

Question

Consider a quantum system made of one qubit. The system is in the initial state $|\psi\rangle = |1\rangle$. The quantum gate Hadamard is applied on $|\psi\rangle$. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

Sure, I can help you with that!

The Hadamard gate, denoted by the symbol H , is a fundamental quantum gate that operates on a single qubit. It maps the computational basis states $|0\rangle$ and $|1\rangle$ to equal superpositions of those states, also known as the "plus" and "minus" states:

$$H|0\rangle = |+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$$

$$H|1\rangle = |-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$$

Therefore, when the Hadamard gate is applied to the initial state $|1\rangle$, we have:

$$H|1\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$$

This means that the Hadamard gate creates an equal superposition of the $|0\rangle$ and $|1\rangle$ states, with a relative phase shift of $-\pi/2$ on the $|1\rangle$ state. Geometrically, this transformation can be represented as a rotation of the qubit state vector by 90 degrees about the axis that is halfway between the $|0\rangle$ and $|1\rangle$ states.

In terms of a circuit diagram, the operation of the Hadamard gate on a qubit can be represented by the following circuit:

$$|1\rangle \xrightarrow{H} (|0\rangle - |1\rangle)/\sqrt{2}$$

where the qubit is initialized in the $|1\rangle$ state, and the Hadamard gate is applied to it.

Therefore, the final state of the system after the Hadamard gate is applied to the initial state $|1\rangle$ is $(|0\rangle - |1\rangle)/\sqrt{2}$.

Answer 2

Yes, I can describe the action of the Hadamard gate on a qubit in the state $|1\rangle$.

The Hadamard gate is represented by the matrix:

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

When the Hadamard gate is applied to the qubit $|1\rangle$, we can write the operation as:

$$H|1\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

So, the Hadamard gate transforms the qubit in the state $|1\rangle$ into an equal superposition of the states $|0\rangle$ and $|1\rangle$. This is represented graphically as:

$$|1\rangle \xrightarrow{H} \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

The final state of the system is $\frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$, which is an equal superposition of the states $|0\rangle$ and $|1\rangle$.

Answer 3

Certainly! The Hadamard gate, often denoted as H , is a fundamental gate in quantum computing. When applied to a qubit, it performs a specific transformation on its state. Let's consider the initial state $|\psi\rangle = |1\rangle$.

The Hadamard gate operates on a single qubit and is represented by the following matrix:

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

To describe the action of the Hadamard gate on the state $|\psi\rangle = |1\rangle$, we multiply the Hadamard matrix by the state vector:

$$\begin{aligned} H|1\rangle &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= \frac{1}{\sqrt{2}} \begin{bmatrix} 0 + 1 \\ 0 - 1 \end{bmatrix} \\ &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \end{aligned}$$

Therefore, the final state of the system after applying the Hadamard gate on $|\psi\rangle = |1\rangle$ is:

$$\begin{aligned} H|1\rangle &= \frac{1}{\sqrt{2}} * |0\rangle + \frac{1}{\sqrt{2}} * |-1\rangle \\ &= \frac{1}{\sqrt{2}} * |0\rangle - \frac{1}{\sqrt{2}} * |1\rangle \end{aligned}$$

Graphically, the circuit representation for this scenario would be:

$|1\rangle$ --H-- Final State

In this representation, the input state $|1\rangle$ is acted upon by the Hadamard gate H, and the resulting state is the final state of the system.

Therefore, the final state of the system is $\frac{1}{\sqrt{2}} * |0\rangle - \frac{1}{\sqrt{2}} * |1\rangle$.