清華大學 交叉信息研究院
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Probing many-body Bell correlations with variational quantum algorithms

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2-Nov-2024 at JuliaCN Meetup, HKUST Guangzhou

Joint work with my colleagues at IIIS, Tsinghua Univ., H. Wang's superconducting team from Zhejiang Univ., and Jordi's group from Instituut-Lorentz, Leiden Univ.

Wang*, **WKL***, Xu*, Hu, Emonts, Tura, Song+, Wang+, Deng+ et al, arXiv:2406.17841 (2024)
WKL*, Hu*, Deng, Song+, Wang, Emonts+, Tura et al, arXiv:2407.12347 (2024)

Numerical explorations powered by

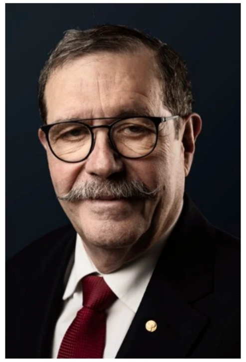
julia

Motivation | Bell correlations

- **Bell correlations** are stronger than entanglement
- **Resource** for device-independent QIP
  - Less assumptions
    - Black-box devices can be untrusted
    - Loophole-free Bell tests performed

    [Hensen *et al*, Giustina *et al*, Shalm *et al*, Li *et al*, Rosenfeld *et al*, Storz *et al*]
  - Device-independent applications
    - Quantum key distribution
    - Randomness expansion
    - Randomness Amplification
    - Self-testing
  - Unconditional quantum advantages
    - Computational advantanges [Bravyi *et al*, Science 362, 308 (2018); Le Gall, CCC 2019]
    - Generative models [Gao et al, Phys. Rev. X 12, 021037 (2022)]
    - Supervised learning [Zhang, Gong, **WKL**, Deng, arxiv:2405.00770 (2024)]

The Nobel Prize in Physics 2022



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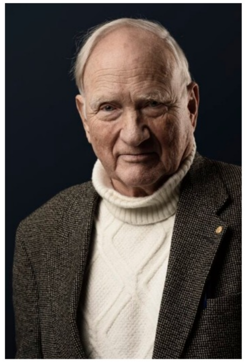

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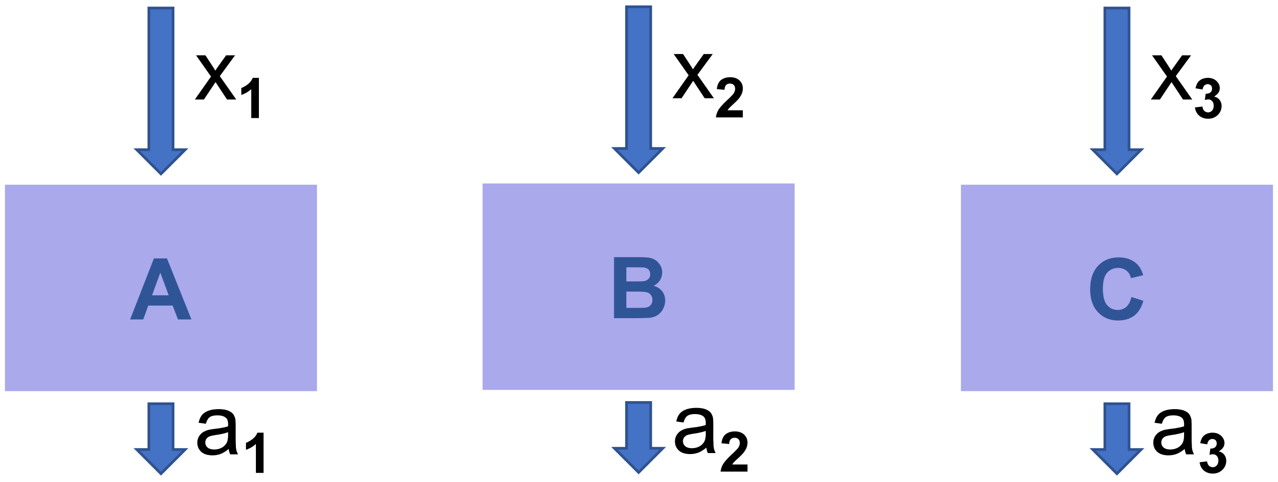
- **Less studied** due to
  - Mathematical complexity
  - Experimentally demanding
  - Quantum description of multipartite states grows exponentially

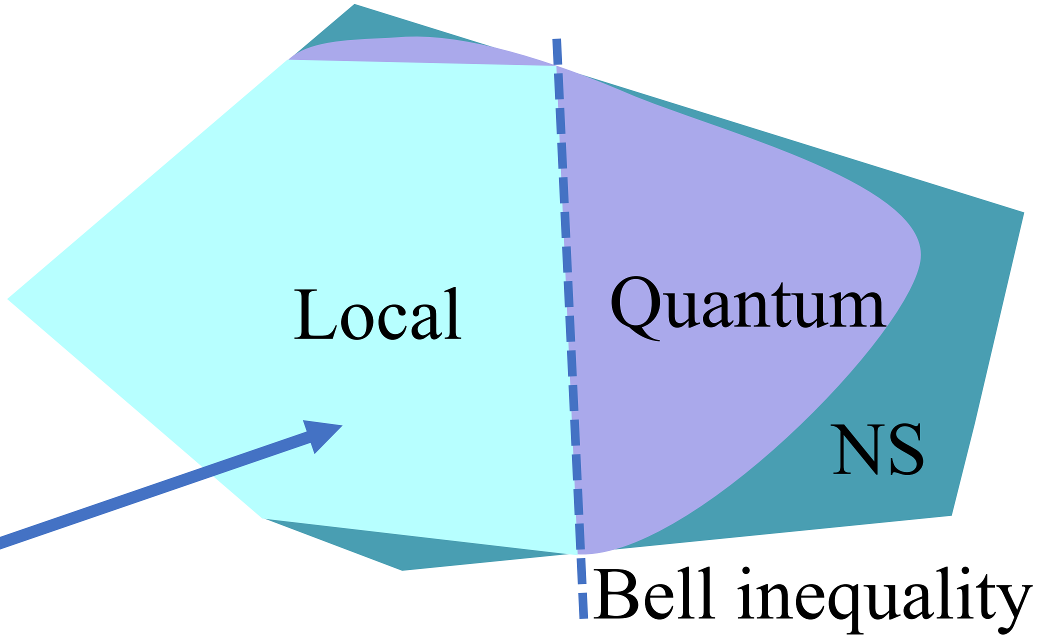
- **Recent developments**
  - **Theory**: Permutationally invariant systems [Tura et al, Science 344 1256 (2014)]
  - **Experiment**: [Schmied et al, Science 352 441 (2016), Engelsen *et al*, Phys. Rev. Lett. 118, 140401 (2017)]

- **Limitations** of previous works
  - Previous experiments focused on ensembles of atoms **without individual addressing**
  - Previous works rely on carefully designed Bell correlation witnesses suitable for spin-squeezed states and thus lack **general applicability** to other Bell inequalities
  - Previous experimental works mainly reveal Bell correlations in quantum many-body systems, while the Bell correlation **depth** has not been studied



$$P(a_1 \dots a_n | x_1 \dots x_n)$$

$$\vec{v} = \{P(\vec{a} \mid \vec{x}) \ \ \forall \vec{a}, \vec{x}\}$$



A simple example, the CHSH inequality: $\mathcal{I} = \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle$

- Local hidden variable model (LHVM): $|\mathcal{I}| \leq 2$
- Sharing quantum correlations: **A conflicting result**

$$\mathcal{J} = \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle$$

Let the two parties share a singlet state

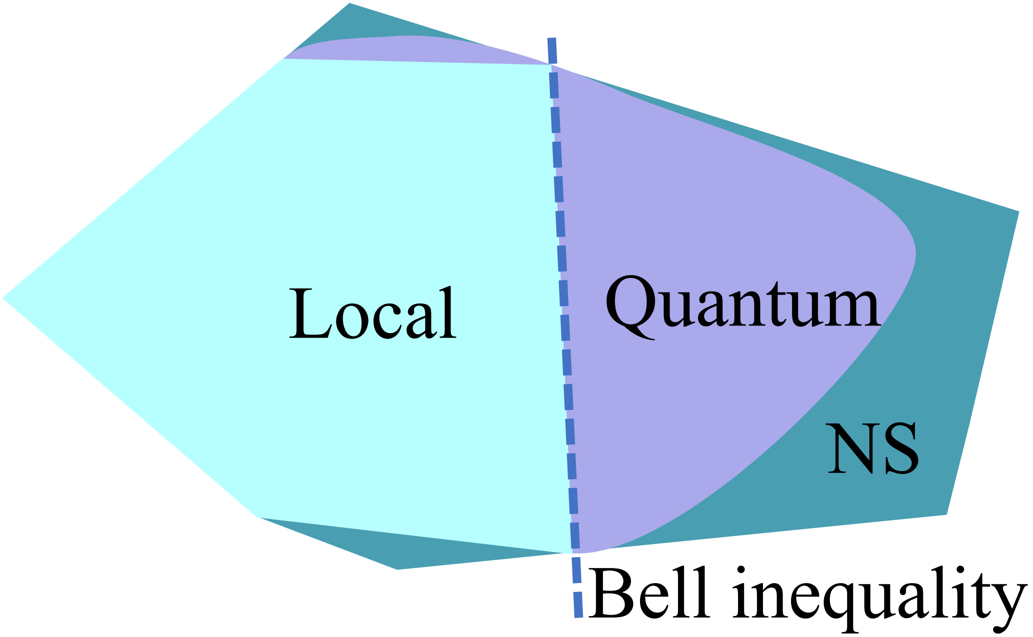
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

Party A perform measurements

$$A_0 = \sigma_z, A_1 = \sigma_x,$$

and Party B perform measurements

$$B_0 = \frac{1}{\sqrt{2}}(\sigma_z + \sigma_x), B_1 = \frac{1}{\sqrt{2}}(\sigma_z - \sigma_x)$$



