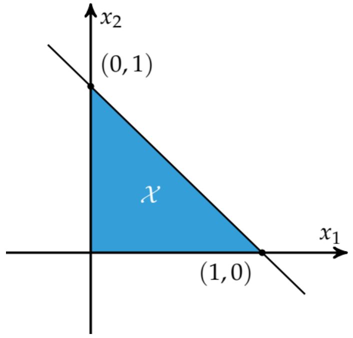
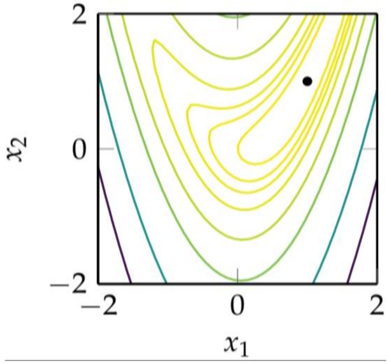
Tecnológico Nacional de México

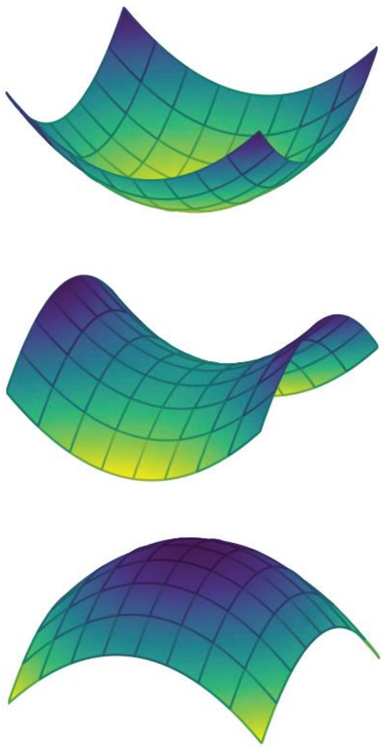
Instituto Tecnológico de Orizaba/Celaya

Modern Computing Algorithms for Process Optimization with Julia Programming. Part I





“1 - Introduction”



By:

Dr. Kelvyn Baruc Sánchez Sánchez

Postdoctoral Researcher/I.T. Celaya

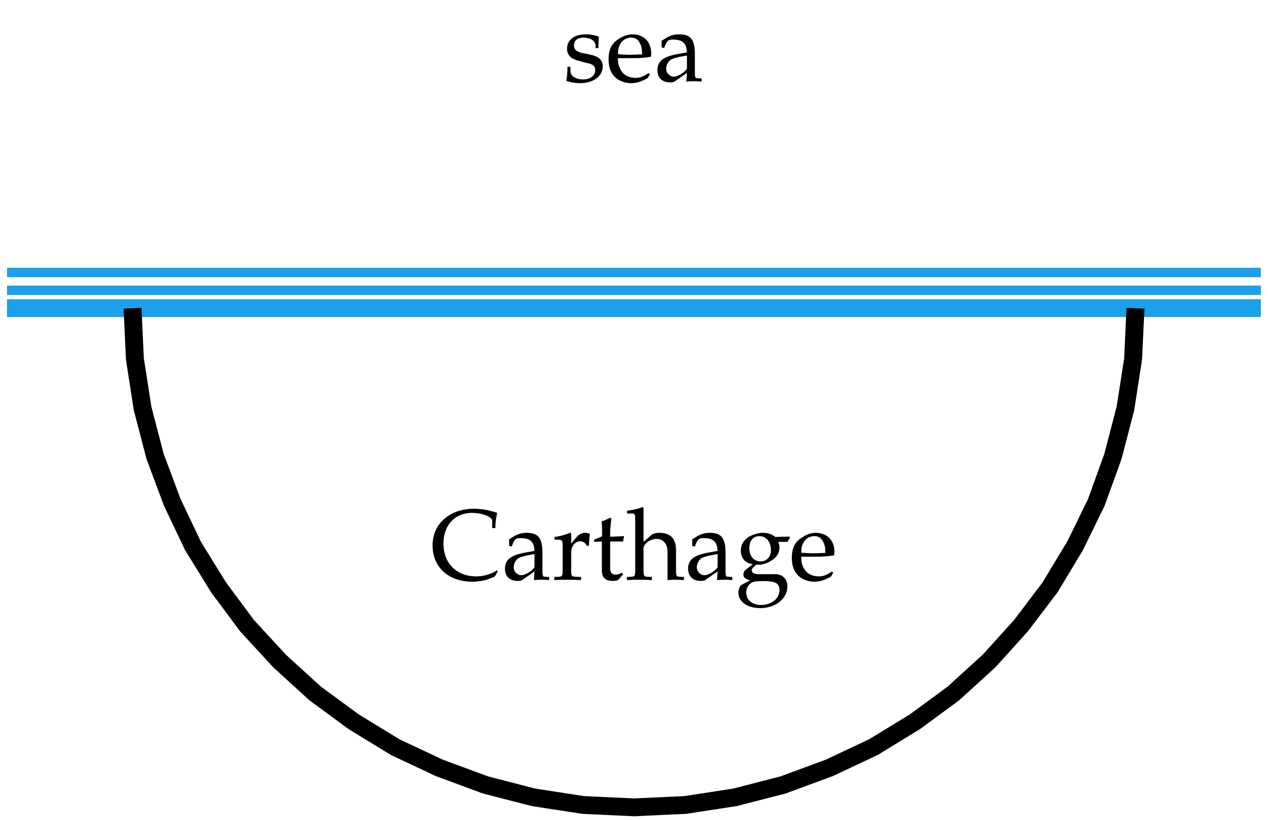
Introduction

- Applications of Optimization
  - Physics
  - Business
  - Biology
  - Engineering
- Objectives to Optimize
  - Efficiency
  - Safety
  - Accuracy

- Constraints
  - Cost
  - Weight
  - Structural Integrity
- Challenges
  - High-Dimensional Search Spaces
  - Multiple Competing Objectives
  - Model Uncertainty

