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FlowFPX: Nimble Tools for Debugging Floating-Point Exceptions

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ABSTRACT

Reliable numerical computations are central to scientific computing, but the floating-point arithmetic that enables large-scale models is error-prone. Numeric exceptions are a common occurrence and can propagate through code, leading to flawed results. This paper presents FlowFPX, a toolkit for systematically debugging floating-point exceptions by recording their flow, coalescing exception contexts, and fuzzing in select locations. These tools help scientists discover when exceptions happen and track down their origin, smoothing the way to a reliable codebase.

Keywords

Julia, floating-point, debugging

1. Introduction

Reliable numeric computations are central to high-performance computing, machine learning, and scientific applications. Yet the floating-point arithmetic that powers these computations is fundamentally unreliable (section 2). Exceptional values, such as Not a Number (`NaN`) and infinity (`Inf`), can and often do arise thanks to culprits such as roundoff error, catastrophic cancellation, singular matrices, and vanishing derivatives [4, 5, 9, 11, 23, 34]. Developers are responsible for guarding against exceptions, but this task is difficult because many operations can generate and propagate exceptions. Worst of all, operations such as less-than (<) can kill an exceptional value, leaving no trace of the problem. There is little tool support to assist in exception debugging, thus (unsurprisingly) a quick GitHub search reports over 4,000 open issues related to `NaN` or `Inf` exceptions [10].

This paper introduces FlowFPX, a toolkit for debugging floating-point exceptions (section 3) that has helped improve a variety of applications, from ocean simulations to heat modes (section 4). FlowFPX consists of three tools that can work together to debug floating-point computations:

(1) The centerpiece of FlowFPX is `TrackedFloats.jl`, a dynamic analysis tool written in Julia for Julia code that monitors exceptions and fuzzes for vulnerabilities.

(2) To visualize results, FlowFPX uses coalesced stack trace graphs (CSTGs, or stack graphs) [15]. Our CSTG library is written in `C++` and accepts text-format stack traces as input.

(3) For programs that offload work to CUDA GPUs, the binary instrumentation tool GPU-FPX [24] adds logging to kernels.

The toolkit is online: `https://utahplt.github.io/flowfpx`

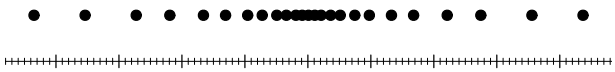

Fig. 1. Floats are spread across the real number line

2. Floating-Point Exception Primer

Floating-point numbers use a finite number of bits to represent a spectrum of points along the real number line (fig. 1). The implementation strategy is essentially that followed in scientific notation. A floating-point number packs a sign bit, an exponent, and a fraction part (also called the "significand" or "mantissa") into a bit string. Typical strings are 64 or 32 bits long, but 16-bit and 8-bit formats are on the rise [17, 26, 29]. This representation supports very small and very large numbers in a narrow range of bits:

```
julia> floatmax(Float64)
1.7976931348623157e308
julia> floatmin(Float64)
2.2250738585072014e-308
```

The flip side is that most real numbers fall into the gaps between floating-point numbers and must be rounded, which introduces error and can lead to surprising results. For example, adding the tiny Planck constant to the large Avogadro constant results in the Avogadro constant after rounding:

```
julia> planck = 6.62607015e-34
6.62607015e-34
julia> avogadro = 6.02214076e23
6.02214076e23
julia> avogadro + planck == avogadro
true
```

This is an extreme example, but many operations on floating-point numbers closer in magnitude induce a loss of accuracy. Refer to the literature for more details, e.g., [21, 35, 28].

