Question
Consider a quantum system made of one qubit. The system is in the initial state $|\psi\rangle = 1/\sqrt{N}\ (a|0\rangle + b|1\rangle)$, where a and b are complex numbers and N is a normalization factor. A quantum gate Identity is applied on $|\psi\rangle$. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1
The quantum gate Identity, often denoted as I, is a trivial gate that does not alter the quantum state of a qubit. It effectively leaves the qubit unchanged. Mathematically, the Identity gate can be represented by the 2x2 identity matrix:

I = |1 0|
|0 1|

When the Identity gate is applied to the initial state $|\psi\rangle = 1/\sqrt{N} (a|0\rangle + b|1\rangle)$, the resulting state is simply the same as the initial state. Therefore, the final state of the system after applying the Identity gate is:

$|\psi'\rangle = 1/\sqrt{N} (a|0\rangle + b|1\rangle)$

In terms of the complex numbers a and b, the final state remains unchanged.

Graphically, the circuit representation of applying the Identity gate to the qubit can be depicted as follows:

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                ┌──────────┐
                 │ I │
|ψ⟩ ──────────┤    ├──────── |ψ'⟩
                └──────────┘
```

Here, the initial state $|\psi\rangle$ enters the Identity gate (I) and results in the final state $|\psi'\rangle$, which is the same as the initial state. The circuit diagram shows the flow of the quantum state through the gate.

Note that while the Identity gate does not have any effect on the qubit state, it is often included in quantum circuits for the purpose of standardization or to maintain the desired structure of a larger quantum algorithm.

Answer 2

When the Identity gate is applied to a qubit, it does nothing to the state of the qubit. That is, the Identity gate is equivalent to applying the identity matrix, which leaves the state unchanged.

In this case, when the Identity gate is applied to the initial state $|\psi\rangle = 1/\sqrt{N} (a|0\rangle + b|1\rangle)$, the resulting state is simply the same as the initial state: $|\psi\rangle = 1/\sqrt{N} (a|0\rangle + b|1\rangle)$.

A graphical representation of the circuit would look like:

$|\psi\rangle$ --- Id --- $|\psi\rangle$

where $|\psi\rangle$ represents the initial state and Id represents the Identity gate.

The final state of the system is the same as the initial state: $|\psi\rangle = 1/\sqrt{N} (a|0\rangle + b|1\rangle)$.

Answer 3

The Identity gate, often denoted as I or simply as an empty box, is a quantum gate that has no effect on the qubit it acts upon. It leaves the state of the qubit unchanged. Mathematically, the action of the Identity gate can be described as follows:

$I|0\rangle = |0\rangle$
$I|1\rangle = |1\rangle$

Since the Identity gate doesn't alter the state of the qubit, applying it to the initial state $|\psi\rangle = 1/\sqrt{N} (a|0\rangle + b|1\rangle)$ would yield the same state:

$I(1/\sqrt{N} (a|0\rangle + b|1\rangle)) = 1/\sqrt{N} (a|0\rangle + b|1\rangle)$

Therefore, the final state of the system remains as $|\psi\rangle = 1/\sqrt{N} (a|0\rangle + b|1\rangle)$.

Graphically, the circuit representation of applying the Identity gate to a qubit can be depicted as:

|ψ⟩ ----[I]----
Here, the |ψ⟩ state is input to the Identity gate [I], and the output remains the same.

It's worth noting that the Identity gate is often used as a placeholder or a no-operation (NOP) operation in quantum circuits.