A History

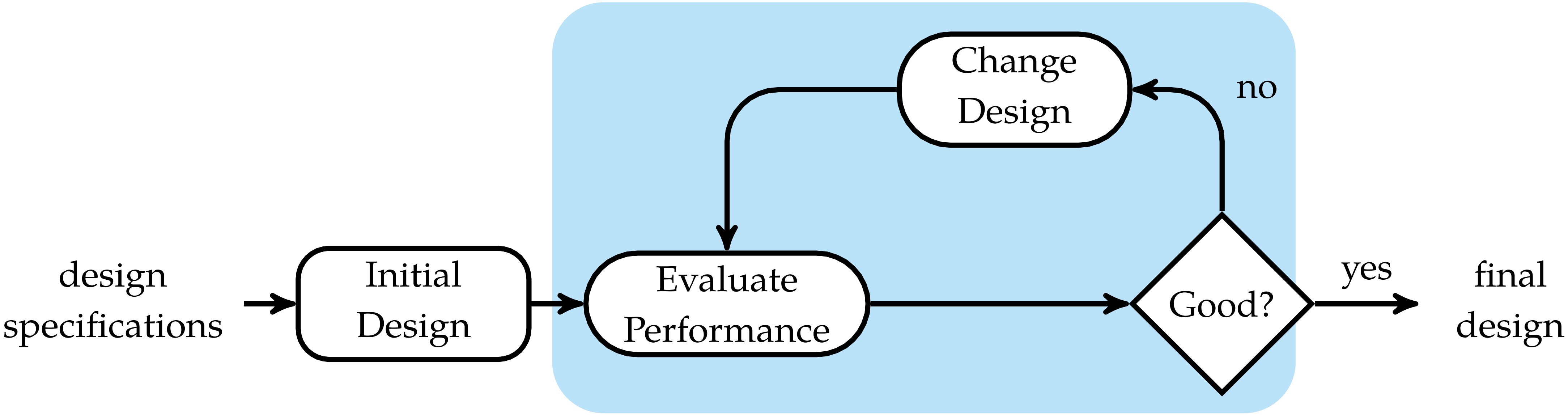
- Queen Dido’s Optimization Problem



A History

- Calculus
- Numerical Algorithms
- Artificial Intelligence

Optimization Process



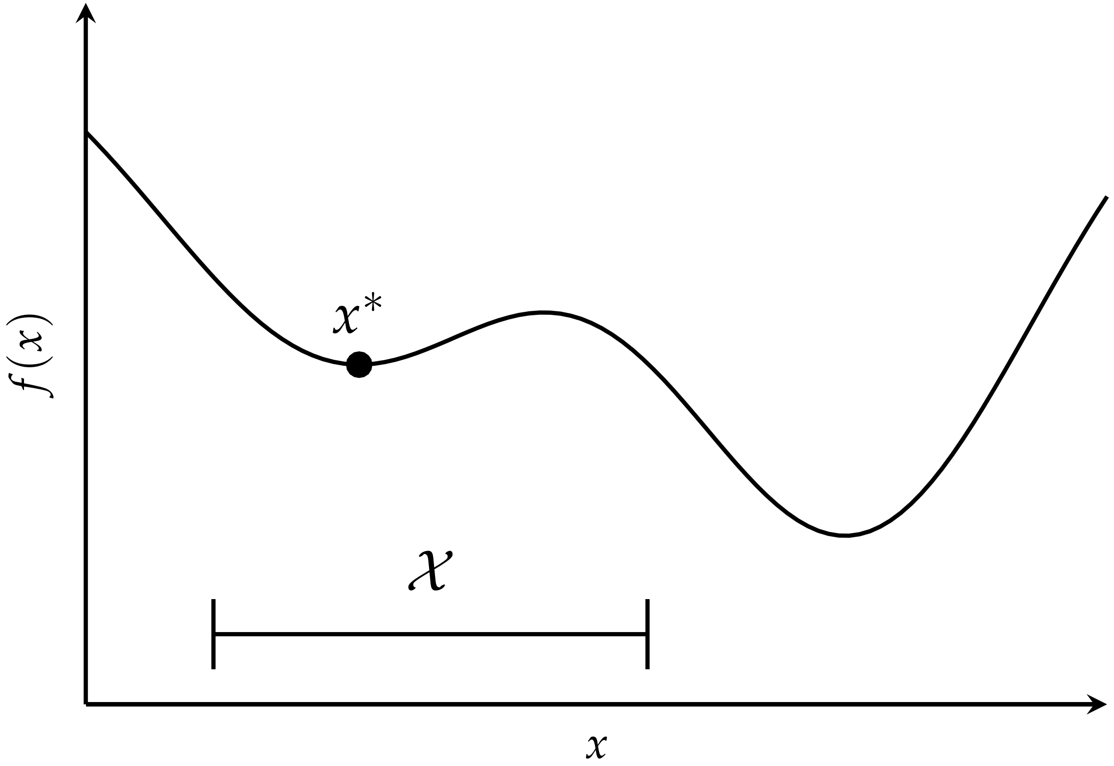
Basic Optimization Problem

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & \mathbf{x} \in \mathcal{X} \end{aligned}$$

- Design Point
- Design Variables
- Objective Function
- Feasible Set
- Minimizer