2.1 Exceptions and Exceptional Values

The IEEE 754 floating-point standard [16] defines exceptions and exceptional values as the outcome of operations that have "no single universally acceptable result" [31]. For example, dividing by zero and exceeding the `Float64` range both lead to exceptions:

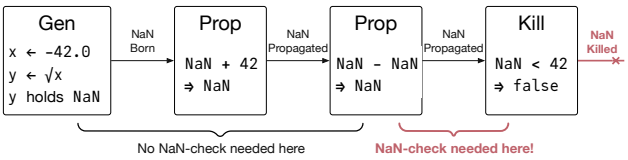

Fig. 2. Gen, Prop, Kill: Lifetime of an exceptional value

```
julia> 0 / 0
NaN
julia> avogadro^avogadro
Inf
julia> log(0)
-Inf
julia> Inf + NaN
NaN
```

IEEE 754 defers the question of how to handle such exceptions to application code. It is up to developers to watch for `NaN`, `Inf`, and subnormal numbers (underflow) and implement an appropriate repair. This is a difficult task because of the approximations inherent to floating point. Even an apparently-safe division could result in a `NaN` if its denominator gets truncated to zero. Some `NaN`s might be spurious while others might be fatal, but in any event anticipating the various exceptions is a burden. All too often, exceptional values go unhandled and flow through the code.

2.2 Lifetime of an Unhandled Exceptions

Unhandled exceptional values have a lifetime: they are born, or *generated*, by some operation; they *propagate* through other operations; and they either appear in the program output, go out of scope, or get *killed* by a numeric operation. Figure 2 summarizes this *gen-prop-kill* process. The gens and props are straightforward; see above for examples (section 2.1). The kills often arise from numeric comparisons (`<`, `=`, etc.), but exponents (`1^NaN`) and over-eager matrix optimizations [5] can kill exceptions as well.

To illustrate the perils of killed exceptions, consider the following two ways of finding the maximum value in a list. The first compares numbers with `<=` while the second uses the built-in `max` function:

```
function max1(lst)
  max_seen = 0.0
  for x in lst
    # swap if x is not too small
    if ! (x <= max_seen)
      max_seen = x
    end
  end
  max_seen
end

function max2(lst)
  foldl(max, lst)
end
```

For lists with a `NaN` inside, the functions can give different results because `<=` kills `NaN`s whereas `max` propagates them:

```
julia> max1([1, 5, NaN, 4])
4.0
julia> max2([1, 5, NaN, 4])
NaN
```

Exn. Input?	⇒	Exn. Output?	=	Event
×	⇒	✓	=	gen
✓	⇒	✓	=	prop
✓	⇒	×	=	kill

Fig. 3. How to classify operations that see exceptions

Not only is the result from `max1` problematic for obscuring the fact that there was a `NaN` in the list, the result is arguably wrong! The result from `max2` at least shows that a `NaN` was in the works, though in a realistic setting it may not be clear where the `NaN` came from. Both versions would thus benefit from tools that track exceptions across their lifetime. FlowFPX can help.

3. FlowFPX

FlowFPX (FPX: Floating Point eXception) is a toolkit for tracking down floating-point exceptions. The primary tools in the FlowFPX toolkit are `TrackedFloats.jl`, which records lifetimes and enables fuzzing, and stack graphs (more precisely, CSTGs), which visualize the flow of exceptions. GPU-FPX is a third component of FlowFPX that tracks floating-point exceptions inside GPU kernels [24]. This section covers each tool in detail.

3.1 `TrackedFloats.jl`

`TrackedFloats.jl` tracks exceptional values in Julia programs across their *gen-prop-kill* lifetime and can fuzz code for vulnerabilities by injecting a `NaN` or `Inf` as the result of an operation. `TrackedFloats.jl` is implemented in Julia as a library and is available on JuliaHub:

`https://juliahub.com/ui/Packages/TrackedFloats`

3.1.1 Tracking Exceptional Values. `TrackedFloats.jl` monitors `NaN` and `Inf` exceptions by overloading arithmetic operations and logging key events. When the input to an operation is exception-free but the output is exceptional, the operation is a gen event (fig. 3). When the input and output contain exceptions, the operation is a prop event. And when the input contains an exception but the output does not, the operation is a kill event. Put together, the log of all gens, props, and kills sheds light of how various exceptions traveled across the program. The logs also record the call context (stack trace) and the arguments to the operation as a starting point for debugging efforts.

`TrackedFloats.jl` writes logs to three files: one for gen events, one for props, and one for kills. This way, discovering where a `NaN` came from is a matter of sifting through the gen file. Users can also lower the overhead of logging by turning it off for prop events.