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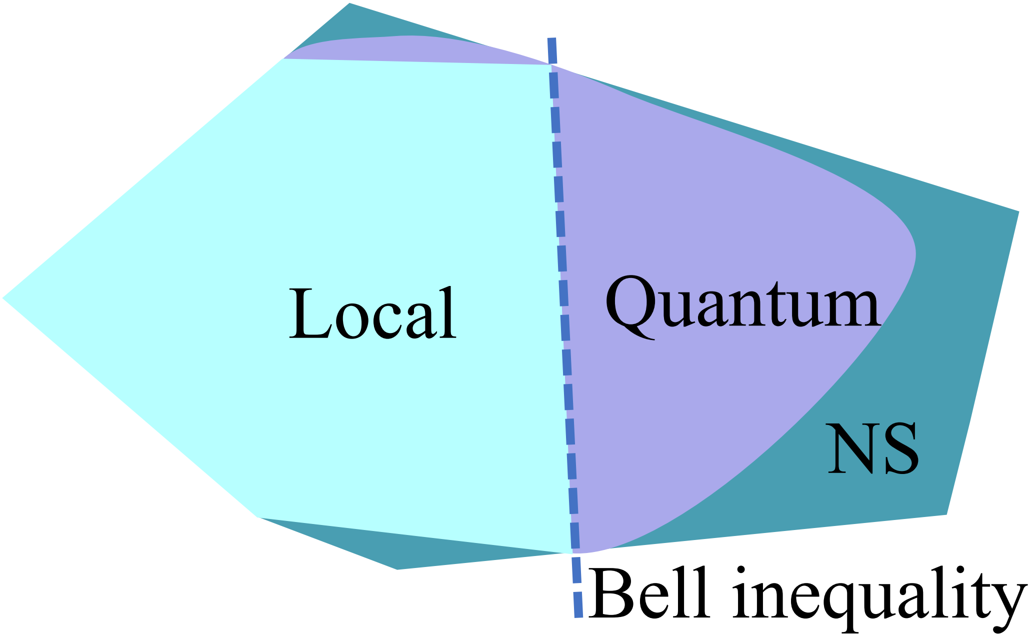
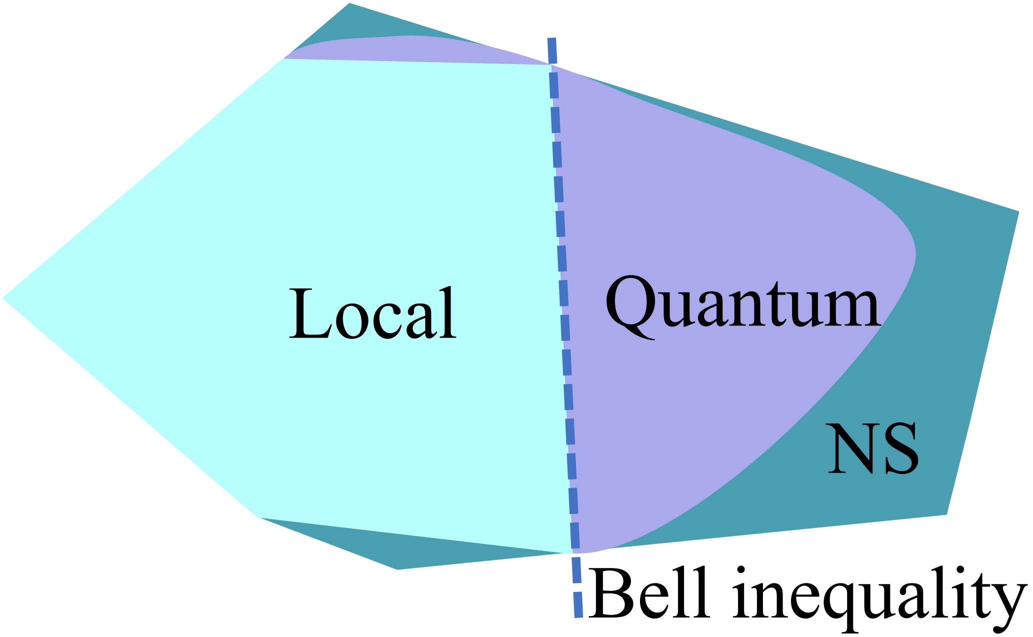


$\mathcal{J} = 2\sqrt{2}$, **a violation**

$$A_0 = \sigma_z, A_1 = \sigma_x,$$

$$\mathcal{J} = \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle$$

$$B_0 = \frac{1}{\sqrt{2}}(\sigma_z + \sigma_x), B_1 = \frac{1}{\sqrt{2}}(\sigma_z - \sigma_x)$$



$\mathcal{J}$ ⇒ A Bell operator $\sigma_x^A \sigma_x^B + \sigma_z^A \sigma_z^B$

⇔ A Hamiltonian

⇒ Low-energy states certify nonlocality

Overview

Hamiltonian
$\mathcal{H}$

Bell operator
$\mathcal{B}$

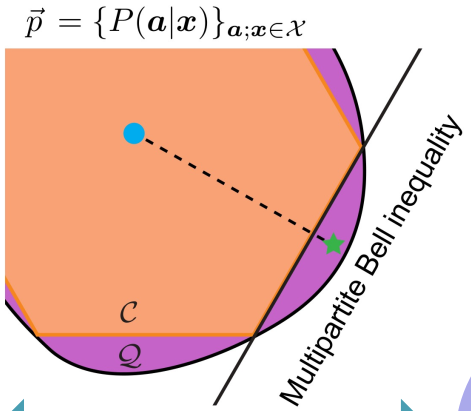
Inequality coefficients

Suitable observables

Bell inequality
I

- Jordan-Wigner
- DMRG

- Dynamic programming

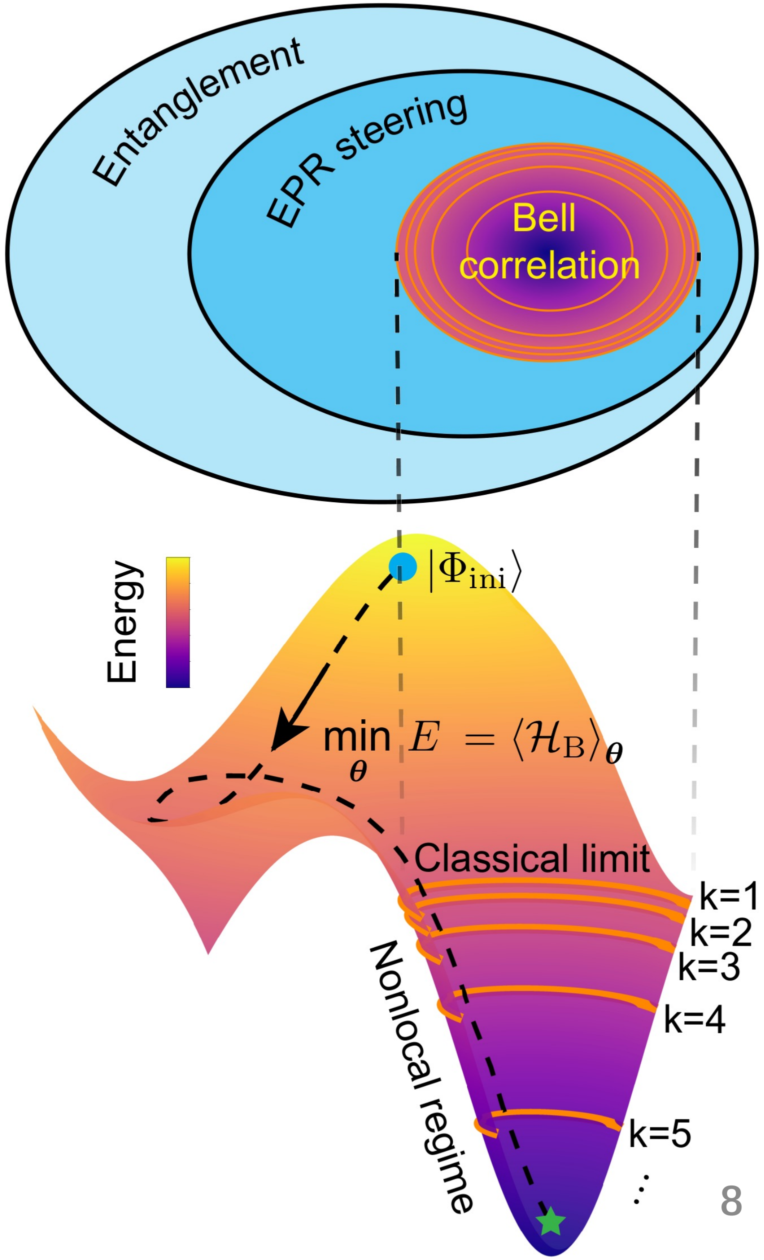


Ground state energy

Quantum value

Violations?