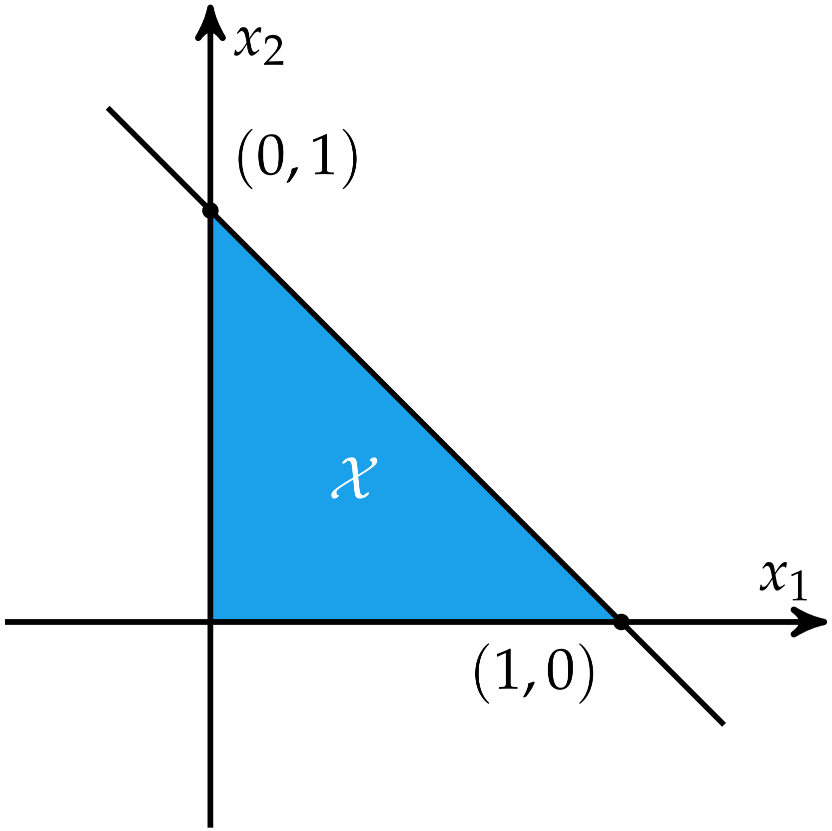
Basic Optimization Problem

$$\underset{\mathbf{x}}{\text{minimize}} \quad f(\mathbf{x})$$
$$\text{subject to} \quad \mathbf{x} \in \mathcal{X}$$



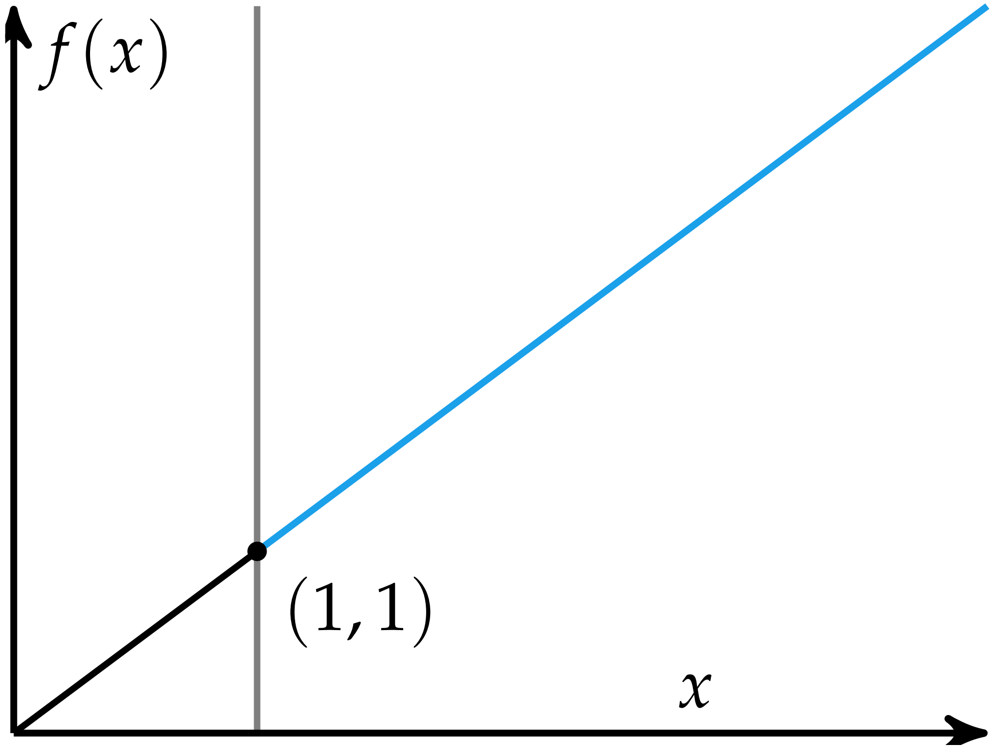
Constraints

$$
\begin{aligned}
\underset{x_1, x_2}{\text{minimize}} \quad & f(x_1, x_2) \\
\text{subject to} \quad & x_1 \geq 0 \\
& x_2 \geq 0 \\
& x_1 + x_2 \leq 1
\end{aligned}
$$