The instrumentation works through custom floating-point types: `TrackedFloat64`, `TrackedFloat32`, and `TrackedFloat16`. Developers must opt in to `TrackedFloats.jl` by wrapping numbers in a custom type. From then on, tracking is automatic and extends transitively to all outputs of tracked operations. Multiplying a `Float64` value with a `TrackedFloat64`, for example, yields a `TrackedFloat64` to continue the logging trail.

Crucially, `TrackedFloats.jl` does nothing to modify the bit-level representation of floats or exceptions. The tracked version of, say, a `NaN`, points to the original exceptional value. Thus `NaN`-packing and other creative uses of the payload continue to work—provided the code does not use some other Julia operation that does not preserve `NaN`s bit-for-bit.[1]

[1] `https://github.com/JuliaLang/julia/issues/48523`

3.1.2 Fuzzing. Julia's operator overloading lets us fuzz code from the inside out. Each overloaded function serves as a hook where `TrackedFloats.jl` can decide whether to observe the operation or replace the original result with an exceptional value. Injecting faults in a random way [12], also known as fuzzing, can discover vulnerabilities in a large codebase. Demmel et al. propose essentially the same idea for BLAS and LAPACK [5].

When fuzzing is enabled, every time `TrackedFloats.jl` intercepts a floating-point operation, it decides whether or not it should return a `NaN` instead of the actual computed value. This decision is based on several parameters:

— `active::Bool` fuzz only when set to true.

— `odds::Int64` inject at each candidate location with probability $1/$`odds`.

— `n_inject::Int64` upper bound on the number of `NaN`s to inject.

— `functions::ArrayString` limit fuzzing to the dynamic extent of the listed functions.

— `libraries::ArrayString` limit fuzzing to functions from the following libraries

`TrackedFloats.jl` looks at these parameters in conjunction with stack trace at the point where the operation was intercepted to make the decision. When fuzzing is active (`active == true`) and there are `NaN`s remaining to be injected (`n_inject > 0`) and `rand(1:odds)` returns `1`, then `TrackedFloats.jl` inspects the current stack trace to see if the point is in the dynamic extent of the declared `functions` or `libraries` (if any). When all these conditions are met, `TrackedFloats.jl` returns a `NaN` instead of calling the overloaded operation. The value `n_inject` is decremented before the next operation gets intercepted.

Every intercepted operation is a candidate for injection, which means that injected `NaN`s can appear deep inside of an algorithm instead of merely at its toplevel inputs. This can reveal vulnerabilities that are difficult to catch with input-based testing. The parameters `functions` and `libraries` let users control how deep to fuzz; for example, it is often helpful to test specific libraries and avoid trusted standard ones.

When fuzzing reveals an error, the next step is to craft a regression test to guide repairs. `TrackedFloats.jl` therefore records the sequence of injections that it makes during a fuzzing run and enables a replay of any recording after the fact. Replay runs proceed deterministically so that developers can harden their code and check that the fixes remove the error. When running a replay, each intercepted operation consults the reply file (instead of the stack trace and fuzzing parameters) to decide whether or not to return a `NaN` from the computation.

3.1.3 Internals. `TrackedFloats.jl` takes advantage of Julia's operator overloading to track exceptions and fuzz for vulnerabilities. For example, below is a variant of `+` that is overloaded for `TrackedFloat64`. It calls the basic `+` (or injects a `NaN`) and checks for exceptions before returning: This lets `TrackedFloats.jl` intercept all floating-point operations involving `TrackedFloat` types.

```
function +(x::TrackedFloat64, y::TrackedFloat64)
  result = run_or_inject(+, x.val, y.val)
  check_error(+, result, x.val, y.val)
  TrackedFloat64(result)
end
```

