
Function	Description
<code>ilsenc</code>	general function for solving interval lin. systems
<code>ilsjacobienc</code>	enclosure based on the Jacobi method
<code>ilsgsenc</code>	enclosure based on the Gauss–Seidel method
<code>ilsgeenc</code>	enclosure based on Gaussian elimination
<code>ilskrawczykenc</code>	enclosure based on Krawczyk’s method
<code>ilshbrenc</code>	the Hansen–Bielek–Rohn enclosure
<code>ilshullver</code>	verified hull
<code>ilshull</code>	unverified hull (faster)
<code>ige</code>	Gaussian elimination
<code>ibacksubst</code>	backward substitution
<code>ilsprecondinv</code>	preconditioning
<code>vsol</code>	verified solution of a real linear system
<code>isuns</code>	unsolvability test
<code>issolvable</code>	solvability test

12.4.3 oils

This package defines methods for overdetermined interval linear systems.

Function	Description
<code>oilsenc</code>	enclosure of an overdetermined int. lin. system
<code>oilshull</code>	same as <code>ilshull</code>
<code>oilsgeenc</code>	enclosure based on Gaussian elimination
<code>oilsrohnenc</code>	enclosure based on Rohn’s method
<code>oilssubsqenc</code>	enclosure by subsquares method
<code>oilsmultijacenc</code>	the multi-Jacobi method
<code>oilslsqenc</code>	enclosure of the least squares

12.4.4 idet

The package is devoted to determinants of interval matrices. Some of the functions were written by Josef Matějka.

Function	Description
<code>idet</code>	main function for computing an int. determinant
<code>idethull</code>	hull of determinant
<code>idethad</code>	determinant enclosure by Hadamard's inequality
<code>idetcram</code>	determinant enclosure by Cramer's rule
<code>idetgauss</code>	determinant enclosure by Gaussian elimination
<code>idetgersch</code>	determinant enclosure by Gershgorin discs
<code>idetencsym</code>	determinant enclosure for symmetric matrices

12.4.5 iest

This package covers various interval data regressions and estimations. Most of the functions in this package were implemented by Petra Pelikánová.

Function	Description
<code>iestlsq</code>	the least squares regression
<code>iest</code>	outer estimation
<code>iesttol</code>	tolerance interval regression

12.4.6 ieig

This package contains a few methods regarding eigenvalues. They are usually needed by other methods.

Function	Description
<code>eigsymdirect</code>	direct method for computing eigenvalues of a sym. matrix
<code>eigsymrohn</code>	fast outer enclosure of eigenvalues of a sym. matrix
<code>ieigbauerfike</code>	eigenvalues enclosure based on Bauer–Fike theorem
<code>igersch</code>	interval Gershgorin discs
<code>vereigsym</code>	verified eigenvalues of a real sym. matrix
<code>l1upperb</code>	upper bound on the largest eigenvalue

12.4.7 useful

This package contains useful methods that do not fit into other packages.

Function	Description
<code>area</code>	computes a generalized volume of an interval vector
<code>compareenc</code>	compares two interval enclosures
<code>generateyn</code>	generates all Y_n vectors
<code>latextablesimple</code>	prints a L ^A T _E Xtable from data
<code>radfi</code>	uniformly random number from an interval
<code>randfim</code>	uniformly random matrix from an interval matrix
<code>randseln</code>	selects n random elements from a list

12.4.8 iviz

LIME contains also various methods enabling display of interval results.

Function	Description
<code>plotboxes</code>	plotting interval boxes
<code>plotilssol</code>	plot solution set of an interval linear system

12.4.9 ocdoc

OcDoc is our own light-weight documentation system. To generate an `.html` documentation, go to a desired folder using the command `cd` in the Octave command line. Then simply call `ocdoc`. The function searches the current folder for `.m` files and for each such a file it generates an `.html` file containing documentation. It also generates a common `.html` index file for the whole folder. This way each package can be documented separately. To make OcDoc work it is necessary to keep the prescribed format of documentation in each `.m` file. A template `.m` file with the documentation structure is attached in the `doc` package. An example of an automatically generated documentation can be seen in Figure 12.3.

