

Question

Consider a quantum system made of two qubits. The system is in the initial state $|\psi\rangle = 1/\sqrt{N} (a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)$. A CNOT gate is applied on the system with the first qubit as control qubit and the second qubit as target qubit. Can you describe the action of this gate on the system? Can you give a graphical representation of the circuit? What is the final state of the system?

Answer 1

The CNOT gate, also known as the controlled-NOT gate, is a two-qubit gate in quantum computing. It acts on two qubits: a control qubit and a target qubit. The gate flips the state of the target qubit if and only if the control qubit is in the state $|1\rangle$.

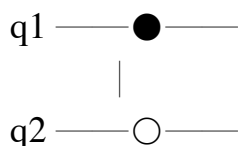
In the given system, the CNOT gate is applied with the first qubit (qubit 1) as the control qubit and the second qubit (qubit 2) as the target qubit. Let's analyze the action of the CNOT gate on the initial state $|\psi\rangle$:

- 1 If the control qubit (qubit 1) is in the state $|0\rangle$, there is no change in the target qubit (qubit 2). So the coefficients for $|10\rangle$ and $|11\rangle$ remain the same.
- 2 If the control qubit (qubit 1) is in the state $|1\rangle$, the target qubit (qubit 2) is flipped. Therefore, the coefficients for $|01\rangle$ and $|11\rangle$ are swapped.

Taking these into account, the action of the CNOT gate on the initial state $|\psi\rangle$ can be described as follows:

$$\text{CNOT}(|\psi\rangle) = 1/\sqrt{N} (a|00\rangle + b|01\rangle + c|11\rangle + d|10\rangle)$$

Now, let's provide a graphical representation of the circuit:



In this representation, the qubit labeled as q1 is the control qubit, and the qubit labeled as q2 is the target qubit. The \bullet represents the CNOT gate, with the control qubit being q1 and the target qubit being q2.

Finally, let's calculate the final state of the system after applying the CNOT gate:

$$\text{CNOT}(|\psi\rangle) = 1/\sqrt{N} (a|00\rangle + b|01\rangle + c|11\rangle + d|10\rangle)$$

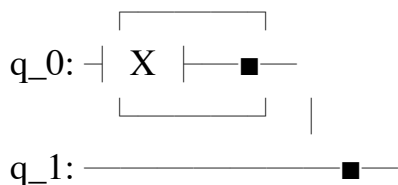
The final state of the system after applying the CNOT gate is given by the above expression. The coefficients for the states $|00\rangle$ and $|01\rangle$ remain the same, while the coefficients for $|11\rangle$ and $|10\rangle$ are swapped compared to the initial state.

I hope this clarifies the action of the CNOT gate and the final state of the system in your given scenario!

Answer 2

The CNOT (Controlled NOT) gate is a two-qubit gate that acts on the target qubit, conditional on the state of the control qubit. The gate flips the state of the target qubit if the control qubit is in the state $|1\rangle$, and leaves it unchanged if the control qubit is in the state $|0\rangle$. The CNOT gate is often used in quantum computation as a basic building block for more complex circuits.

The graphical representation of the CNOT gate is:



In this circuit, qubit 0 is the control qubit, and qubit 1 is the target qubit. The X gate on qubit 0 corresponds to a classical NOT gate, which flips the state of the control qubit. The vertical line represents a quantum controlled operation, which acts on the target qubit if the control qubit is in the state $|1\rangle$.

To apply the CNOT gate to the initial state $|\psi\rangle$, we need to evaluate the action of the gate on each basis state:

$$\text{CNOT } |00\rangle = |00\rangle$$

$$\text{CNOT } |01\rangle = |01\rangle$$

$$\text{CNOT } |10\rangle = |11\rangle$$

$$\text{CNOT } |11\rangle = |10\rangle$$

Using these results, we can write the final state of the system as:

$$\text{CNOT } |\psi\rangle = 1/\sqrt{N} (a|00\rangle + b|01\rangle + c|11\rangle + d|10\rangle)$$

The action of the CNOT gate flips the state of the second qubit if the first qubit is in the state $|1\rangle$, but leaves it unchanged if the first qubit is in the state $|0\rangle$.

Therefore, the final state of the system depends on the state of the first qubit. If the first qubit is in the state $|0\rangle$, the final state is unchanged, and if the first qubit is in the state $|1\rangle$, the state of the second qubit is flipped.

Note that the final state is not entangled, since the state of each qubit can be described independently.

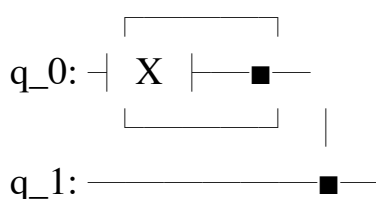
Answer 3

Certainly! The CNOT gate, also known as the controlled-NOT gate, is a two-qubit gate that operates on a control qubit and a target qubit. It performs a NOT operation (bit-flip) on the target qubit if and only if the control qubit is in the state $|1\rangle$. Otherwise, it leaves the target qubit unchanged.

In the given system, the CNOT gate is applied with the first qubit (qubit 1) as the control qubit and the second qubit (qubit 2) as the target qubit. The action of the CNOT gate on the initial state $|\psi\rangle$ is as follows:

- If the control qubit (qubit 1) is in the state $|0\rangle$, no operation is performed on the target qubit (qubit 2). So the state remains the same.
- If the control qubit (qubit 1) is in the state $|1\rangle$, a NOT operation is performed on the target qubit (qubit 2). So the state of the target qubit is flipped.

To represent this circuit graphically, we can use a quantum circuit diagram. The circuit diagram for the given system would look like this:



Here, the gate "X" represents the NOT gate, which is equivalent to applying a Pauli-X gate to the target qubit. The vertical line with the circle on top represents the control qubit, and the horizontal line represents the target qubit.

Now, let's determine the final state of the system after applying the CNOT gate.

The initial state of the system is given as:

$$|\psi\rangle = 1/\sqrt{N} (a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)$$

Let's go through each possible case:

1 If the control qubit (qubit 1) is in the state $|0\rangle$:

The state of the system remains the same.

$$\text{Final state: } |\psi\rangle = 1/\sqrt{N} (a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)$$

1 If the control qubit (qubit 1) is in the state $|1\rangle$:

The target qubit (qubit 2) undergoes a NOT operation.

$$\text{Final state: } |\psi'\rangle = 1/\sqrt{N} (a|00\rangle + b|01\rangle + d|10\rangle + c|11\rangle)$$

So, the final state of the system after applying the CNOT gate is $|\psi'\rangle = 1/\sqrt{N} (a|00\rangle + b|01\rangle + d|10\rangle + c|11\rangle)$.

Please note that the normalization factor $1/\sqrt{N}$ ensures that the state remains normalized, where $N = |a|^2 + |b|^2 + |c|^2 + |d|^2$.