Classical bound



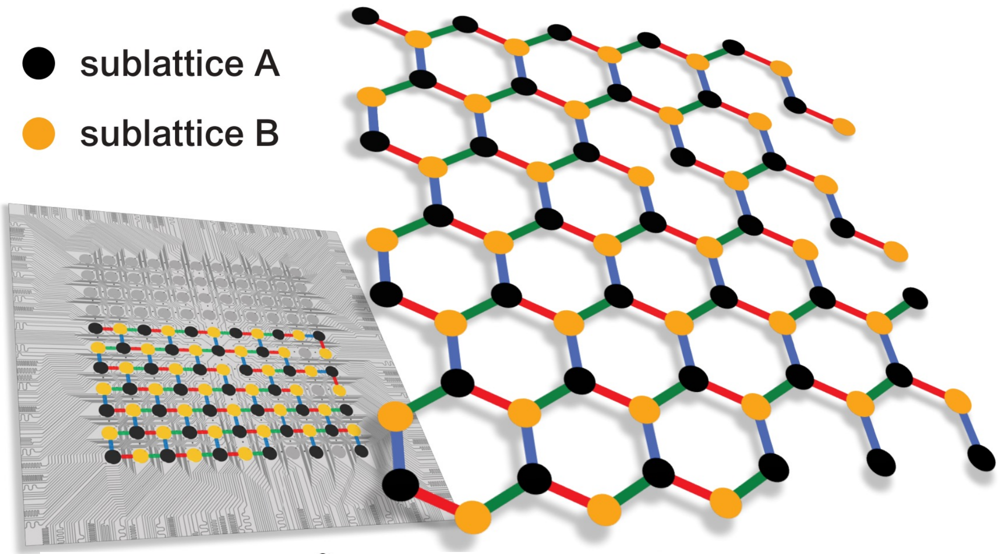
Bell corr. in a honeycomb model

Bell inequality:

$$\mathcal{I} = \sum_{\tau-\text{link}} J_\tau(\epsilon) \sum_{x,y} (-1)^{x \cdot y} \langle A_x B_y \rangle_\tau \geq \beta_{\mathrm{C}}$$

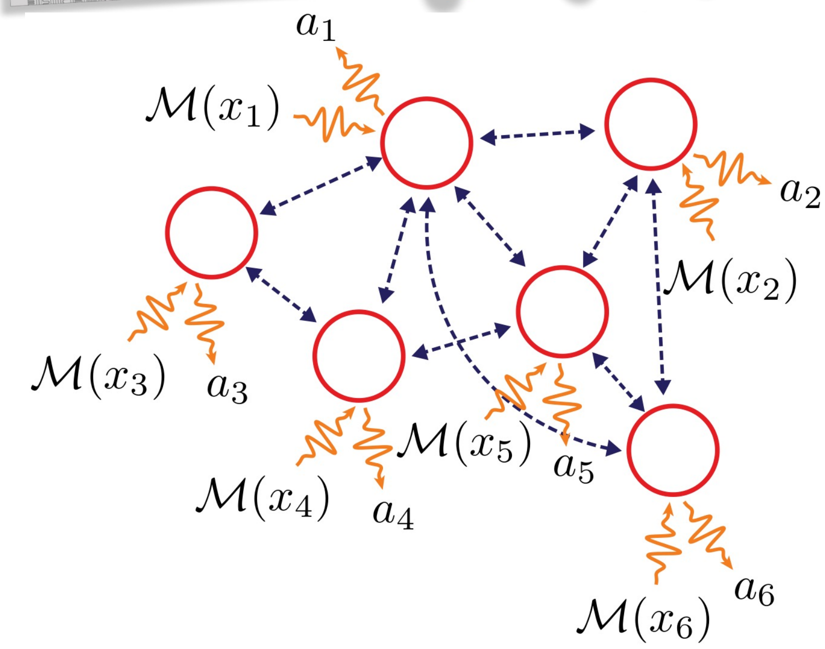
$J_\tau(\epsilon) = 1 + \epsilon$ and $\frac{1-\epsilon}{2}$ for $\tau = \mathrm{r}$ and $\tau = \mathrm{b, g}$, respectively

$\beta_{\mathrm{C}} = -2 \sum_\tau J_\tau(\epsilon)$

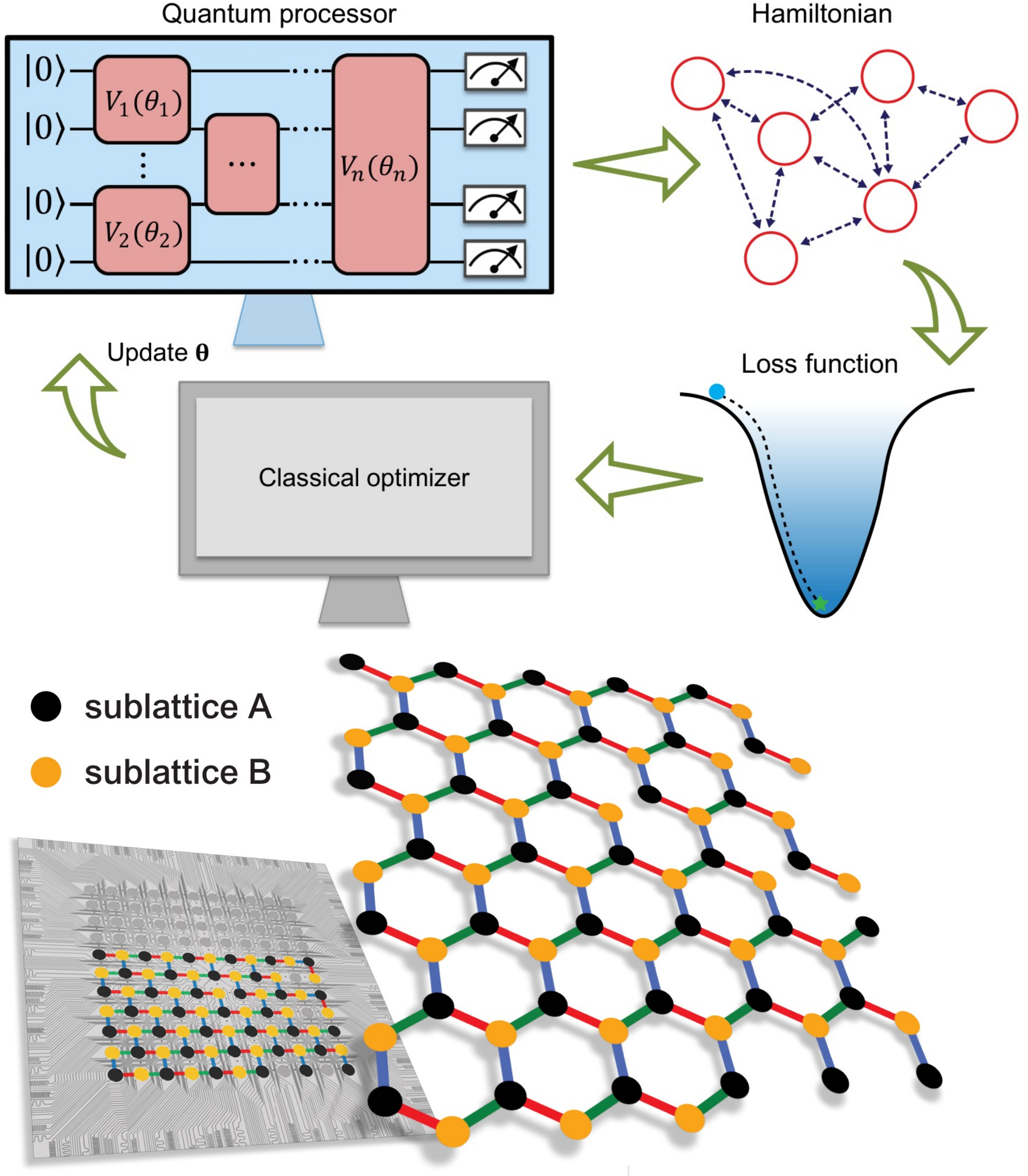


Bell operator: an XZ-type Hamiltonian:

$$\mathcal{H}_{\mathrm{B}} = \sum_{\tau-\text{link}} J_\tau(\epsilon) (\sigma_x^A \sigma_x^B + \sigma_z^A \sigma_z^B)_\tau$$



Phase 1 Variational quantum algorithm



$$|\psi(\boldsymbol{\theta})\rangle = U(\boldsymbol{\theta})|\mathbf{0}\rangle$$

$$E(\boldsymbol{\theta}) = \langle\psi(\boldsymbol{\theta})|\mathcal{H}|\psi(\boldsymbol{\theta})\rangle$$

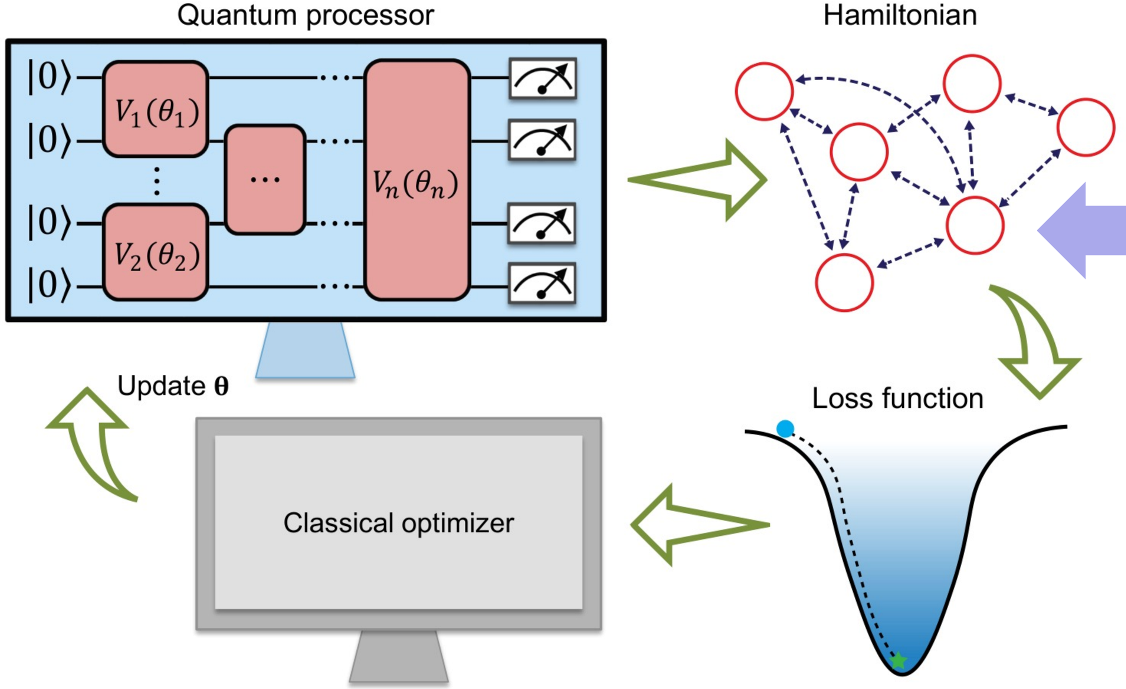
$$L(\boldsymbol{\theta}) = E(\boldsymbol{\theta}) = \langle\mathbf{0}|U^\dagger(\boldsymbol{\theta})\mathcal{H}U(\boldsymbol{\theta})|\mathbf{0}\rangle$$