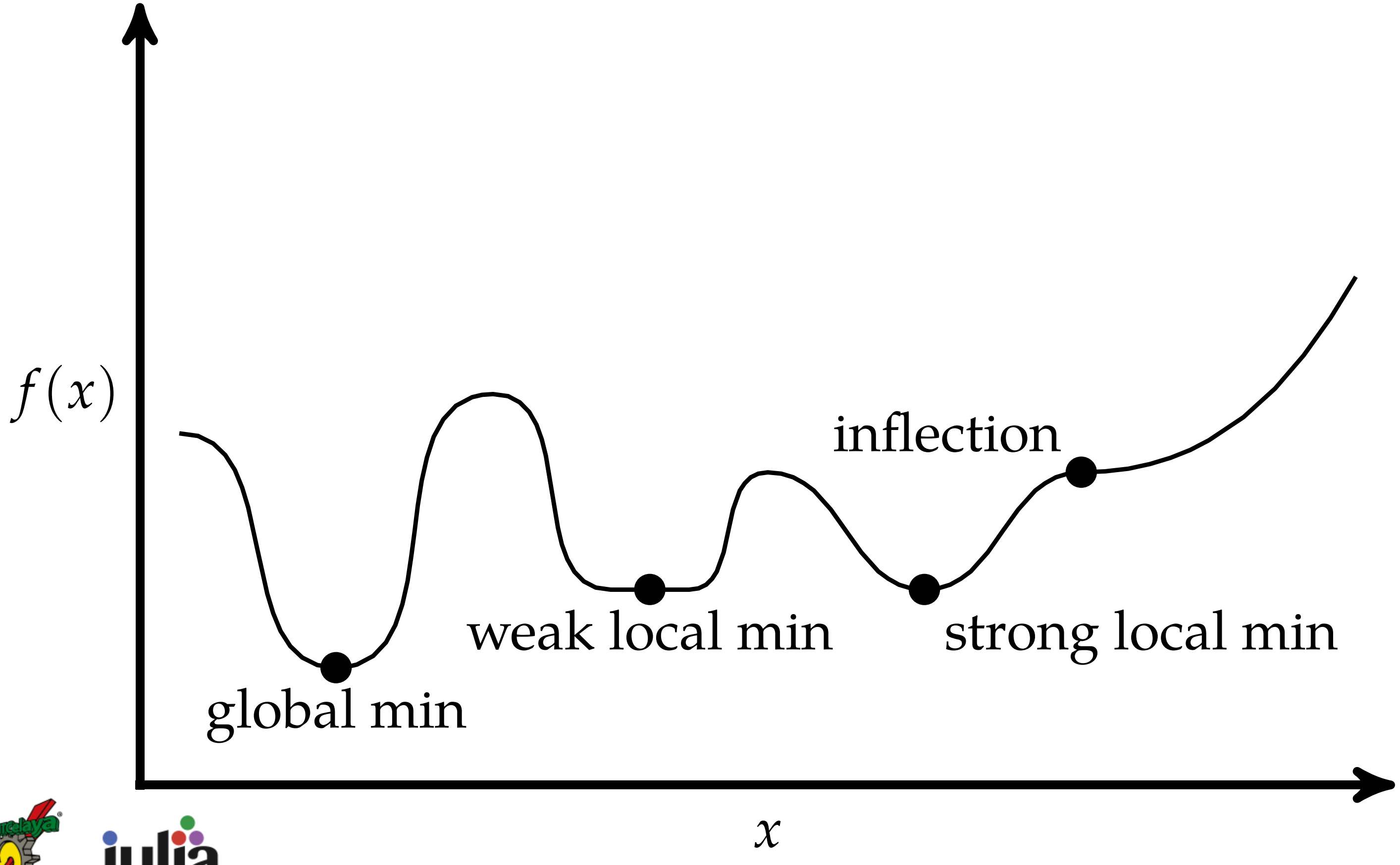
Constraints

$$\underset{x}{\text{minimize}} \quad x$$
$$\text{subject to} \quad x > 1$$



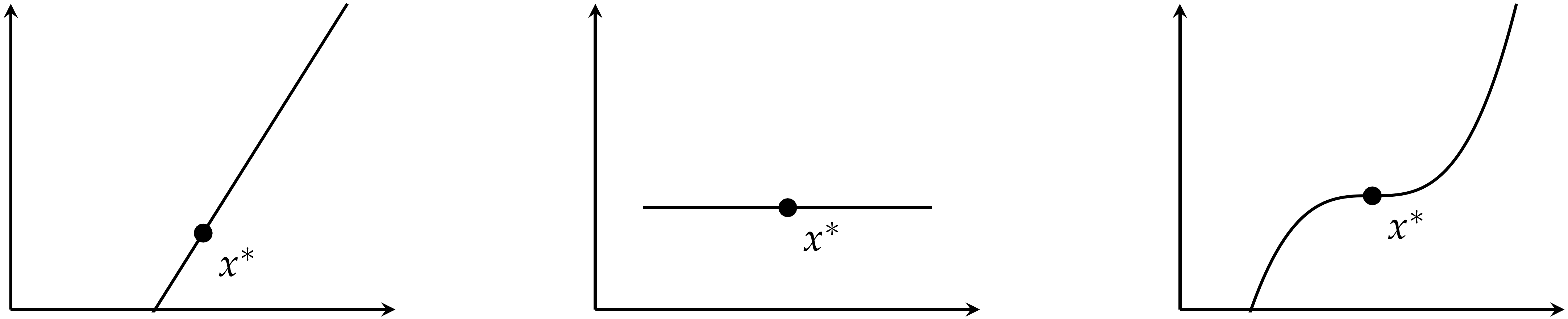
Critical Points

- Univariate Function



Conditions for Local Minima

