OSTBAYERISCHE
TECHNISCHE HOCHSCHULE
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Formelsammlung Felder, Wellen und Leitungen

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1 Grundlagen

1.1 mathematische

Divergenz/Rotation/Gradient

div: macht aus einem Vektor ein Skalar.
rot: bildet ein Vektor auf Vektorfeld ab.
grad: bildet ein Skalar-/Gradientenfeld in ein Vektorfeld ab. Zeigt Richtung stärkster Zunahme des Feldes.

$$\operatorname{div}\vec{G} = \nabla \cdot \vec{G} = \frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y} + \frac{\partial G_z}{\partial z}$$

$$\begin{aligned} &= 0 \quad \Rightarrow \text{Volumen} \\ &> 0 \quad \Rightarrow \text{Quelle} \\ &< 0 \quad \Rightarrow \text{Senke} \end{aligned}$$

$$\operatorname{rot}\vec{G} = \nabla \times \vec{G} = \begin{pmatrix} \frac{\partial G_z}{\partial y} - \frac{\partial G_y}{\partial z} \\ \frac{\partial G_x}{\partial z} - \frac{\partial G_z}{\partial x} \\ \frac{\partial G_y}{\partial x} - \frac{\partial G_x}{\partial y} \end{pmatrix}$$

$$\operatorname{grad} G = \nabla \cdot G = \begin{pmatrix} \frac{\partial G}{\partial x} \\ \frac{\partial G}{\partial y} \\ \frac{\partial G}{\partial z} \end{pmatrix}$$

Nabla Operator

$$\nabla = \vec{\nabla} = \left(\frac{\partial G}{\partial x}, \frac{\partial G}{\partial y}, \frac{\partial G}{\partial z} \right)$$

Feldänderung bei Bewegung

$$\begin{aligned} \Delta G &= \frac{\partial G}{\partial x}\Delta x + \frac{\partial G}{\partial y}\Delta y + \frac{\partial G}{\partial z}\Delta z \\ &= dG = \operatorname{grad} G \cdot d\vec{s} \end{aligned}$$

1.2 Randbedingung

Dirichlet-RB	Funktion nimmt an den Rändern einen bestimmten Wert an (Bsp.: $\rho_r = 5V$)
Neumann-RB	Die Normalableitung der Fkt. nimmt an den Rändern einen bestimmten Wert an

1.3 Begriffe

	Begriff	Beschreibung
ρ	Raumladungsdichte	

1.4 Vergleich/Umrechnung

Kart.	Zyl.	Kug.
x	$r\cos\alpha$	$r\sin\vartheta\cos\alpha$
y	$r\sin\alpha$	$r\sin\vartheta\sin\alpha$
z	z	$r\cos\vartheta$
$\sqrt{x^2+y^2}$	r	
$\arctan\frac{y}{x}$	α	
z	z	
$dx\cos\alpha + dy\sin\alpha$	dr	
$dy\cos\alpha - dx\sin\alpha$	$r d\alpha$	
dz	dz	
$\sqrt{x^2+y^2+z^2}$		r
$\arctan\frac{y}{x}$		α
$\arctan\frac{\sqrt{x^2+y^2}}{z}$		ϑ
$dx\sin\vartheta\cos\alpha$ + $dy\sin\vartheta\sin\alpha$ + $dz\cos\vartheta$		dr
$dy\cos\alpha - dx\sin\alpha$		$r\sin\vartheta d\alpha$
$dx\cos\vartheta\cos\alpha$ + $dy\cos\vartheta\sin\alpha$ − $dz\sin\vartheta$		$r d\vartheta$

1.5 Kartesische Koordinaten

Einheitsvektoren:

$$\vec{e_x}, \vec{e_y}, \vec{e_z}$$

Rechtssystem:

$$\vec{e_x} \times \vec{e_y} = \vec{e_z}$$

Linienelemente:

$$ds = \sqrt{dx^2 + dy^2 + dz^2}$$

Nabla Operator:

$$\nabla = \frac{\partial}{\partial x}\vec{e_x} + \frac{\partial}{\partial y}\vec{e_y} + \frac{\partial}{\partial z}\vec{e_z}$$