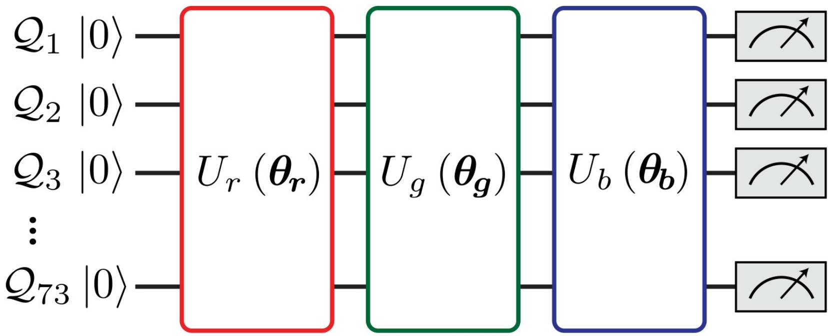
$$\frac{\partial L(\boldsymbol{\theta})}{\partial\theta_k} = \frac{L(\boldsymbol{\theta}\backslash\theta_k, \theta_k + \pi/2) - L(\boldsymbol{\theta}\backslash\theta_k, \theta_k - \pi/2)}{2}$$

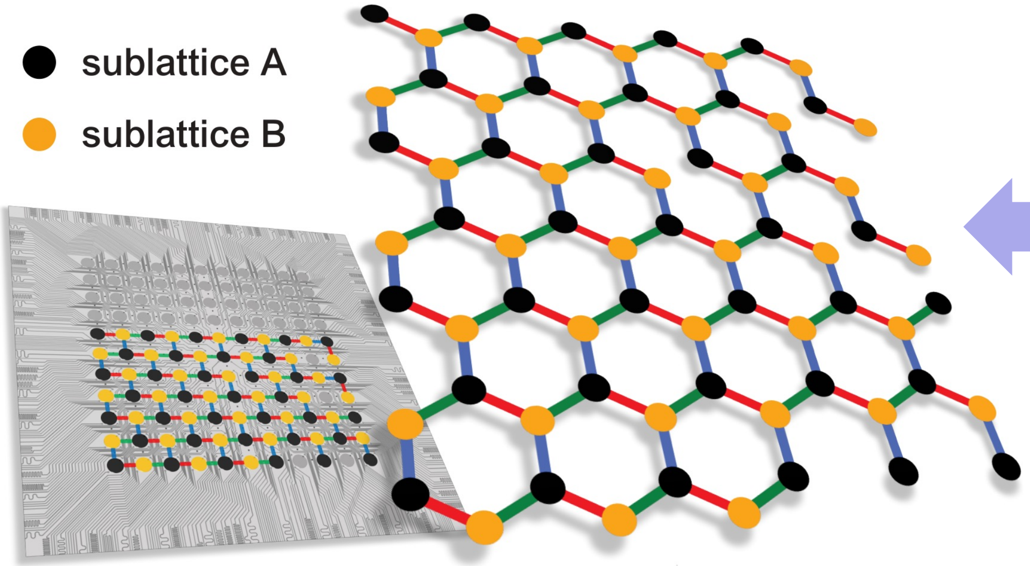
$$\boldsymbol{\theta}_{n+1} = \boldsymbol{\theta}_n - \epsilon\nabla_{\boldsymbol{\theta}}L(\boldsymbol{\theta}_n)$$

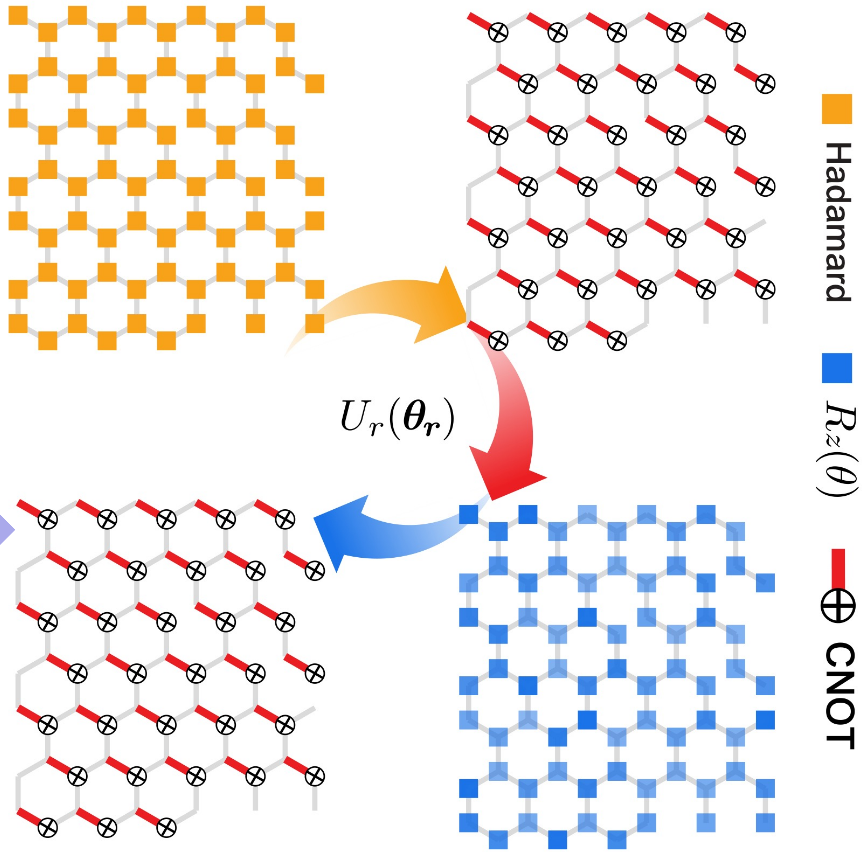
Phase 1

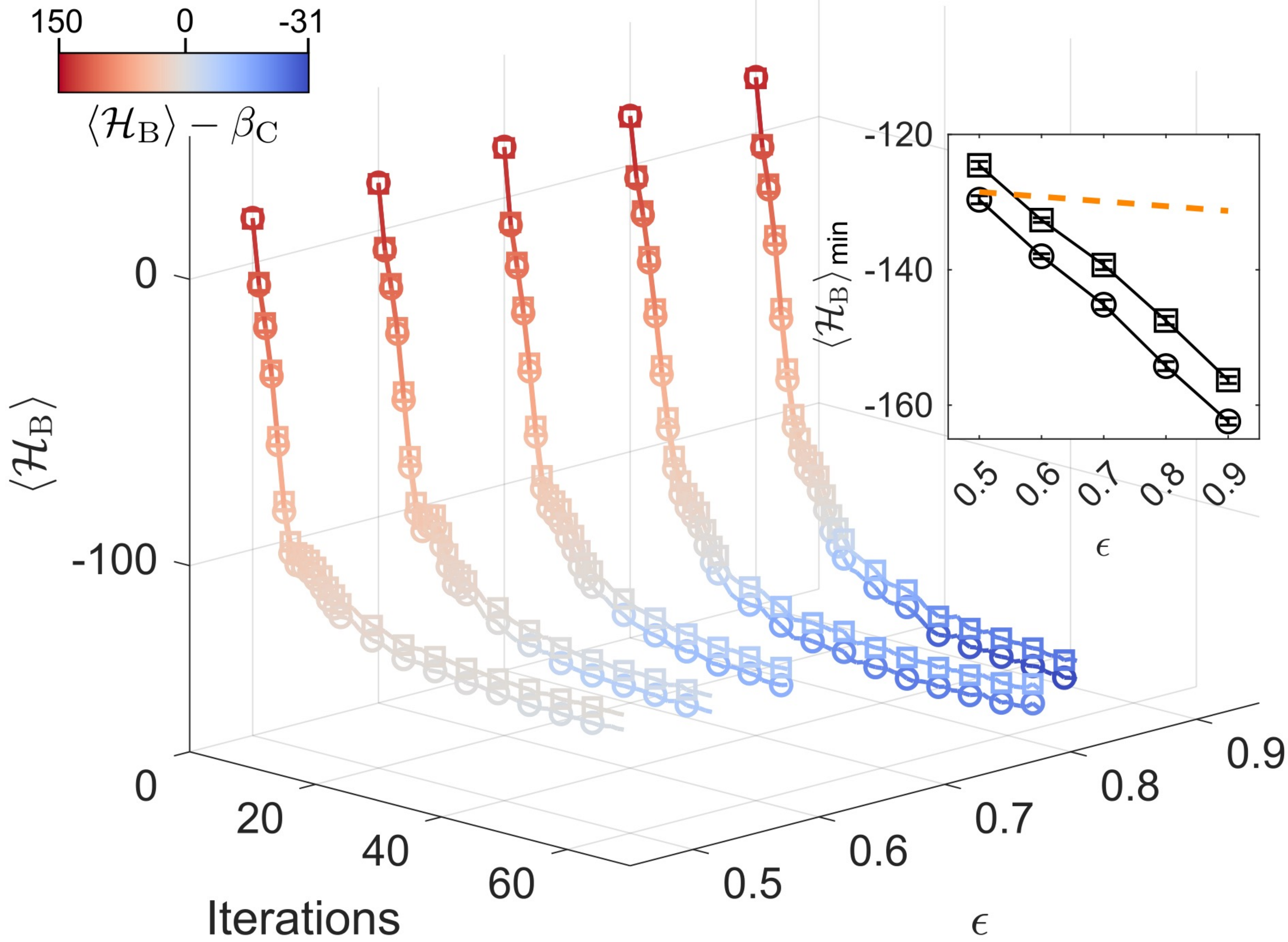
Variational quantum algorithm

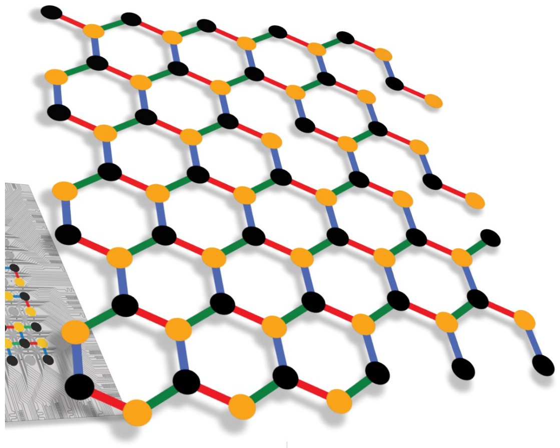









150
0
-31
$\langle \mathcal{H}_{\rm B} \rangle - \beta_{\rm C}$
$\langle \mathcal{H}_{\rm B} \rangle$
0
-100
0
20
40
60
Iterations
0.5
0.6
0.7
0.8
0.9
ϵ
-120
-140
-160
$\langle \mathcal{H}_{\rm B} \rangle_{\rm min}$
0.5
0.6
0.7
0.8
0.9
ϵ



What is Bell correlation depth?