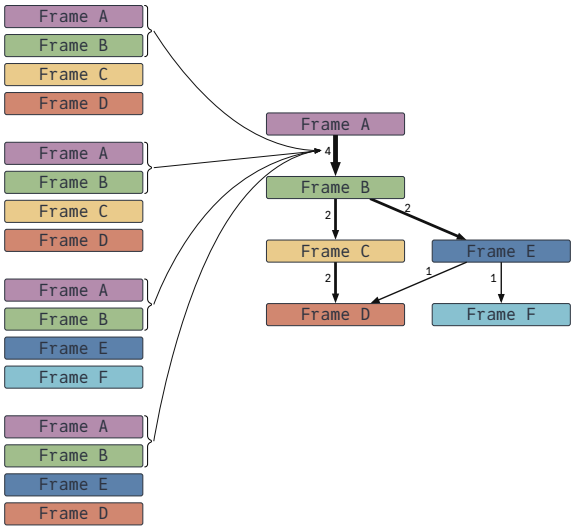

Fig. 4. From stack traces (left) to stack graph (right)

Every arithmetic operation requires a similar overloading. The implementation of `TrackedFloats.jl` uses a metaprogramming technique adapted from `Sherlogs` [20] to abstract over the common patterns. For every binary operation, the library creates an overloading similar to the one for `+` above. Unary operations and others work analogously. This approach saved thousands of lines of code. The implementation weighs in at 218 lines and defines 645 overloaded function; assuming 5 lines of code per function, a handwritten version would require over 3,000 lines.

3.2 Stack Graphs

`TrackedFloats.jl` can produce copious amounts of log files which can be challenging to sift through manually. Coalesced stack trace graphs (CSTGs or stack graphs for short) provide a way to visualize large amounts of stack traces in a compact form [15]. This technique pairs well with `TrackedFloats.jl` and eases the task of analyzing log files.

Figure 4 illustrates the construction of a stack graph from a collection of stack traces. Each trace on the left contributes nodes and edges to the graph on the right. Repeated edges get emphasized with darker lines and larger counts.

Reading bottom-up, a stack graph based on the *gen* events in a program highlights the contexts that frequently produced exceptional values. Using fig. 4 as an example, `Frame D` produced three exceptions, two of which arose under `Frame C`. In a large program with many exceptions, stack graphs offer a way to prioritize debugging efforts: go for the heavily-trodden paths first.

3.3 GPU-FPX

Many programs offload work onto GPUs, which are no less susceptible to floating-point exceptions than CPUs. In fact, GPU programs are worse off because they lack exception-handling mechanisms from the CPU world [23]. Since `TrackedFloats.jl` instruments Julia programs, it cannot help directly; however, the companion tool GPU-FPX instruments GPU kernels (specifically CUDA) to detect and report floating-point exceptions [24]. There is no formal connection between `TrackedFloats.jl` and GPU-FPX. Developers who choose to apply both tools to the respective Julia and CUDA components of their code can gain insights for accelerated programs.

4. Case Studies

FlowFPX has helped to debug exceptions and fuzz for issues in a variety of settings, some synthetic and some realistic. The case studies include a shallow water simulation, the `OrdinaryDiffEq.jl` solver, and a Bayesian inference library.

4.1 `ShallowWaters.jl`

`ShallowWaters.jl` simulates the flow of water over a seabed [18, 19]. The library has dozens of parameters that a scientist can experiment with. One notable parameter is the Courant-Friedrichs-Lewy (CFL) number, which roughly describes the size of the time step to take in running the simulation. A small CFL number makes the simulation run slowly, but accurately; a large number speeds it up but loses precision because the system does not get enough time to propagate information.

Normal values for the CFL number range between zero and one. With a CFL of 0.9, the shallow water simulation produces the graph on the left half of fig. 5 showing current speed in a rectangular basin. Raising the CFL too high, to 1.6, causes trouble (fig. 5, right) with large, white regions where the current speed is `NaN` instead of a normal value.

