Question

Consider a quantum system made of one qubit. The system is in the initial state $|\psi\rangle = 1/\sqrt{N}$ ($a|0\rangle + b|1\rangle$), where a and b are complex numbers and N is a normalization factor. A quantum gate Pauli Y is applied on $|\psi\rangle$. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

The Pauli Y gate is a single-qubit gate that acts on the state vector of the qubit by performing a rotation around the y-axis of the Bloch sphere. Specifically, it maps the state $|0\rangle$ to the state $i|1\rangle$ and the state $|1\rangle$ to the state $-i|0\rangle$. Therefore, the action of the Pauli Y gate on the initial state $|\psi\rangle$ is:

$Y |\psi\rangle = 1/\sqrt{N}$ ($a\, Y|0\rangle + b\, Y|1\rangle$)

$= 1/\sqrt{N}$ ($a\, i|1\rangle - b\, i|0\rangle$)

To represent this circuit graphically, we can use a circuit diagram where the initial state $|\psi\rangle$ is input into the Pauli Y gate and the output state is measured:

$|\psi\rangle$ -- Y -- measurement

The final state of the system after the application of the Pauli Y gate is $1/\sqrt{N}$ ($a\, i|1\rangle - b\, i|0\rangle$).

Answer 2

The Pauli Y gate is a single-qubit gate in quantum computing that acts as a rotation around the y-axis of the Bloch sphere, which is a geometric representation of the state space of a single qubit. The Pauli Y gate is defined as:

Y = [0   -i]
   [i   0]

When the Pauli Y gate is applied to a qubit in the state $|0\rangle$, it transforms the state into $i|1\rangle$. Similarly, when it is applied to a qubit in the state $|1\rangle$, it transforms the state into $-i|0\rangle$. Thus, the Pauli Y gate introduces a phase shift of π radians between the $|0\rangle$ and $|1\rangle$ states.

Now, let's consider the initial state $|\psi\rangle = 1/\sqrt{N}$ ($a|0\rangle + b|1\rangle$). The action of the Pauli Y gate on this state can be represented by the following circuit:

$|\psi\rangle$ --- Y --- $\rangle$

The Y gate acts on the input state $|\psi\rangle$ to give the output state:

$Y|\psi\rangle = 1/\sqrt{N}$ ($aY|0\rangle + bY|1\rangle$)
$= 1/\sqrt{N}$ ($a(0 - i)|1\rangle + b(i|0\rangle)$)
$= 1/\sqrt{N}$ ($-ai|1\rangle + bi|0\rangle$)0

Therefore, the final state of the system after the application of the Pauli Y gate is $1/\sqrt{N}$ ($-ai|1\rangle + bi|0\rangle$). This state can be represented as a point on the Bloch sphere located on the negative y-axis.

Answer 3
The Pauli Y gate, denoted as Y, is one of the fundamental single-qubit gates in quantum computing. It is represented by the following matrix:

Y = [[0, -i],
[i, 0]],

where i is the imaginary unit.

To determine the action of the Pauli Y gate on the initial state $|\psi\rangle = 1/\sqrt{N}$ ($a|0\rangle + b|1\rangle$), we simply multiply the state vector by the Y gate matrix:

$Y|\psi\rangle = Y(1/\sqrt{N}\ (a|0\rangle + b|1\rangle))$
$= 1/\sqrt{N}\ (Y(a|0\rangle) + Y(b|1\rangle))$
$= 1/\sqrt{N}\ (aY|0\rangle + bY|1\rangle)$.

The action of the Pauli Y gate on the basis states $|0\rangle$ and $|1\rangle$ is as follows:

$Y|0\rangle$ = [[0, -i],

$[i, 0]]\ |0\rangle$
$= [[0, -i],$
$[i, 0]]\ [[1],$
$[0]]$
$= [[0],$
$[i]]$
$= i|1\rangle,$

$Y|1\rangle = [[0, -i],$
$[i, 0]]\ |1\rangle$
$= [[0, -i],$
$[i, 0]]\ [[0],$
$[1]]$
$= [[-i],$
$[0]]$
$= -i|0\rangle.$

Therefore, we can express the final state after applying the Pauli Y gate as:

$Y|\psi\rangle = 1/\sqrt{N}\ (aY|0\rangle + bY|1\rangle)$
$= 1/\sqrt{N}\ (a(i|1\rangle) + b(-i|0\rangle))$
$= 1/\sqrt{N}\ (ia|1\rangle - ib|0\rangle).$

Graphical representation of the circuit:

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        ┌───┐
q_0: ───┤ Y ├
        └───┘
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The final state of the system is $|\psi\rangle = 1/\sqrt{N}\ (ia|1\rangle - ib|0\rangle)$, where a and b are complex numbers and N is the normalization factor.