A notion that quantifies the number of particles sharing genuinely nonlocal correlations in a multipartite system.

- Local correlations:

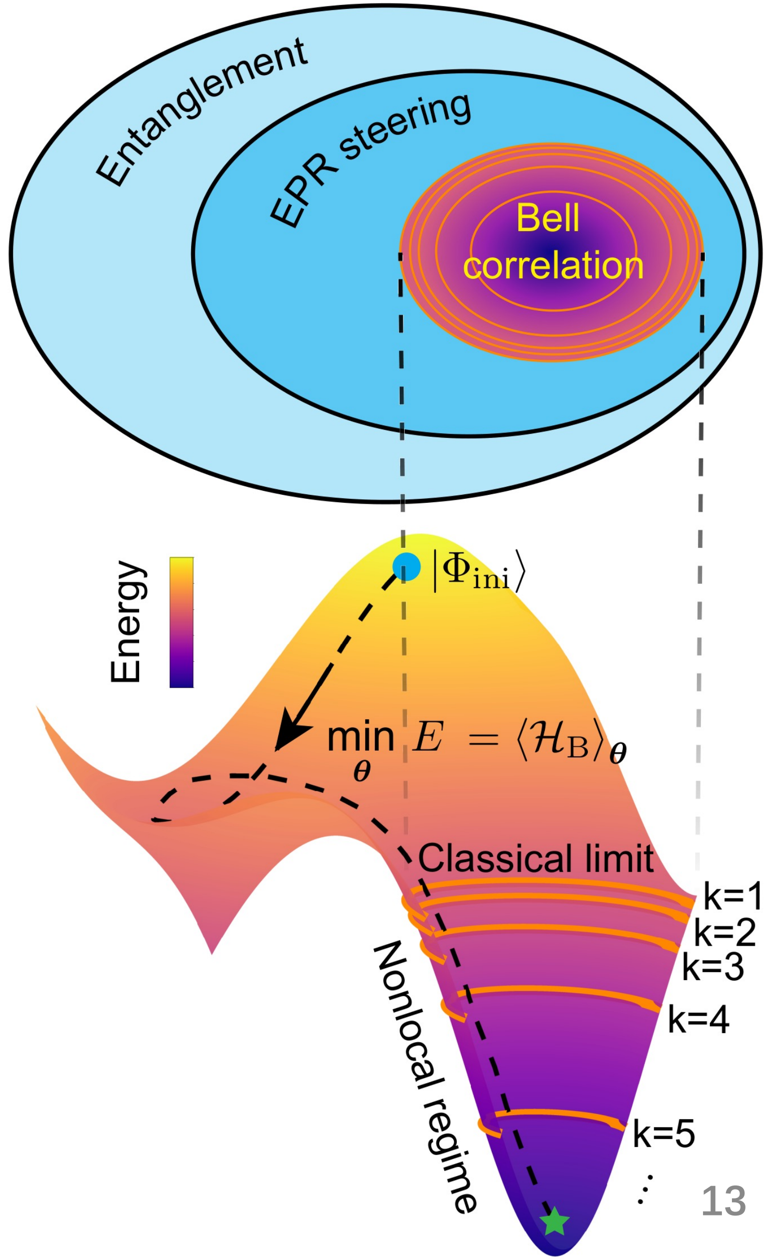
$$p(a_1, \ldots, a_N | x_1, \ldots, x_N) = \sum_\lambda p_\lambda \prod_{i=1}^{N} p(a_i | x_i, \lambda)$$

- Correlations that allow k parties to share genuinely nonlocal correlations:

$$p(\mathbf{a}|\mathbf{x}) = \sum_\lambda p(\lambda) p_1(\mathbf{a}_{\mathcal{A}_1} | \mathbf{x}_{\mathcal{A}_1}, \lambda) \times \cdots \times p_L(\mathbf{a}_{\mathcal{A}_L} | \mathbf{x}_{\mathcal{A}_L}, \lambda)$$

- More generally, the k-producible correlations:

$$p(\mathbf{a}|\mathbf{x}) = \sum_{L_k \in S_k} q_{L_k} p_{L_k}(\mathbf{a}|\mathbf{x})$$



The Svetlichny inequality

$$\mathcal{I}_N^{\mathrm{Sv}} = 2^{-\frac{N}{2}}\left[\sum_{\mathbf{x}|s=0}(-1)^{\frac{s}{2}}\langle\mathbf{A}\rangle_{\mathbf{x}} + \sum_{\mathbf{x}|s=1}(-1)^{\frac{s-1}{2}}\langle\mathbf{A}\rangle_{\mathbf{x}}\right] \geq -2^{\left(N-\lceil\frac{N}{k}\rceil\right)/2},$$

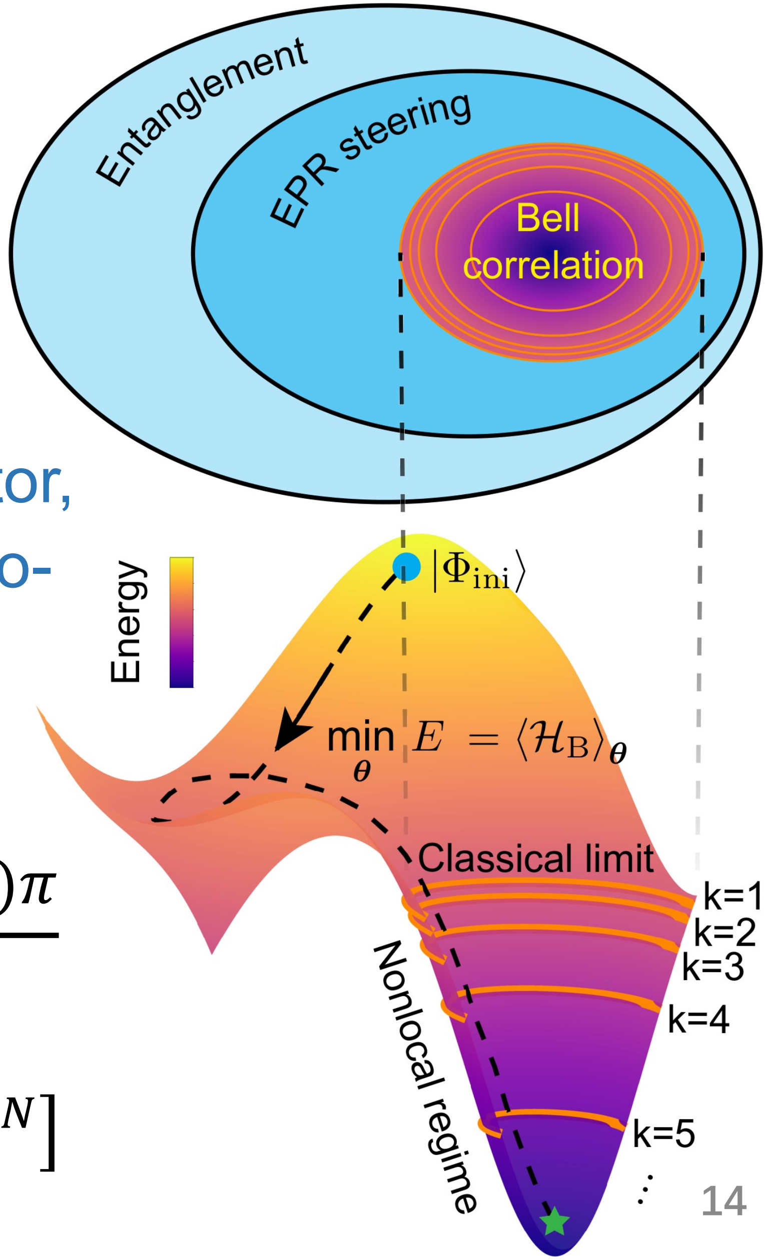
where $\langle\mathbf{A}\rangle_{\mathbf{x}} = \langle A_{1,x_1} \dots A_{N,x_N}\rangle$ denotes the N-partite correlator, and s is the parity of $\sum_i x_i$. Within the k parties, we allow no-signaling correlations.

By assigning

$$A_{i,0} = cos(\phi_1)\sigma_x + \sin(\phi_1)\sigma_y, \phi_1 = -\frac{\pi}{4N}$$

