Question

Consider a quantum system made of two qubits. The system is in the initial state $|\psi\rangle = |10\rangle$. A CNOT gate is applied on the system with the first qubit as target qubit and the second qubit as control qubit. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

The CNOT gate, also known as the Controlled-NOT gate or CX gate, is a commonly used quantum gate in quantum computing. It acts on two qubits, with one qubit serving as the control qubit and the other as the target qubit. The CNOT gate applies the Pauli-X gate (or NOT gate) to the target qubit if and only if the control qubit is in the state $|1\rangle$.

In your case, the initial state of the system is $|\psi\rangle = |10\rangle$. This means the first qubit (leftmost) is in the state $|1\rangle$, and the second qubit (rightmost) is in the state $|0\rangle$. The CNOT gate is applied with the first qubit as the target and the second qubit as the control.

The graphical representation of the circuit is as follows:

$|\psi\rangle = |1\rangle \otimes |0\rangle$ ---[CNOT]---> $|\psi'\rangle$

Here, the CNOT gate is represented by the symbol [CNOT]. The target qubit is on the bottom, and the control qubit is on the top.

Now let's analyze the action of the CNOT gate on the system. Since the control qubit is in the state $|0\rangle$, there is no change applied to the target qubit. Thus, the final state of the system after applying the CNOT gate is still $|10\rangle$.

Therefore, the final state of the system remains unchanged: $|\psi'\rangle = |10\rangle$.

Answer 2

The CNOT gate is a two-qubit gate commonly used in quantum computing. It acts on two qubits, a control qubit and a target qubit, and flips the target qubit if the control qubit is in the state $|1\rangle$. The gate can be represented graphically as follows:

```
control qubit ----●----
                  |
target qubit -----X----
```

In this case, the first qubit is the target qubit and the second qubit is the control qubit, so the gate is applied as follows:

```
|1⟩ -----●----
         |
|0⟩ -----X----
```

The state of the two qubits before the gate is applied is |10⟩, which means the first qubit is in the state |1⟩ and the second qubit is in the state |0⟩. Applying the CNOT gate as described above results in the following state:

```
|1⟩ -----●----
         |
|0⟩ -----X----
          |
          |
          v
```

Since the second qubit is in the state |0⟩ (which is the state |+⟩ of the computational basis), the gate does not flip the target qubit and the final state of the system is:

```
|1⟩
 |
|0⟩
```

So the final state of the system is still |10⟩.

Answer 3

In a quantum system with two qubits, denoted as Qubit 1 and Qubit 2, the CNOT gate (controlled-NOT gate) is a commonly used gate in quantum computing. It operates on two qubits, where one qubit acts as the control qubit (Qubit 1 in this case) and the other qubit acts as the target qubit (Qubit 2 in this case). The CNOT gate flips the state of the target qubit (Qubit 2) if and only if the control qubit (Qubit 1) is in the state |1⟩.

Given that the initial state of the system is $|\psi\rangle = |10\rangle$, let's go step by step:

1. The state $|10\rangle$ represents Qubit 1 in the state $|1\rangle$ and Qubit 2 in the state $|0\rangle$.
2. Applying the CNOT gate with Qubit 1 as the target qubit and Qubit 2 as the control qubit, the gate checks the state of the control qubit (Qubit 2) and flips the state of the target qubit (Qubit 1) if the control qubit is in the state $|1\rangle$. In this case, the control qubit (Qubit 2) is in the state $|0\rangle$, so no flipping occurs.
3. As a result, the final state of the system remains the same: $|10\rangle$.

Graphical representation of the circuit:

```
|1⟩ --●--
      |
|0⟩ --X--
```

Here, the vertical line represents the control qubit (Qubit 2), and the horizontal line represents the target qubit (Qubit 1). The ● symbol denotes the CNOT gate, with the control qubit entering from the top and the target qubit entering from the left.

The final state of the system is $|10\rangle$, which means Qubit 1 remains in the state $|1\rangle$ and Qubit 2 remains in the state $|0\rangle$.