

### Question

Consider a quantum system made of two qubits. The system is in the initial state  $|\psi\rangle = |11\rangle$ . A Swap gate is applied on the system. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

### Answer 1

Certainly! The Swap gate is a quantum gate that exchanges the states of two qubits. In the case of a quantum system with two qubits, applying a Swap gate will interchange the states of the qubits.

Let's denote the two qubits as qubit 1 and qubit 2. Initially, the system is in the state  $|\psi\rangle = |11\rangle$ , which means that qubit 1 is in the state  $|1\rangle$  and qubit 2 is also in the state  $|1\rangle$ .

The action of the Swap gate can be represented by the following circuit:

----Swap----  
|            |  
Qubit 1   Qubit 2

After applying the Swap gate, the final state of the system will be:

$$|\psi\rangle = |11\rangle \rightarrow \text{Swap} \rightarrow |\psi'\rangle = |11\rangle$$

Therefore, the final state of the system remains the same as the initial state,  $|11\rangle$ .

Note that the Swap gate does not change the individual states of the qubits, but it exchanges their positions or labels. In other words, it swaps the state of qubit 1 with qubit 2.

### Answer 2

The Swap gate exchanges the state of two qubits. In this case, if we apply the Swap gate on the two qubits in the state  $|11\rangle$ , we will get the following transformation:

$$|11\rangle \rightarrow \text{Swap}|11\rangle = |11\rangle.$$

The Swap gate leaves the state of the system unchanged since the two qubits are

already in the same state.

The graphical representation of the circuit is as follows:

$|11\rangle \text{ -- Swap -- } |11\rangle$

where the two qubits are represented by the vertical lines, and the Swap gate is represented by the horizontal line crossing them.

The final state of the system is the same as the initial state, i.e.,  $|11\rangle$ .

### Answer 3

The Swap gate, also known as the SWAP or CNOT gate, is a two-qubit gate that exchanges the states of two qubits. It is commonly represented as a circuit with two lines representing the qubits and a crossover symbol indicating the swap operation. In the case of the initial state  $|11\rangle$ , the action of the Swap gate can be described as follows:

The Swap gate swaps the states of the two qubits, resulting in the final state:

$|11\rangle \rightarrow |11\rangle$ .

Graphical representation of the circuit:

--X--  
--|--  
--X--

Explanation of the circuit:

The circuit consists of two gates represented by "X" symbols. Each gate acts on one of the qubits, effectively swapping their states. The vertical line represents the two qubits.

Final state of the system:

After applying the Swap gate, the final state remains unchanged as  $|11\rangle$ . The Swap gate essentially exchanges the states of the two qubits, but since both qubits are in the state  $|1\rangle$  initially, the result is the same state.