- *Univariate Function*
  1. $f'(x^*) = 0$, the first-order necessary condition (FONC)
  2. $f''(x^*) \geq 0$, the second-order necessary condition (SONC)



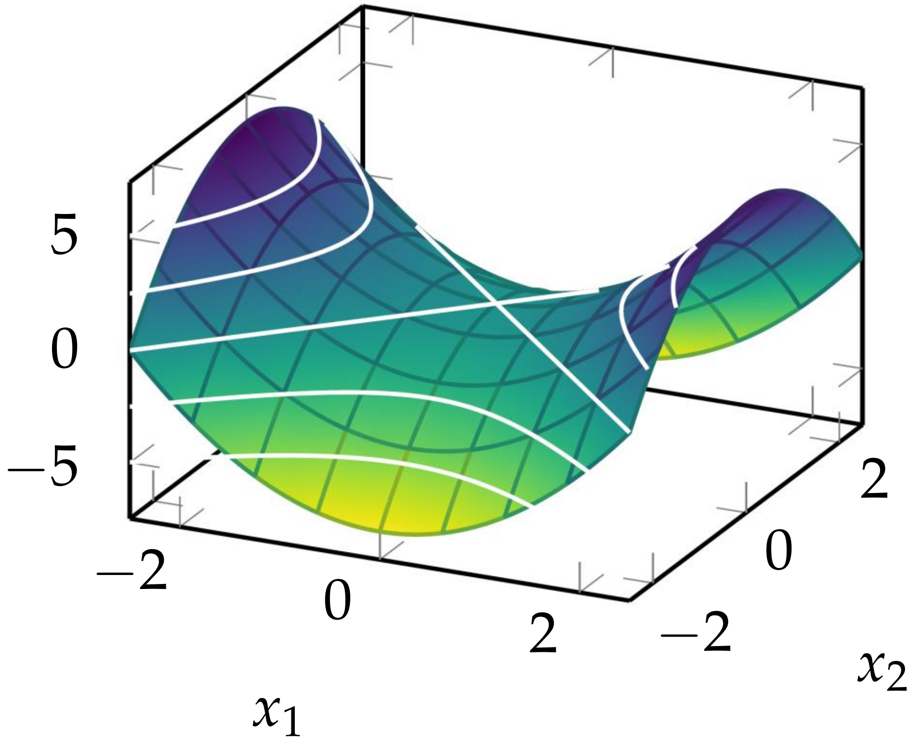
SONC but not FONC — FONC and SONC — FONC and SONC

