Question

Consider a quantum system made of one qubit. The system is in the initial state |ψ⟩ = |1⟩. The quantum gate Pauli Y is applied on |ψ⟩. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

The Pauli Y gate is represented by the following matrix:

Y = [[0, -i],

  [i, 0]]

When applied to a single-qubit state, this gate performs a rotation around the y-axis of the Bloch sphere, mapping the initial state |1⟩ to the state |0⟩ and vice versa. Mathematically, we can represent this as follows:

Y|1⟩ = [[0, -i], [i, 0]]|1⟩ = [i, 0] = i|0⟩

Thus, the action of the Pauli Y gate on the initial state |1⟩ is to rotate it to the state |0⟩, up to a global phase of i. The final state of the system is therefore:

|ψ⟩ = Y|1⟩ = i|0⟩

A graphical representation of this circuit can be given by the following diagram:

|1⟩ ---- Y ---- i|0⟩

Here, the state |1⟩ is input into the Y gate, which outputs the state i|0⟩. Note that the global phase of i does not affect any observable quantities, so this final state is equivalent to the state |0⟩ up to an overall phase factor.

In terms of the Bloch sphere representation, the Pauli Y gate rotates the qubit state by 180 degrees around the y-axis, reflecting it across the x-z plane.

Answer 2

The Pauli Y gate is a single-qubit gate that corresponds to a rotation around the Y-axis of the Bloch sphere by 180 degrees. The matrix representation of the Pauli Y gate is:

```
Y = [[0, -i],
    [i,  0]]
```

Applying the Pauli Y gate to the initial state |1⟩ gives:

```
Y|1⟩ = [[0, -i],
        [i,  0]] |1⟩
     = [[-i],
        [0]]
     = -i|0⟩
```

Therefore, the final state of the system is -i|0⟩. Geometrically, the Pauli Y gate corresponds to a reflection of the state vector about the Y-axis of the Bloch sphere. The circuit representation of applying the Pauli Y gate to the initial state |1⟩ is:

```
┌─────────┐
│  Y  │
└─────────┘
 │
┌────┴────┐
│    │
│  1  │
│    │
└─────────┘
```

Here, the gate Y is applied to the qubit in the initial state |1⟩. The final state of the system is the output of this gate, which is -i|0⟩.

Answer 3

The Pauli Y gate is one of the basic quantum gates, which acts on a single qubit. It is represented by the following matrix:

Y = [[0, -i], [i, 0]]

where 'i' represents the imaginary unit.

The action of the Pauli Y gate on the initial state |ψ⟩ = |1⟩ can be calculated by multiplying the matrix representation of the gate with the vector representation of the state. In this case:

Y * |1⟩ = [[0, -i], [i, 0]] * [[0], [1]] = [[-i], [0]]

So, the final state of the system after applying the Pauli Y gate is |-i⟩, which can also be expressed as -i|0⟩.

Graphical representation of the circuit:

|1⟩ ----- Y ----- |-i⟩

In the circuit diagram, the qubit starts in the state |1⟩, then the Pauli Y gate is applied, and the final state is |-i⟩.

Note: In quantum mechanics, we often represent the states as unit vectors, but for simplicity, I have used the Dirac notation to represent the states as kets (e.g., |1⟩, |-i⟩).