Running `TrackedFloats.jl` on the high-CFL simulation reveals where the simulation drifted from numbers to `NaN` exceptions. The first step in applying `TrackedFloats.jl` is to convert relevant floats to tracked floats, e.g., `Float32` to `TrackedFloat32`. For `ShallowWaters.jl`, this step is easy because the simulation is parameterized by a floating-point type for internal use. Swapping in a tracked type is enough:

```
run_model(T=TrackedFloat32,
          cfl=10,
          nx=100,
          Ndays=100,
          L_ratio=1,
          bc="nonperiodic",
          wind_forcing_x="double_gyre",
          topography="seamount")
```

With tracking, the simulation runs as before, producing an ugly graph. It additionally outputs logs for all gen, prop, and kill events as they happen. Below is one event from the gen logs:

```
-([-Inf, -Inf])    TF/TrackedFloat.jl:106
momentum_u!        SW/rhs.jl:246
rhs_nonlinear!     SW/rhs.jl:50
rhs!               SW/rhs.jl:14 [inlined]
time_integration   SW/time_integration.jl:77
run_model          SW/run_model.jl:37
top-level
```

We can see that the `NaN` appeared as the result of subtracting two infinities (`-Inf - -Inf`). The trace further shows that this subtraction happens inside the function `momentum_u!` on line 246 of the `rhs.jl` file. This solves the mystery of where one `NaN` came from, but raises a new question about the source of the `Inf` value. The logs for `Inf` gens have an answer:

```
^([-1.515f31, 2])          TF/TrackedFloat.jl:138
literal_pow()              intfuncs.jl:325
...
materialize(^)             broadcast.jl:860
top-level getproperty(...) examples/sw_nan_tf.jl:14
```

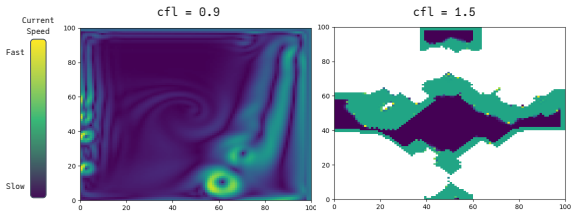

Fig. 5. Raising the CFL number creates white gaps due to `NaN`s

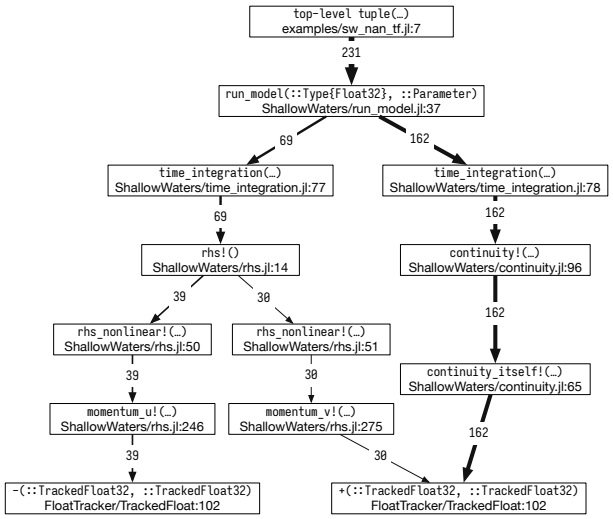

Fig. 6. Stack graph for `NaN` gens in `ShallowWaters.jl`

This `Inf` came from an exponent that overflowed the float type (`-1.515e31^2`). `TrackedFloats.jl` has shown exactly which numeric values in which operation caused exceptions to occur.

4.1.1 Stack Graphs for a Bigger Picture. While the logs for `ShallowWaters.jl` contain useful information, there is an overwhelming amount of it. There are over ten thousand lines in the gen file alone. Converting these logs to a stack graph gives a quick overview of the most common paths to exceptional values.

Figure 6 presents the stack graph for `NaN` gen events in `ShallowWaters.jl` with the high CFL number. Reading bottom-up, every `NaN` came from calls to the '`-`' and '`+`' operations. Calls to '`+`' account for most of the `NaN`s. These `NaN`s arose in two different contexts: a momentum calculation (30 `NaN`s) and a continuity step (162 `NaN`s). Moving to the top of the graph, it shows that the function `run_model` drove the entire simulation. With this overview of the program, a promising next step is to guard against `NaN`s in the momentum and continuity functions.