Even though, it is demanding to fully keep the structure of the file, it is favorable to do so, at least for the sake of future users. The documentation comments contain the following blocks:

- `.Author`. – name of the author(s),
- `.Input parameters`. – description of input parameters,
- `.Output parameters`. – description of output parameters,
- `.Implementation details`. – explanation of tricky details with references to papers and literature,
- `.History`. – history of changes,
- `.Todo`. – known mistakes, errors, future to do's and improvements.

```

function [c, d, state] = gausselim(A, b)
%BEGINDOC=====
% .Author.
%
% Jaroslav Horáček
%
%-----
% .Description.
%
% Gaussian elimination
%
% Function provides Gaussian elimination on an interval matrix [A\b]
% that represents a system Ax=b
%
%-----
% .Input parameters.
%
% A ... m x n matrix
% b ... right-hand side n-dimensional vector (or matrix of multiple
%      right-hand sides)
%
%-----
% .Output parameters.
%
% C ... eliminated matrix in upper triangular form with [1,1] intervals
%      on diagonal
% d ... right hand side
% state ... state is a string flag containing 'reg' if A is regular and
%           'sin' if A is possibly singular
%
%-----

```

[<< back to list of functions](#)

function gausselim

Author

Jaroslav Horáček

Description

Gaussian elimination

Function provides Gaussian elimination on an interval matrix [A\b] that represents a system Ax=b

Input parameters

A ... m x n matrix
b ... right-hand side n-dimensional vector (or matrix of multiple right-hand sides)

Output parameters

C ... eliminated matrix in upper triangular form with [1,1] intervals on diagonal
d ... right hand side
state ... state is a string flag containing 'reg' if A is regular and 'sin' if A is possibly singular

Figure 12.3: Example of OcDoc text documentation structure within an .m file (left) and the resulting .html page (right).

12.5 Access, installation, use

12.5.1 Installation

LIME is accessible online at

<https://kam.mff.cuni.cz/~horacek/lime>¹

To install it run `install.m` file. The only action it executes is adding the LIME directories into Octave PATH variable.

To make LIME work, one needs to have Octave Interval package installed. Detailed information, how to do that is provided at

https://octave.sourceforge.io/interval/package_doc/index.html²

12.5.2 User modifications

For the sake of modifying the existing code of LIME we give some recommendations that can contribute to preserving the overall structure of the toolbox.

To get a basic idea how each file is structured, there are prepared empty template files in the folder `ocdoc`.

LIME is divided into packages. Each package has its distinct functionality. Although, for some functions it might be arguable where they should be placed. If new functions are of common special functionality are designed, then a new package

¹Accessed on March 22nd, 2019.

²Accessed on March 22nd, 2019.

(folder) is recommended to be created. Remember, that the OcDoc documentation tool creates an `.html` documentation for each file in a given folder.

Functions have a simple naming convention – the name is composed using lower case abbreviations to describe its functionality. No upper case, no dashes, no hyphens are used. The first part of a function name usually consists of the name of the package the function comes from – `ilsgeenc` (comes from the package `ils`), `imatinv` (comes from the package `imat`). Then the rest of the function name is composed of functionality specification (e.g., `norm` for a function computing a norm, `hull` for a function computing the hull) – `imatnorm`, `idethull`. Also the specification of implementation of a function is usually added – `ilsjacobien`, `ilshbrenc`.

We remind again, that in order to make the automatic documentation work, the structure of the file must be kept.

Here we add some more recommendations:

- Methods do not always succeed. To indicate the state of the result we use the output variable `state`. We use short (mostly three-letter) string flags. The most common flags are '`vec`' – a finite vector or scalar is returned, '`sin`' – possible singularity occurred, '`zer`' – zero division occurred or pivot contains zero, '`inf`' – infinite result returned, '`exc`' – maximum number of iterations exceeded, '`empty`' – empty solution returned. For flags that can be returned by a given method see its `.m` file or documentation.
- Methods do not always return verified results. We use the output variable `ver` to indicate verified result. The value 1 means verification, the value 0 means the opposite.
- To indicate that a variable is an interval variable we use the prefix `i`. Hence `ix` (`iA`) is an interval vector (matrix).
- Most of the interval methods work only for interval input, when they are to be used for a real input, it must be intervalized first. It is a responsibility of a user to properly handle that (simply calling the Octave Interval function `infsupdec` on a real input might not be sufficient).
- If a method is implemented that has similar functionality to some existing method, first see its input and return parameters to make it consistent for other methods that can possibly use this method too.

Here is an example of a function definition satisfying the above recommendations:

```
[ix, state, it] = ilskrawczykenc(iA, ib, e, maxit, ioldx)
```

12.5.3 LIME under Matlab

The new version LIME² has not been migrated back to Matlab. Even though, we tried to keep the things similar. In LIME some Intlab names of interval functions can be

used. Such mirror functions are contained in folder `octave`. Some of the methods cause problems and cannot be simply renamed (it is mainly the case of `intval`, `isnan`).

