Question
Consider a quantum system made of two qubits. The system is in the initial state |ψ⟩ = |01⟩. A Control Z gate is applied on the system with the first qubit as target qubit and the second qubit as control qubit. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1
In a quantum system with two qubits, the Control Z gate, also known as the Controlled Pauli Z gate, applies a phase shift of -1 to the target qubit if and only if the control qubit is in the state |1⟩. The gate has no effect if the control qubit is in the state |0⟩.

The initial state of the system is given as |ψ⟩ = |01⟩, where the first qubit is in state |0⟩ and the second qubit is in state |1⟩.

To represent this gate operation graphically, we can use the following quantum circuit notation:

```
|0⟩ --[CZ]-- |0⟩
        |
        |
|1⟩ --[ ]-- |1⟩
```

In this representation, the first qubit is the target qubit and the second qubit is the control qubit. The CZ gate is applied between them. The lines connecting the qubits represent the entangling operation of the CZ gate.

Now, let's determine the final state of the system after applying the Control Z gate.

The state |01⟩ can be expressed as the tensor product of the individual qubit states: |0⟩ ⊗ |1⟩. Applying the CZ gate to this initial state gives us:

```
|0⟩ ⊗ |1⟩ --[CZ]-- |0⟩ ⊗ CZ(|1⟩)
|
|
--[ ]-- |1⟩ ⊗ |1⟩
```

Applying the CZ gate between the two qubits, we get:

$|0\rangle \otimes |1\rangle$ --[CZ]-- $|0\rangle \otimes -|1\rangle$
|
|
--[ ]-- $|1\rangle \otimes |1\rangle$

Simplifying the notation, we can write the final state as:

$|\psi\rangle = |01\rangle$ --[CZ]-- $|0\rangle \otimes -|1\rangle = |0\rangle \otimes (-|1\rangle) = |0\rangle \otimes -|1\rangle = -|01\rangle$

Therefore, the final state of the system after applying the Control Z gate is $-|01\rangle$.

Answer 2
The Control Z gate is a two-qubit gate that applies a phase shift of -1 to the target qubit when the control qubit is in the state $|1\rangle$, and leaves it unchanged when the control qubit is in the state $|0\rangle$. In other words, it flips the sign of the $|11\rangle$ state and leaves the other states unchanged.

The circuit diagram for this gate is:

```
|0⟩———————@———————
    |
|1⟩———————○———————
```

where the control qubit is represented by the $|1\rangle$ state and the target qubit by the $|0\rangle$ state. The symbol "@" represents the Control Z gate.

In the initial state $|\psi\rangle = |01\rangle$, the first qubit (the target qubit) is in the $|0\rangle$ state and the second qubit (the control qubit) is in the $|1\rangle$ state. Applying the Control Z gate to this state gives:

Control Z gate: $|01\rangle \rightarrow |01\rangle$
(sign flip only happens for $|11\rangle$ state)

Therefore, the final state of the system is the same as the initial state: $|\psi\rangle = |01\rangle$.

Answer 3

The Control Z gate, also known as the Controlled-Z gate or CZ gate, is a two-qubit gate that applies a phase flip (a rotation by π radians) on the target qubit if and only if the control qubit is in the state $|1\rangle$. In this case, the target qubit is the first qubit, and the control qubit is the second qubit.

Let's denote the basis states for the two qubits as $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$, where the first qubit represents the state of the target qubit, and the second qubit represents the state of the control qubit.

The action of the Control Z gate on the system can be described as follows:

- If the control qubit is in the state $|0\rangle$, there is no effect on the system.
- If the control qubit is in the state $|1\rangle$, the phase of the target qubit is flipped. In this case, the state $|01\rangle$ will acquire a negative phase, resulting in the final state $-|01\rangle$.

Now, let's represent the circuit graphically. The Control Z gate is usually represented by a box with a small "Z" inside, with the control qubit as the line with the small circle (control indicator) and the target qubit as the line without the circle.

$|\psi\rangle$ --[Control Z]-- $|\psi'\rangle$

The initial state of the system is $|\psi\rangle = |01\rangle$, and applying the Control Z gate, we get:

$|\psi'\rangle = -|01\rangle$

Therefore, the final state of the system is $-|01\rangle$.