4.1.2 Stack Graph Differences. Graph diffing works well for stack graphs; it shows how flows in the program evolved from one stage to another. Figure 7 illustrates one diff in the context of `NaN` gens for `ShallowWaters.jl`: it compares the first 10% of gen events to the latter 90% of gens. The positive numbers and green lines indicate flows that are new in the latter part, and the negative numbers and red lines show flows that disappeared in the latter part. A domain expert might use these clues to find where an instability started in the first part of the program. In this case, the function

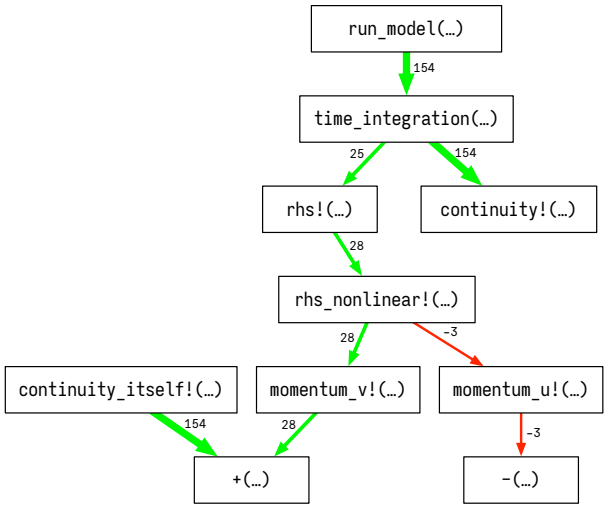

Fig. 7. Stack graph diff

`momentum_u!` showed up only in the beginning of the logs—it may be an effective point to check for `NaN`s.

4.2 Fuzzing from NBody to ODE

We applied the fuzzing abilities of `TrackedFloats.jl` to the `NBodySimulator.jl` package.[2] To avoid injecting in the standard library or other dependencies, we initially configured the fuzzer to inject `NaN`s only when inside of a function from the simulator. Surprisingly, fuzzing injected *zero* exceptional values even when the fuzzer's odds were configured to force injection at the first available opportunity. It turns out that all the floating-point operations for the simulation happened within the `OrdinaryDiffEq.jl` solver.[3]

Fuzzing on `OrdinaryDiffEq.jl` led to a curious situation. The library itself reported a `NaN` and printed a message stating that it would exit. However, after printing that message, the program went into an infinite loop. Using stack graphs to guide our search, we found a `NaN` kill that manifested repeatedly in the logs. The stack traces for that kill originated from inside the file `solve.jl`:

```
<([NaN, 3.0e6])   at TF/TrackedFloat.jl:193
solve!            at ODE/solve.jl:515
...
```

The relevant part of `solve.jl` contains a pair of loops. With injection, the variable `tdir` holds a `NaN`, stopping all productivity:

```
while !isempty(time_stops)
  while tdir * t < first(time_stops)
    # do integration work
    # pop_off(time_stops)
  end
end
```

In more detail, the `NaN` for `tdir` propagates though the multiplication and gets killed by the `<` comparison. Hence the condition for inner `while` loop is always false, which means the outer loop never ends up with an empty list.

This is a real-world example of how `NaN` kills can affect control flow, and we filed an issue for it.[4] Fortunately, the problem in this case is benign as the code was already trying to halt.

4.3 Finch

Finch is a domain-specific language for specifying PDEs [14].[5] In the spirit of FEniCS [2] and related tools [13, 25, 7, 33], Finch helps scientists quickly convert math into code. What sets Finch apart is its flexibility. It supports multiple discretization methods (finite element and finite volume) and multiple backends (Julia, `C++`, DENDRO [8]). Furthermore it strives to output code that humans can easily fine-tune.

Fuzzing with `TrackedFloats.jl` revealed two places where Finch needed protection against user input. The first was when reading an input mesh.[6] A `NaN` injected in the mesh led to a crash further on:

```
BoundsError: attempt to access 1-element
 Vector{Int64} at index [2]
```

Before accessing the input, Finch needed to check for `NaN`s. The second place was in setting bounds for the solver.[7] Here, a `NaN` could leave bounds uninitialized, leading to a bounds error. Additionally, `TrackedFloats.jl` and stack graphs have been useful for identifying `NaN`s that appear in unstable heat simulations written in Finch.