For the case when there is a need to migrate LIME methods to Matlab and Intlab we could provide a few hints:

1. First it may be favorable to rename or delete the folder `octave`.
2. Some methods that have different names in Intlab and Octave may cause problems (for the list of such methods see Octave WIKI (https://wiki.octave.org/Interval_package)).³
3. We use Octave way of constructing intervals with flavors (`infsupdec` function), earlier versions of Intlab did not have such a method. However, if we can do without flavors, we can replace it with `infsup`.
4. There are some Matlab functions that are not currently implemented in Octave yet⁴.

³Accessed on March 22nd, 2019.

⁴See <https://www.gnu.org/software/octave/missing.html>, Accessed on March 22nd, 2019.

13

Additional materials

-
- ▶ List of author's publications
 - ▶ Supervised students
-

13.1 List of author's publications

13.1.1 Journal papers

- [83] J. Horáček and M. Hladík. Computing enclosures of overdetermined interval linear systems. *Reliable Computing*, 19:143, 2013.
- [86] J. Horáček, M. Hladík, and J. Matějka. Determinants of interval matrices. *Electronic Journal of Linear Algebra*, 33:99–112, 2018.
- [89] J. Horáček, V. Koucký, and M. Hladík. Novel approach to computerized breath detection in lung function diagnostics. *Computers in Biology and Medicine*, 101:1–6, 2018.

13.1.2 Conference and workshop papers

- [80] M. Hladík and J. Horáček. Interval linear programming techniques in constraint programming and global optimization. In M. Ceberio and V. Kreinovich, editors, *Constraint Programming and Decision Making*, pages 47–59. Springer, 2014.
- [81] M. Hladík and J. Horáček. A shaving method for interval linear systems of equations. In R. Wyrzykowski, J. Dongarra, K. Karczewski, and J. Waśniewski, editors, *Parallel Processing and Applied Mathematics*, volume 8385 of *Lecture Notes in Computer Science*, pages 573–581. Springer, 2014.
- [84] J. Horáček and M. Hladík. Subsquares approach – A simple scheme for solving overdetermined interval linear systems. In R. Wyrzykowski, J. Dongarra, K. Karczewski, and J. Waśniewski, editors, *Parallel Processing and Applied Mathematics*, volume 8385 of *Lecture Notes in Computer Science*, pages 613–622. Springer, Springer, 2014.

- [85] J. Horáček, M. Hladík, and M. Černý. Interval linear algebra and computational complexity. In N. Bebiano, editor, *International Conference on Matrix Analysis and its Applications*, pages 37–66. Springer, 2015.
- [87] J. Horáček, J. Horáček, and M. Hladík. Detecting unsolvability of interval linear systems. In M. Martel, N. Damouche, and J. A. D. Sandretto, editors, *TNC’18. Trusted Numerical Computations*, volume 8 of *Kalpa Publications in Computing*, pages 54–69. EasyChair, 2018.
- [88] J. Horáček, V. Koucký, and M. Hladík. Children lung function diagnostics – New methods for handling of clinical data. In *Proceedings of the 9th EAI International Conference on Bio-inspired Information and Communications Technologies (formerly BIONETICS)*, pages 174–176. ICST (Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering), 2016.

13.1.3 Unpublished work

- [90] J. Horáček, V. Koucký, and M. Hladík. Contribution of interval linear algebra to the ongoing discussions on multiple breath washout test. *arXiv preprint arXiv:1902.09026*, 2019.

13.2 Defended students

In this section defended Bachelor's theses supervised by the author of this work are listed. The joint research collaboration influenced the further research [90] and created a starting point for extension and further publication [86].

- [125] M. Mečiar. Visualisation of interval data (in czech). Bachelor's thesis, Charles University, Faculty of Mathematics and Physics, Prague, Czech Republic, 2015.
- [127] M. Milota. Psychologically-plausible and connectionism-friendly implementation of long-term memory (in czech). Bachelor's thesis, Charles University, Faculty of Mathematics and Physics, Prague, Czech Republic, 2016.
- [123] J. Matějka. Determinants of interval matrices (in czech). Bachelor's thesis, Charles University, Faculty of Mathematics and Physics, Prague, Czech Republic, 2017.
- [148] P. Pelikánová. Estimating data with use of interval analysis (in czech). Bachelor's thesis, Charles University, Faculty of Mathematics and Physics, Prague, Czech Republic, 2017.
- [210] A. Szabo. Application of branch and bound approach to parametric interval linear systems (in czech). Bachelor's thesis, Charles University, Faculty of Mathematics and Physics, Prague, Czech Republic, 2018.

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