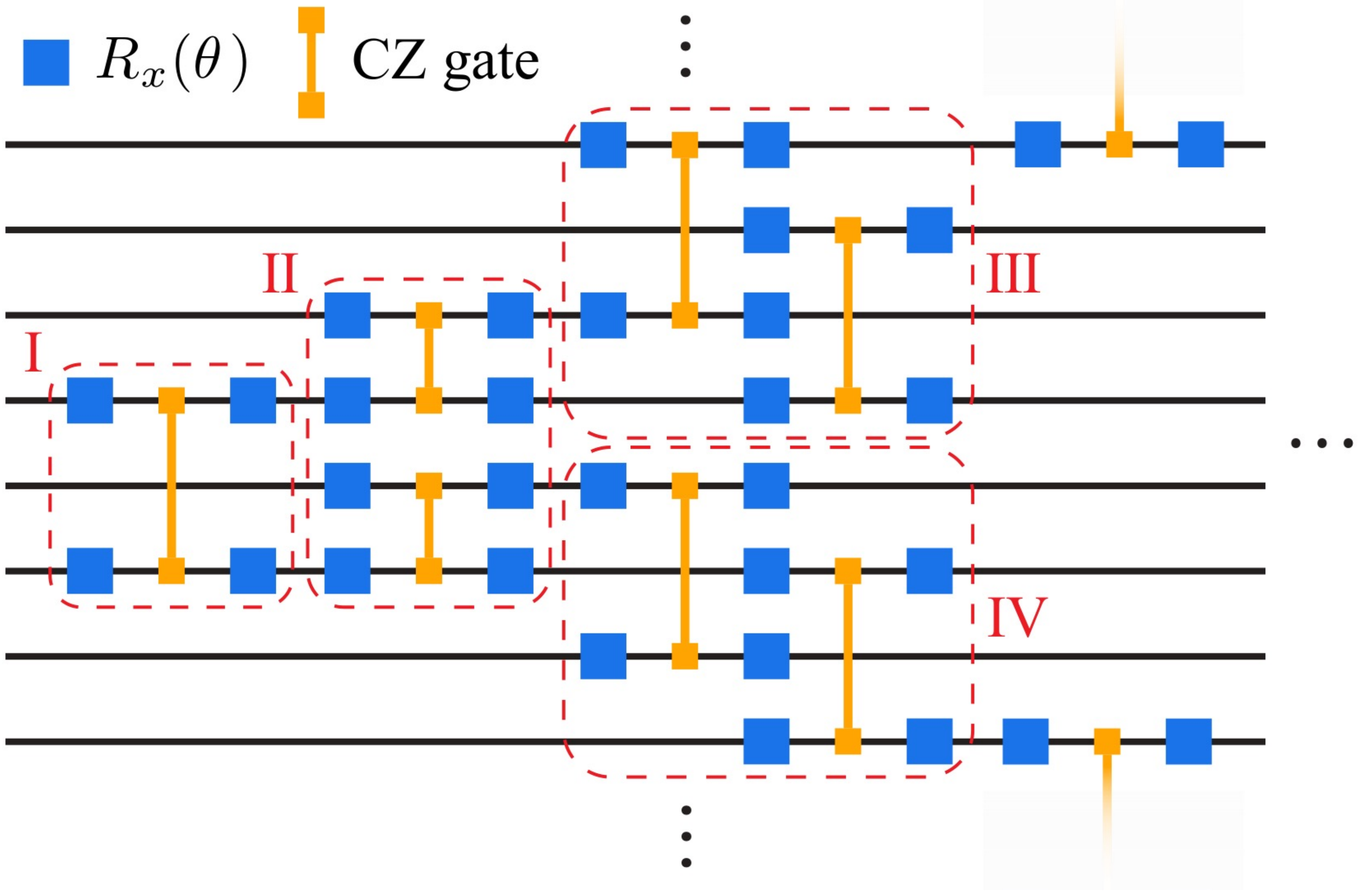
$$A_{i,1} = cos(\phi_2)\sigma_x + sin(\phi_2)\sigma_y, \phi_2 = \frac{(2N-1)\pi}{4N}$$

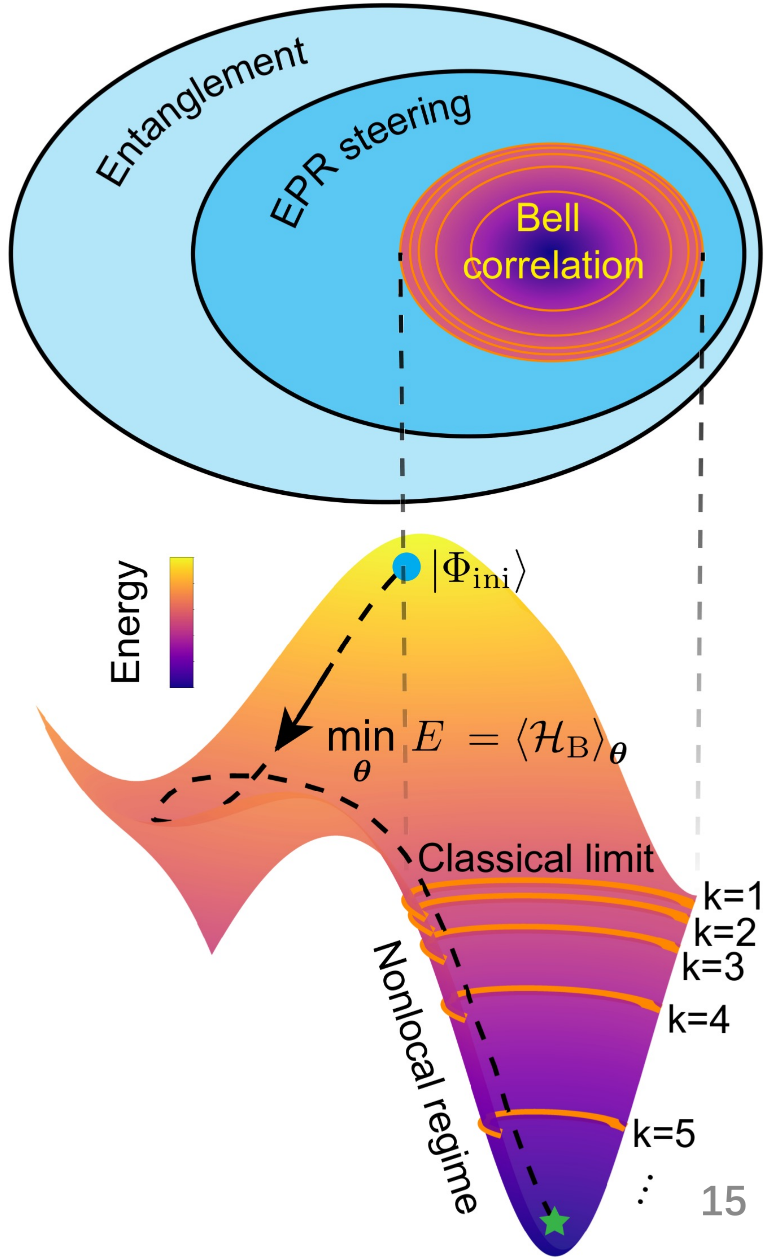
Hamiltonian $\mathcal{H}_{\mathrm{B}}(N) = 2^{\frac{N-1}{2}}\left[(|0\rangle\langle 1|)^{\otimes N} + (|1\rangle\langle 0|)^{\otimes N}\right]$