Gradient:

$$\operatorname{grad}\varphi \equiv \nabla\varphi = \frac{\partial\varphi}{\partial x}\vec{e_x} + \frac{\partial\varphi}{\partial y}\vec{e_y} + \frac{\partial\varphi}{\partial z}\vec{e_z}$$

Divergenz:

$$\operatorname{div}\vec{D} \equiv \nabla\vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

Rotation:

$$\operatorname{rot}\vec{E} \equiv \nabla \times \vec{E} = \left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right]\vec{e_x} + \left[\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right]\vec{e_y} + \left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right]\vec{e_z}$$

Laplace Operator:

$$\Delta = \frac{\partial^2 \dots}{\partial x^2} + \frac{\partial^2 \dots}{\partial y^2} + \frac{\partial^2 \dots}{\partial z^2}$$

$$\begin{aligned}\Delta\vec{E} &= \operatorname{grad}\operatorname{div}\vec{E} - \operatorname{rot}\operatorname{rot}\vec{E} = \Delta E_x\vec{e_x} + \Delta E_y\vec{e_y} + \Delta E_z\vec{e_z} \\ &= \left[\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}\right]\vec{e_x} + \left[\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2}\right]\vec{e_y} + \left[\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2}\right]\vec{e_z}\end{aligned}$$

1.6 Zylinderkoordinaten

Variablen:

$$r, \alpha, z$$

Einheitsvektoren:

$$\vec{e_r}, \vec{e_\alpha}, \vec{e_z}$$

Rechtssystem:

$$\vec{e_r} \times \vec{e_\alpha} = \vec{e_z}$$

Linienelemente:

$$ds = \sqrt{dr^2 + r^2 d\alpha^2 + dz^2}$$

Volumenelemente:

$$dv = r dr d\alpha dz$$

Nabla Operator:

$$\nabla = \frac{\partial}{\partial r}\vec{e_r} + \frac{1}{r}\frac{\partial}{\partial\alpha}\vec{e_\alpha} + \frac{\partial}{\partial z}\vec{e_z}$$

Gradient:

$$\operatorname{grad}\varphi \equiv \nabla\varphi = \frac{\partial\varphi}{\partial r}\vec{e_r} + \frac{1}{r}\frac{\partial\varphi}{\partial\alpha}\vec{e_\alpha} + \frac{\partial\varphi}{\partial z}\vec{e_z}$$

Divergenz:

$$\operatorname{div}\vec{D} \equiv \nabla\vec{D} = \frac{1}{r}\frac{\partial\left(r\vec{D}_r\right)}{\partial r} + \frac{1}{r}\frac{\partial\vec{D}_\alpha}{\partial\alpha} + \frac{\partial\vec{D}_z}{\partial z}$$

Rotation:

$$\operatorname{rot}\vec{E} \equiv \nabla \times \vec{E} = \left[\frac{1}{r}\frac{\partial E_z}{\partial\alpha} - \frac{\partial E_\alpha}{\partial z}\right]\vec{e_r} + \left[\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r}\right]\vec{e_\alpha} + \left[\frac{1}{r}\frac{\partial\left(rE_\alpha\right)}{\partial r} - \frac{1}{r}\frac{\partial E_r}{\partial\alpha}\right]\vec{e_z}$$