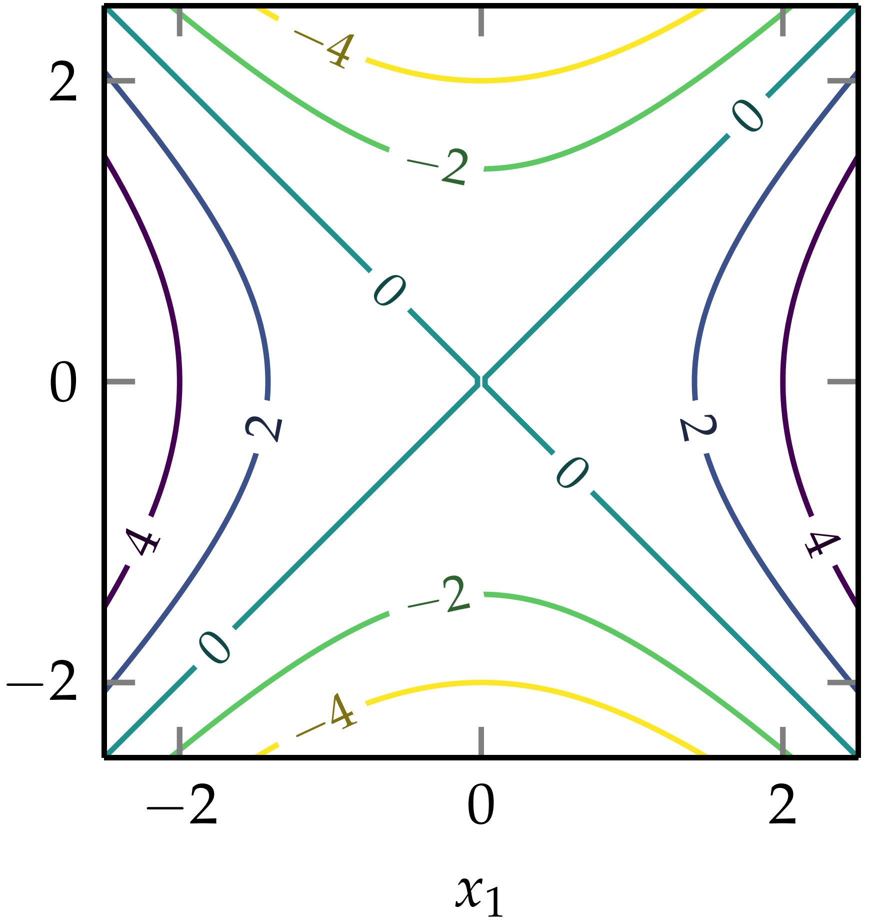
Conditions for Local Minima

- *Multivariate Functions*
    1. $\nabla f(\mathbf{x}^*) = 0,$ the first-order necessary condition (FONC)
    2. $\nabla^2 f(\mathbf{x}^*) \geq 0,$ the second-order necessary condition (SONC)

Contour Plots

- $f(x_1, x_2) = {x_1}^2 - {x_2}^2$





Overview

2. Derivatives → 4. Descent → 5. First-Order, 6. Second-Order

3. Bracketing

7. Direct

8. Stochastic → 9. Population → 20. Expressions

10. Constraints → 11. Linear → 19. Discrete

10. Constraints → 12. Multiobjective

13. Sampling Plans → 14. Surrogate Models → 15. Probabilistic Surrogate Models → 16. Surrogate Optimization

17. Optimization under Uncertainty → 18. Uncertainty Propagation

21. Multidisciplinary Design Optimization

Dr. Kelvyn B. Sánchez Sánchez
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Summary

- Optimization in engineering is the process of finding the best system design subject to a set of constraints
- Optimization is concerned with finding global minima of a function
- Minima occur where the gradient is zero, but zero-gradient does not imply optimality