4.4 `Oceananigans.jl`

`Oceananigans.jl` [32] is simulation package for incompressible fluid dynamics that can generate code for Nvidia GPUs. For example, the following program (from the project readme) simulates turbulence:

```
using Oceananigans
grid = RectilinearGrid(GPU(),
  size=(128, 128), x=(0, 2π), y=(0, 2π),
  topology=(Periodic, Periodic, Flat))
model = NonhydrostaticModel(; grid,
  advection=WENO())
ϵ(x, y, z) = 2rand() - 1
set!(model, u=ϵ, v=ϵ)
simulation = Simulation(model;
  Δt=0.01, stop_time=4)
run!(simulation)
```

GPU-FPX provides detailed feedback on this program. The output in fig. 8 shows that 21 kernels appear and generate six floating-point exceptions. There are three `NaN`s, one `Inf`, and two division by zero errors. The report is a starting point for further investigation of the reliability of the example.

4.5 `RxInfer.jl`

We discovered an open issue in the `RxInfer.jl` library related to `NaN` detection and suggested `TrackedFloats.jl` to the develop-

[2] `https://github.com/SciML/NBodySimulator.jl`
[3] `https://github.com/SciML/OrdinaryDiffEq.jl`
[4] `https://github.com/SciML/OrdinaryDiffEq.jl/issues/1939`
[5] Not to be confused with the loop optimizer `Finch.jl` [1].
[6] `https://github.com/paralab/Finch/issues/16`
[7] `https://github.com/paralab/Finch/issues/17`

```
-- FP64 Operations --     -- FP32 Operations --
Total NaN:         2      Total NaN:         1
Total INF:         1      Total INF:         0
Total subnormal:   0      Total subnormal:   0
Total div0:        2      Total div0:        0
                -- Other Stats --
                Kernels:   21
```

Fig. 8. Example GPU-FPX output

ers.[8] Within a day, they were able to track down the location of an important `NaN` gen. This success came with little input from our end; in fact, the application program was closed-source. We merely explained how to use `TrackedFloats.jl` by wrapping inputs in a tracked type.

5. Related Work

5.1 Error Analysis

Demmel et al. [5] examine floating-point exceptional value handling in BLAS and LAPACK, identify several inconsistencies, and propose an API for debugging and adding determinism. The proposal includes an extension to the `INFO_ARRAY` parameter with fields that record *gen-prop-kill* information. It also includes fuzzing, which we realize in `TrackedFloats.jl`.

Toronto and McCarthy [35] propose a test-driven method for detecting numeric error: plot the results of an expression on a range of inputs and look for sharp deviations, or *badlands*. They point out several ways to rewrite code to avoid badlands by rewriting code in a semantics-preserving way.

Herbie [30] automatically rewrites arithmetic expressions to reduce floating-point error. This would combine well with `TrackedFloats.jl`: first identify the location of a `NaN`, ask Herbie to find a repair. The Odyssey [27] workbench provides an interactive interface to Herbie.

5.2 Diagnosing floating-point exceptions

The Julia library `Sherlogs.jl` [20] inspired our use of a custom number type to intercept operations on a number. In contrast to `TrackedFloats.jl` which monitors for exceptional values and logs stack traces at interesting points in their lifetime, `Sherlogs.jl` tracks and reports the range of values seen over the course of a computation. This is intended to provide insight into whether or not a library could tolerate a lower-precision floating-point format.

FPSpy [6] is an `LD_PRELOAD` shared library that works on unmodified x86 binaries. It monitors a program during execution for operations that generate an exception, such as division by zero, underflow, and overflow. By contrast to `TrackedFloats.jl`, it does not track prop or kill events. FPSpy has the advantage of being lightweight enough to run on production code for certain loads.

5.3 Stack Graphs

Our stack graphs utilize the CSTG library for coalesced stack trace graphs [15]. In turn, CSTGs build on the STAT tool from LLNL [3]. STAT collects, analyzes, and visualizes stack traces from concurrent processes to highlight anomalies. It produces visualizations similar to those of CSTG, thought CSTG offers more compact views and supports diffs.