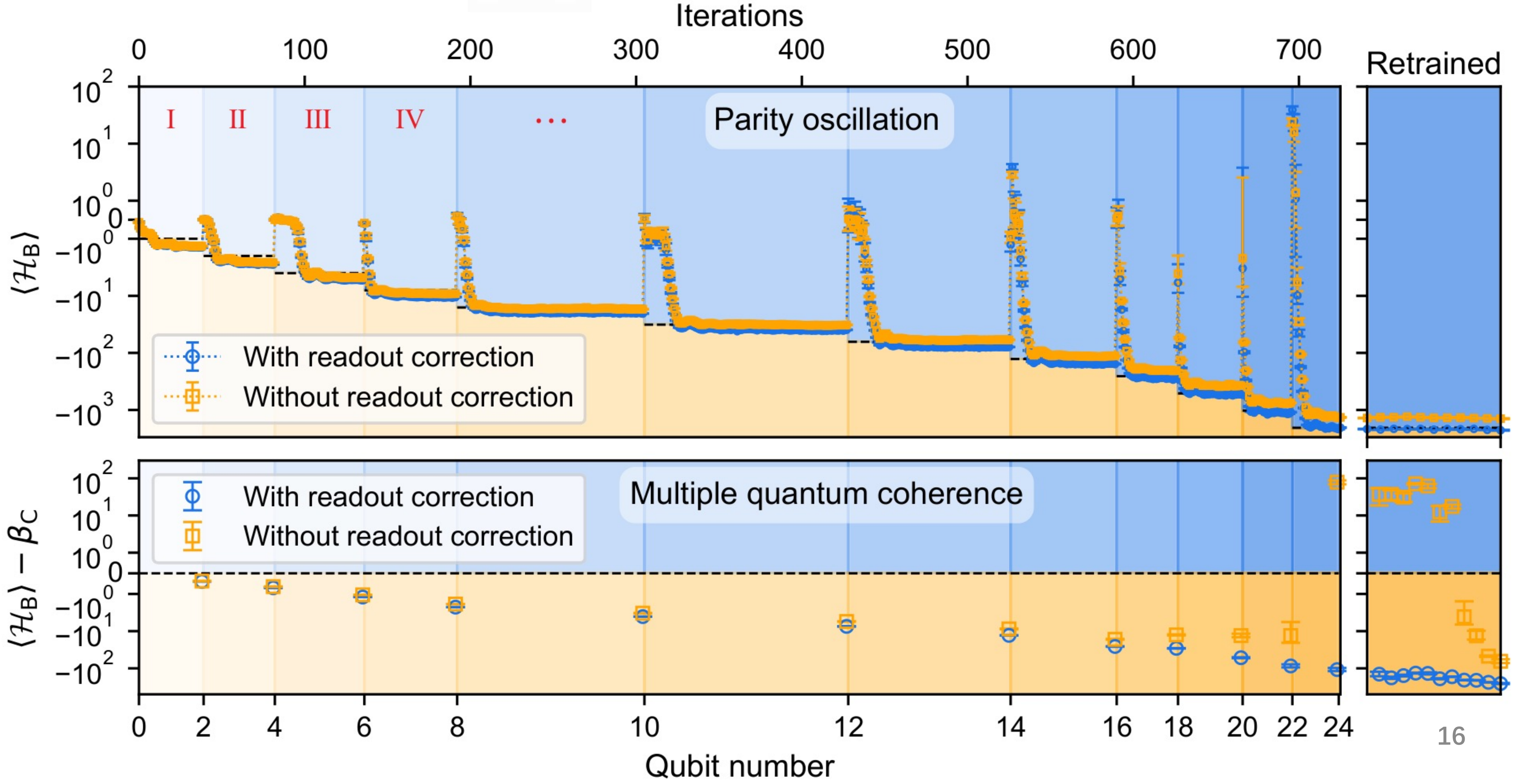
Phase 2

Layer-wised training

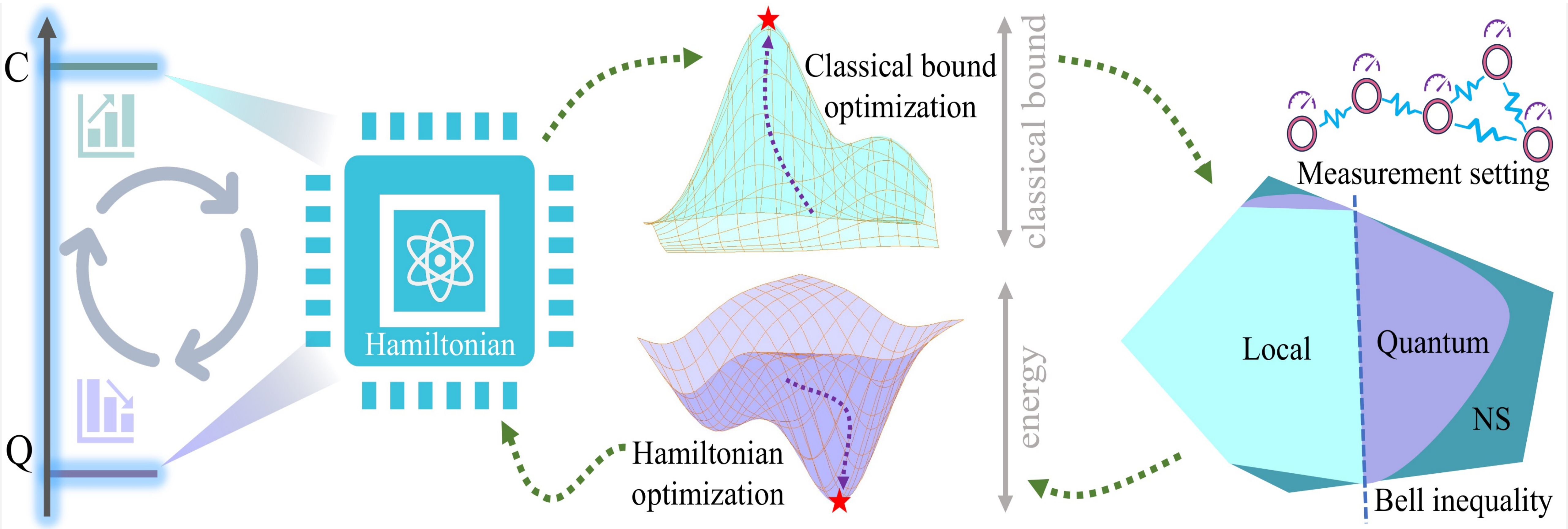




Phase 2 | Layer-wised training



Appendix: Optimizable mapping



WKL*, Hu*, Deng, Song+, Wang, Emonts+, Tura et al, arXiv:2407.12347 (2024)

Appendix: Gisin's elegant inequality

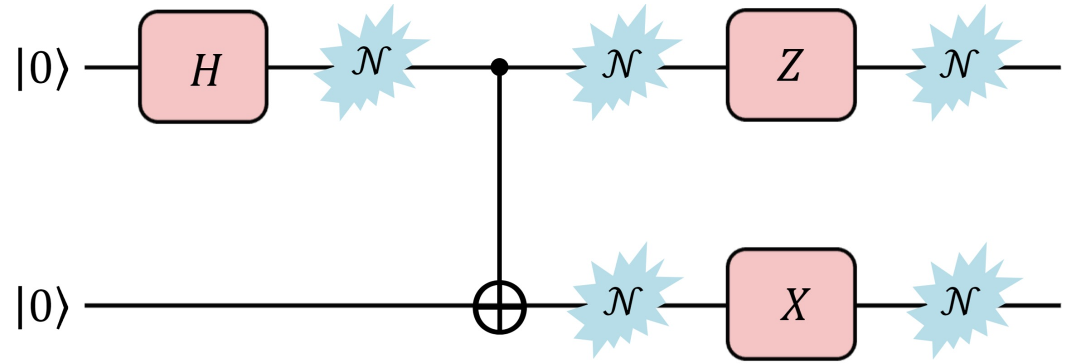
$$\mathcal{I}(\Delta) = A_0B_0 + A_1B_0 - A_2B_0 - A_3B_0 + A_0B_1 - A_1B_1 + A_2B_1 - A_3B_1 + \Delta A_0B_2 - \Delta A_1B_2 - \Delta A_2B_2 + \Delta A_3B_2$$
$$\geqslant -2|\Delta| - |\Delta+2| - |\Delta-2|.$$

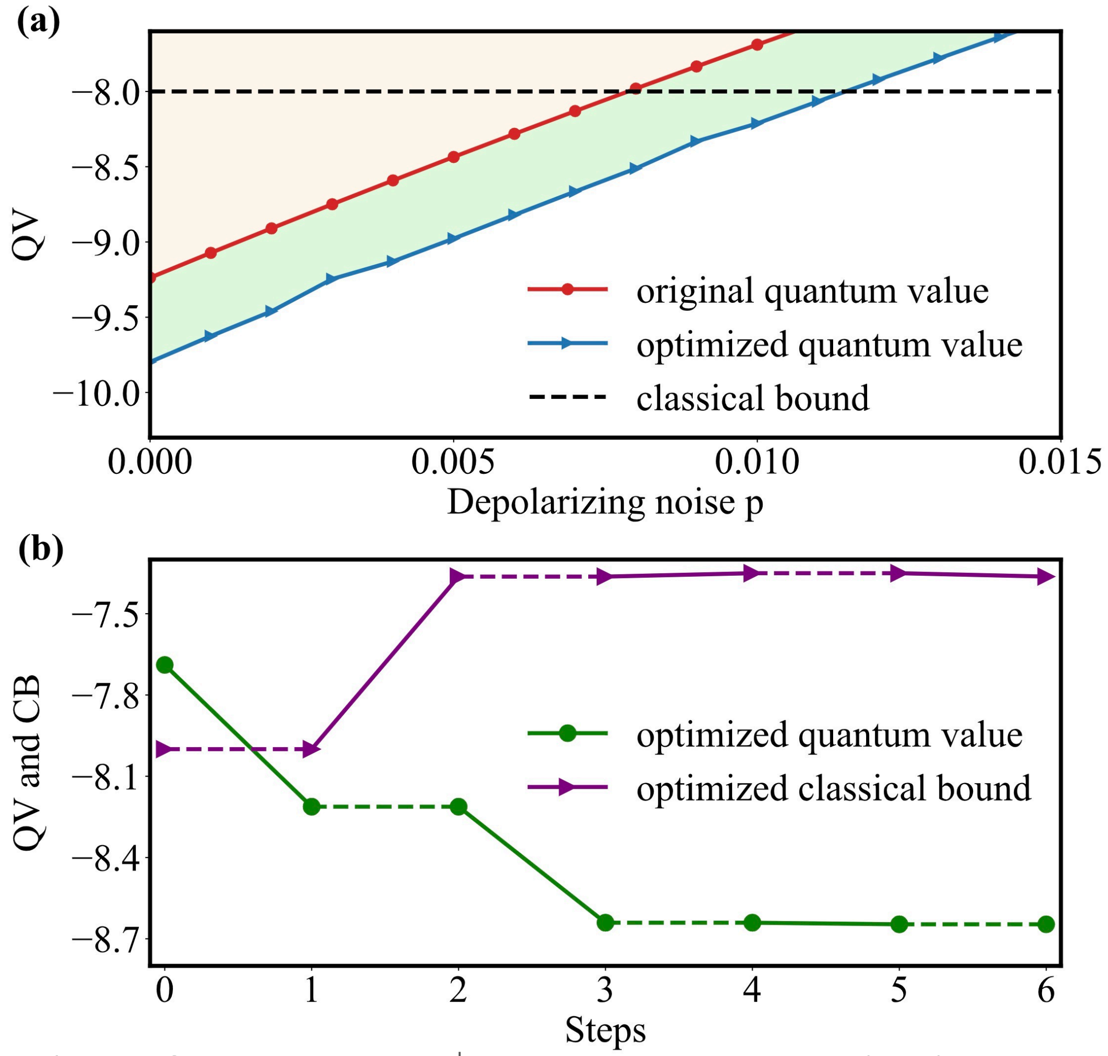
Bell operator:

$$\mathcal{H}_1 = \frac{4}{\sqrt{3}}\left(\sigma_x^{(1)}\sigma_x^{(2)} + \sigma_y^{(1)}\sigma_y^{(2)} + 2\sigma_z^{(1)}\sigma_z^{(2)}\right)$$

Noise model:

$$\mathcal{N}_p(\rho) = (1-3p)\rho + p\left(\sigma_x\rho\sigma_x + \sigma_y\rho\sigma_y + \sigma_z\rho\sigma_z\right)$$