Laplace Operator:

$$\Delta = \frac{1}{r}\frac{\partial\left(r\frac{\partial\dots}{\partial r}\right)}{\partial r} + \frac{1}{r^2}\frac{\partial^2\dots}{\partial\alpha^2} + \frac{\partial^2\dots}{\partial z^2}$$

$$\vec{E} = \left[\Delta E_r - \frac{2}{r^2}\frac{\partial E_\alpha}{\partial\alpha} - \frac{E_r}{r^2}\right]\vec{e_r} + \left[\Delta E_\alpha + \frac{2}{r^2}\frac{\partial E_r}{\partial\alpha} - \frac{E_\alpha}{r^2}\right]\vec{e_\alpha} + \left[\Delta E_z\right]\vec{e_z}$$

1.7 Kugelkoordinaten

Variablen:

$$r, \vartheta, \alpha$$

Einheitsvektoren:

$$\vec{e_r}, \vec{e_\vartheta}, \vec{e_\alpha}$$

Rechtssystem:

$$\vec{e_r} \times \vec{e_\vartheta} = \vec{e_\alpha}$$

Linienelement:

$$ds = \sqrt{dr^2 + r^2 \sin^2 \vartheta d\alpha^2 + r^2 d\vartheta^2}$$

Volumenelement:

$$dv = r^2 \sin \vartheta dr d\vartheta d\alpha$$

Nabla Operator:

$$\nabla = \frac{\partial}{\partial r}\vec{e_r} + \frac{1}{r}\frac{\partial}{\partial \vartheta}\vec{e_\vartheta} + \frac{1}{r \sin \vartheta}\frac{\partial}{\partial \alpha}\vec{e_\alpha}$$

Gradient:

$$\operatorname{grad} \varphi \equiv \nabla \varphi = \frac{\partial \varphi}{\partial r}\vec{e_r} + \frac{1}{r}\frac{\partial \varphi}{\partial \vartheta}\vec{e_\vartheta} + \frac{1}{r \sin \vartheta}\frac{\partial \varphi}{\partial \alpha}\vec{e_\alpha}$$

Divergenz:

$$\operatorname{div} \vec{D} \equiv \nabla \vec{D} = \frac{1}{r^2}\frac{\partial \left(r^2 D_r\right)}{\partial r} + \frac{1}{r \sin \vartheta}\frac{\partial \left(\sin \vartheta \cdot D_\vartheta\right)}{\partial \vartheta} + \frac{1}{r \sin \vartheta}\frac{\partial D_\alpha}{\partial \alpha}$$

Rotation:

$$\operatorname{rot} \vec{E} \equiv \nabla \times \vec{E} = \frac{1}{r \sin \vartheta}\left[\frac{\partial \left(\sin \vartheta \cdot E_\alpha\right)}{\partial \vartheta} - \frac{\partial E_\vartheta}{\partial \alpha}\right]\vec{e_r} + \frac{1}{r}\left[\frac{1}{\sin \vartheta}\frac{\partial E_r}{\partial \alpha} - \frac{\partial r E_\alpha}{\partial r}\right]\vec{e_\vartheta} + \frac{1}{r}\left[\frac{\partial \left(r E_\vartheta\right)}{\partial r} - \frac{\partial E_r}{\partial \vartheta}\right]\vec{e_\alpha}$$

Laplace Operator:

$$\Delta = \frac{1}{r^2}\frac{\partial \left(r^2 \frac{\partial ..}{\partial r}\right)}{\partial r} + \frac{1}{r^2 \sin \vartheta}\frac{\partial \left(\sin \vartheta \frac{\partial ..}{\partial \vartheta}\right)}{\partial \vartheta} + \frac{1}{r^2 \sin^2 \vartheta}\frac{\partial^2 ..}{\partial \alpha^2}$$