[8] `https://github.com/biaslab/RxInfer.jl/issues/116`

5.4 GPU Exception Tracking

FPChecker [22] is a tool to report floating-point exceptions occurring on the GPU. FPChecker relies on LLVM-level instrumentation of GPU kernels, and so cannot run on the plethora of closed-source GPU kernels in usage today. BinFPE [23] is another tool in the same space; BinFPE performs SASS-level analysis of GPU kernels, but is limited in that it is slow and does not catch errors that alter control flow. The latter deficiency is particularly worrisome, as we have seen, silent `NaN` kills can invalidate results without the user noticing. GPU-FPX [24] improves on the work of FPChecker and BinFPE by being more performant and catching a wider set of errors, including those that alter control flow. GPU-FPX is a shared library that, like FPSpy uses `LD_PRELOAD` to work on unmodified binaries. GPU-FPX runs on CUDA cores from NVIDIA and reports total numbers of exceptional values. Like `TrackedFloats.jl`, GPU-FPX can catch `NaN` gens and kills, but it does this on the GPU where `TrackedFloats.jl` doesn't apply. Despite these improvements, GPU-FPX is limited by the closed-source nature of common GPU cores, and cannot report at the rich level of source detail that `TrackedFloats.jl` can.

6. Discussion

Lightweight tools for error analysis that can quickly identify floating-point problems and suggest repairs are an important topic. The number of scale of scientific application has grown tremendously over the years. For small teams that cannot afford a full-time analyst, tools like FlowFPX fill a critical role.

FlowFPX it itself an evolving toolkit. Below we discuss some topics for future work.

6.1 Performance

`TrackedFloats.jl` incurs significant overhead on the order of 100x slower than a non-instrumented run of the same program. It is a debugging tool, not a production tool. To put this number into context, Valgrind runs with a similar level of slowdown.

In addition to the cost incurred by intercepting floating-point operations, gathering stack traces is expensive. We observe a 10x slowdown on `ShallowWaters.jl` with logging disabled. We recommend that users of `TrackedFloats.jl` make use of the `maxLogs` and `exclude_stacktrace` configuration options to limit the number and kind of logs gathered, and thereby reduce the number of calls to `stacktrace()`. Stack traces are essential to decide where to inject a fault when fuzzing, but we defer them as late as possible to maximize performance.

6.2 Enhanced Fuzzing

While fuzzing is useful for discovering issues, its success rate is low because every floating-point operation is a candidate for injection. Even operations that are already well-defended against `NaN`s are candidates. `TrackedFloats.jl` could use two sorts of tools for improving injection. First, fine-grained control to let users decide where not to inject. Second, tools for understanding the context of an injection point after the fact. Program slicing is especially relevant to the latter point and effectively what we did by hand when fuzzing Finch (section 4.3). For each operation, an expert needs to study the values that feed into it to decide whether they are protected or not.

6.3 Tracking Exceptions in External Libraries

`TrackedFloats.jl` is limited to Julia code. It cannot track the lifetime of exceptional values in external libraries, such as GPU kernels

or C programs. For GPUs it relies on GPU-FPX, through the connection between these tools is loose. In the future, `TrackedFloats.jl` would benefit from an API to plug in tools for external libraries, gather their output, and present a comprehensive view of exceptions in a multi-language program.

6.4 Interface Concerns, Multi-Threading

`TrackedFloats.jl` could be extended to monitor events and values that go beyond exceptions, such as very-large or very-small numbers according to bounds supplied by the user. On a similar note, the interface to `TrackedFloats.jl` is primitive: users express interest in a number by wrapping it in a constructor such as `TrackedFloat64`. Helper functions that track data structures and provide a default float size would make experimentation easier.

`TrackedFloats.jl` stores its configuration and event logs in global data structures, making it unsafe for a multithreaded environment. Adding locks would restore safety, but perhaps a richer interface is in order. Individual threads may wish to configure local logging for better data organization and performance.

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