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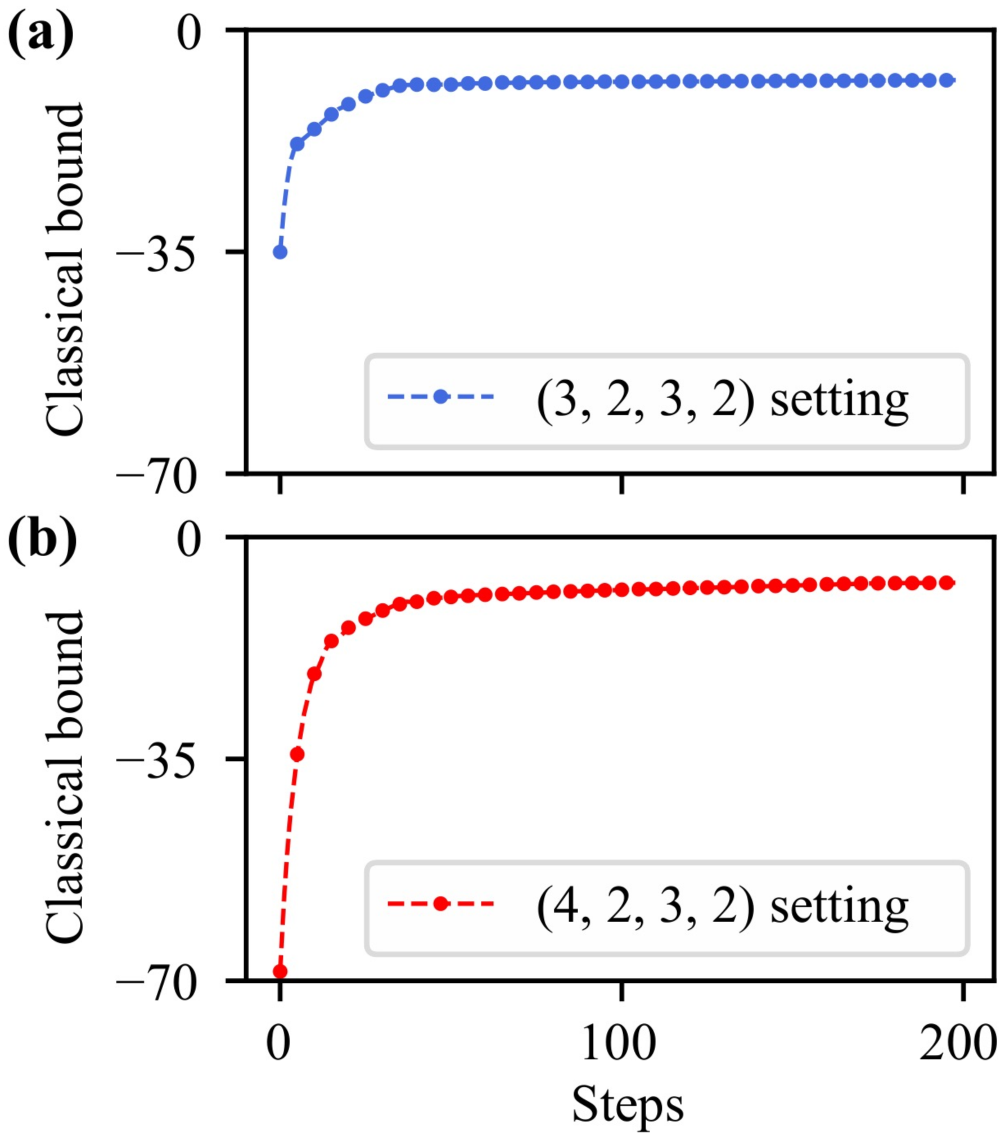
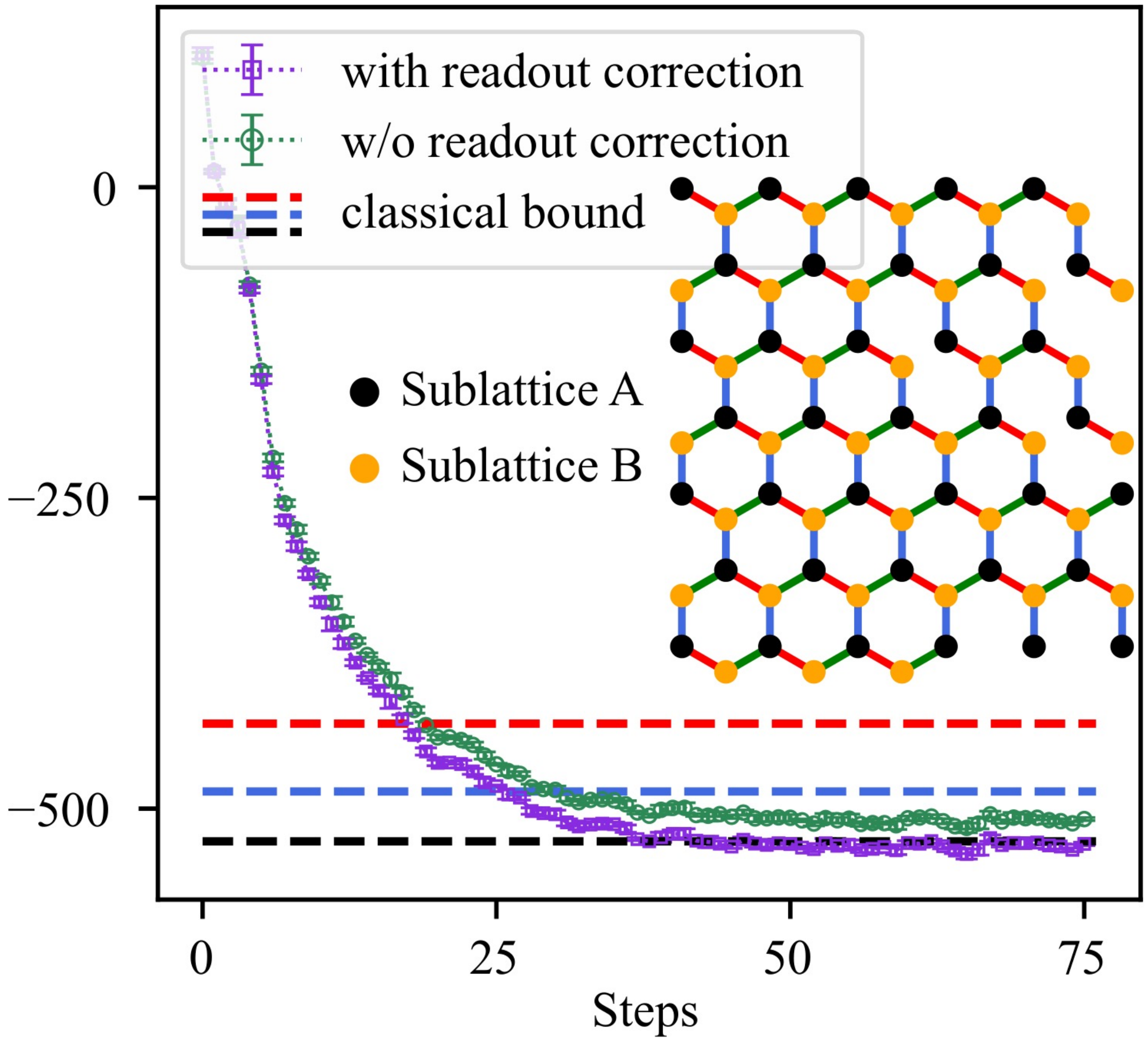
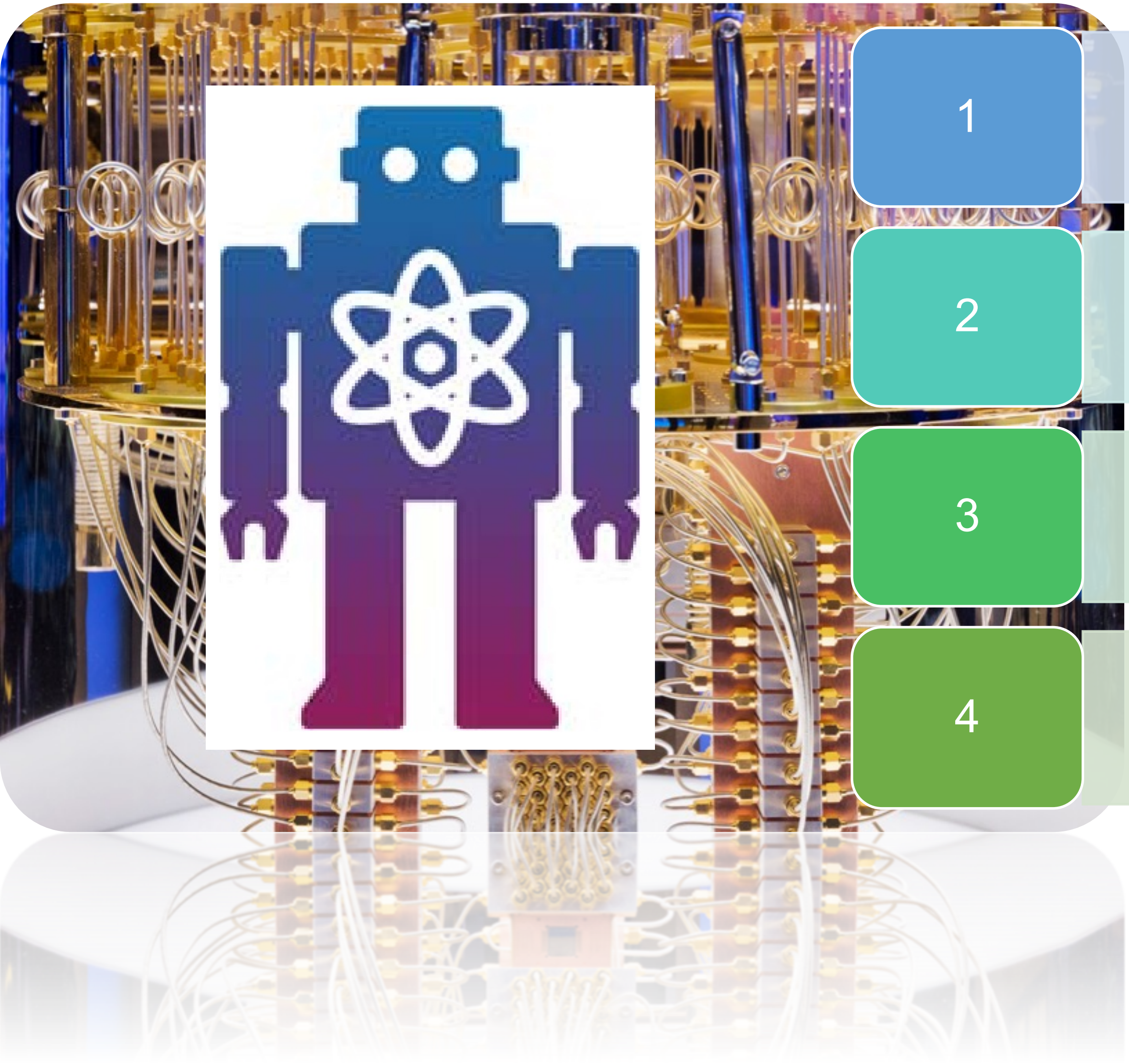




WKL*, Hu*, Deng, Song⁺, Wang, Emonts⁺, Tura et al, arXiv:2407.12347 (2024)

Summary



1. Experimentally revealed Bell correlations on a honeycomb lattice with 73 qubits
2. Observed genuine Bell correlations up to 24 particles
3. (Theoretical framework optimized to make it feasible for experimental realizations)
4. **Outlook: Phase 1-2: exploiting symmetry, dissipative cooling, QAOA, etc; Phase 3: reinforcement learning, architecture search, etc**

Thank you for listening!