Laplace Operator in Kugelkoordinaten, angewandt auf einen Vektor:

$$\begin{aligned}\Delta \vec{E} = &\left[\Delta E_r - \frac{2}{r^2}E_r - \frac{2}{r^2 \sin \vartheta}\frac{\partial \left(\sin \vartheta \cdot E_\vartheta\right)}{\partial \vartheta} - \frac{2}{r^2 \sin \vartheta}\frac{\partial E_\alpha}{\partial \alpha}\right]\vec{e_r} \\ &+ \left[\Delta E_\vartheta - \frac{E_\vartheta}{r^2 \sin^2 \vartheta} + \frac{2}{r^2}\frac{\partial E_r}{\partial \vartheta} - \frac{2 \cot \vartheta}{r^2 \sin \vartheta}\frac{\partial E_\alpha}{\partial \alpha}\right]\vec{e_\vartheta} \\ &+ \left[\Delta E_\alpha - \frac{E_\alpha}{r^2 \sin^2 \vartheta} + \frac{2}{r^2 \sin \vartheta}\frac{\partial E_r}{\partial \alpha} + \frac{2 \cot \vartheta}{r^2 \sin \vartheta}\frac{\partial E_\vartheta}{\partial \alpha}\right]\vec{e_\alpha}\end{aligned}$$

2 Maxwell'schen Gleichungen

2.1 Intergralform I, II

Gauß'sches Gesetz
Induktionsgesetz
Durchflutungsgesetz
Quellenfreiheit B-Feld
Zusammenhang

$$\oiint_{A_{\texttt{Hülle}}} \vec{D} d\vec{A} = Q_{\texttt{eing.}}$$
$$\oint_{\texttt{Rand}} \vec{E} d\vec{s} = -\frac{d\Phi_{\texttt{eing.}}}{dt}$$
$$\oint_{\texttt{Rand}} \vec{H} d\vec{s} = I_{\texttt{eing.}} + I_{\texttt{versch.}}$$
$$\oiint_{A_{\texttt{Hülle}}} \vec{B} d\vec{A} = 0$$

$$\vec{D} = \varepsilon \cdot \vec{E} \qquad \vec{B} = \mu \cdot \vec{H}$$

Bei isotropen Stoffen sind ε u. μ Skalare:

$$\varepsilon = \varepsilon_0 \cdot \varepsilon_r \qquad \mu = \mu_0 \cdot \mu_r$$

2.2 Differentialform I, II

$$\operatorname{div} \vec{D} = \rho$$
$$\operatorname{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\operatorname{rot} \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$
$$\operatorname{div} \vec{B} = 0$$

2.3 stationäre Felder

$$\nabla \cdot \vec{D} = \rho \qquad \vec{D} = \varepsilon \cdot \vec{E}$$
$$\nabla \times \vec{E} = 0$$
$$\nabla \times \vec{H} = \vec{J} \qquad \vec{B} = \mu \cdot \vec{H}$$
$$\nabla \cdot \vec{B} = 0$$
$$\vec{J} = \kappa \vec{E}$$

3 Felder

3.1 Elektrostatik

ist ein wirbelfreies Feld. Elek. Ladungen sind Quellen des Feldes.

$$\operatorname{div} \vec{D} = \nabla \cdot \vec{D} = \rho \qquad \vec{D} = \varepsilon \vec{E}$$
$$\operatorname{rot} \vec{E} = \nabla \times \vec{E} = 0 \quad = \operatorname{rot} \operatorname{grad} E$$
$$\vec{E} = -\operatorname{grad} \varphi$$

3.1.1 Potetialgleichung

$$\operatorname{div} \operatorname{grad} = -\frac{\rho}{\varepsilon}$$

$\Rightarrow$ **Poisson-Gleichung** mit $\rho = 0 \Rightarrow$ **Laplace-Gleichung**

$$\Delta\varphi + \underbrace{\frac{\operatorname{grad} \varepsilon \cdot \operatorname{grad} \varphi}{\varepsilon}}_{=0,\ wenn\ homogen} = -\frac{\rho(x,y,z)}{\varepsilon}$$
$$\frac{d^2\varphi}{dx^2} + \frac{d^2\varphi}{dy^2} + \frac{d^2\varphi}{dz^2} = -\frac{\rho(x,y,z)}{\varepsilon}$$