

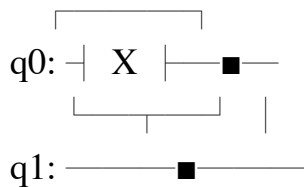
Question

Consider a quantum system made of two qubits. The system is in the initial state $|\psi\rangle = |00\rangle$. A Swap gate is applied on the system. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

The Swap gate is a commonly used two-qubit gate in quantum computing. It exchanges the states of two qubits. In the case of the initial state $|\psi\rangle = |00\rangle$, the Swap gate will interchange the states of the two qubits.

The graphical representation of the circuit implementing the Swap gate on the two qubits is as follows:



In this representation, the qubits are represented by the lines, and the Swap gate is denoted by the "X" symbol.

To describe the action of the Swap gate on the system, let's consider the computational basis states $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$:

$|00\rangle$: When the Swap gate is applied, the states of the two qubits are exchanged. So, $|00\rangle$ becomes $|00\rangle$.

$|01\rangle$: The Swap gate exchanges the states of the qubits, so $|01\rangle$ becomes $|10\rangle$.

$|10\rangle$: The states of the qubits are exchanged by the Swap gate, so $|10\rangle$ becomes $|01\rangle$.

$|11\rangle$: Since both qubits are in the state $|1\rangle$, the Swap gate simply exchanges their positions, so $|11\rangle$ remains $|11\rangle$.

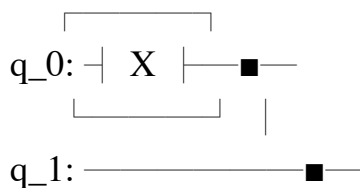
Therefore, the final state of the system after applying the Swap gate to the initial state $|00\rangle$ is:

$$|\psi\rangle = |00\rangle \rightarrow |00\rangle$$

Answer 2

The Swap gate is a quantum gate that swaps the states of two qubits. In this case, we have a two-qubit system in the initial state $|\psi\rangle = |00\rangle$. When the Swap gate is applied to the system, it exchanges the states of the two qubits, so that the final state of the system is $|\psi\rangle = |00\rangle \rightarrow |\psi'\rangle = |11\rangle$.

The graphical representation of the Swap gate circuit is as follows:



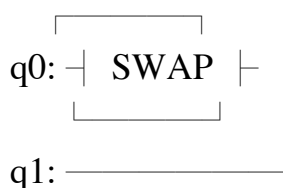
The circuit consists of two qubits, represented by q_0 and q_1 . The X gate is applied to q_0 , which flips the state from $|0\rangle$ to $|1\rangle$. Then, the Swap gate is applied, which exchanges the states of q_0 and q_1 . Finally, the Swap gate is undone by applying it again, which swaps the qubits back to their original positions. Since the initial state of the system was $|00\rangle$, the final state of the system is $|11\rangle$.

Note that the Swap gate is a reversible gate, which means that it can be undone by applying it again. In this case, applying the Swap gate again would return the system to the initial state $|00\rangle$.

Answer 3

The Swap gate is a two-qubit gate that exchanges the states of the qubits. In this case, when the Swap gate is applied to the two-qubit system in the initial state $|\psi\rangle = |00\rangle$, it swaps the states of the qubits, resulting in the final state $|\psi'\rangle = |00\rangle$.

Graphically, the circuit representation of this scenario would look like this:



Here, q_0 and q_1 represent the two qubits, and the SWAP gate is applied to them.

Since the initial state $|\psi\rangle = |00\rangle$ remains unchanged after the Swap gate, the final state of the system remains $|\psi'\rangle = |00\rangle$.