first qubit as the control qubit and the second qubit as the target qubit. When the control qubit is in the state $|1\rangle$, it flips the state of the target qubit, and when the control qubit is in the state $|0\rangle$, it leaves the target qubit unchanged.

In the given scenario, the initial state of the system is $|\psi\rangle = |00\rangle$, which means both qubits are in the state $|0\rangle$. When we apply the CNOT gate with the first qubit as the target qubit and the second qubit as the control qubit, the action of the gate can be described as follows:

If the control qubit (second qubit) is in the state $|0\rangle$, the target qubit (first qubit) remains unchanged. Therefore, the resulting state remains $|00\rangle$.
If the control qubit (second qubit) is in the state $|1\rangle$, the target qubit (first qubit) is flipped. Therefore, the resulting state becomes $|10\rangle$.

A graphical representation of the circuit can be shown as:

```
--C--
 |
-T--
```

Here, the control qubit is represented by "C" and the target qubit is represented by "T." The line connecting the control and target qubits represents the CNOT gate.
Therefore, applying the CNOT gate to the initial state $|\psi\rangle = |00\rangle$ with the first qubit as the target qubit and the second qubit as the control qubit leads to the final state $|10\rangle$.

Answer 2
Yes, I can describe the action of the CNOT gate on the two-qubit system.

The CNOT gate is a two-qubit gate that operates on a target qubit and a control qubit. If the control qubit is in the state |0⟩, the gate does nothing to the target qubit, but if the control qubit is in the state |1⟩, the gate performs a NOT operation on the target qubit.

In this case, the first qubit is the target qubit and the second qubit is the control qubit. Since the system is in the initial state |00⟩, both qubits are in the state |0⟩. Therefore, the control qubit is in the state |0⟩, and the gate does nothing to the target qubit. Thus, the final state of the system is still |00⟩.

A graphical representation of the circuit is shown below:

```
|0⟩ -- CNOT -- |00⟩
        |
        | (control)
        |
|0⟩ -------+
```

In this circuit, the control qubit is represented by the line on top, and the target qubit is represented by the line on the bottom. The CNOT gate is represented by the box in the middle, with the control qubit as the input on top and the target qubit as the output on the bottom. The initial state of the system is represented on the left, and the final state of the system is represented on the right.

In summary, the CNOT gate does not change the state of the system when the control qubit is in the state |0⟩, so the final state of the system is still |00⟩.

Answer 3
Certainly! The CNOT gate, also known as the controlled-X gate, is a two-qubit gate commonly used in quantum computing. It performs a NOT operation (flips the state) on the target qubit (the qubit being acted upon) if and only if the control qubit (the qubit determining the operation) is in the state |1⟩.

In your case, the initial state of the system is |ψ⟩ = |00⟩, where the first qubit is the target qubit (T) and the second qubit is the control qubit (C). Applying the CNOT gate with the first qubit as the target and the second qubit as the control, the action of the gate is as follows:

1. If the control qubit (C) is in the state $|0\rangle$, there is no change in the target qubit (T).
2. If the control qubit (C) is in the state $|1\rangle$, the target qubit (T) undergoes a NOT operation, flipping its state.

Here is a graphical representation of the circuit:

```
|ψ⟩ = |0⟩ -[CNOT]- |ψ'⟩
              |
              v
|ψ⟩ = |0⟩ ---X--- |ψ'⟩
```

The final state of the system can be obtained by applying the CNOT gate on the initial state $|\psi\rangle = |00\rangle$:

Since the control qubit (C) is initially $|0\rangle$, there is no change in the target qubit (T).
Therefore, the final state of the system is $|\psi'\rangle = |00\rangle$.
In other words, the CNOT gate leaves the system unchanged in this particular scenario.