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Lab 1
ECEN 5634

Part I

Q1: If each section of the artificial transmission line is 5 cm long, then the per-unit capacitance and inductance are $C' = \frac{10\ nF}{0.05\ m} = 200\ \frac{nF}{m}$ and $L' = \frac{1\ mH}{0.05\ m} = 20\ \frac{mH}{m}$. The phase velocity is thus $v_{ph} = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{20*10^{-3}*200*10^{-9}}} = 15811\ \frac{m}{s}$, which allows us to calculate the wavelength as $\lambda = \frac{v_{ph}}{f} = \frac{15811}{10000} = 1.58\ m$. The transmission line has a length of $32 * 0.05\ m = 1.6\ m$, which is 1 wavelength. The ratio $\frac{\Delta z}{\lambda} = \frac{0.05}{1.6} = 0.03125$.

If each segment were changed to $\Delta z = 1\ km$, then we would have $C' = \frac{10\ nF}{10^3\ m} = 10\ \frac{pF}{m}$ and $L' = \frac{1\ mH}{10^3\ m} = 1\ \frac{\mu H}{m}$, which would mean $v_{ph} = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{10*10^{-12}*10^{-6}}} = 3.16 * 10^8\ \frac{m}{s}$ (which is practically impossible since it is greater than the speed of light in vacuum). The wavelength would then be $\lambda = \frac{v_{ph}}{f} = \frac{3.16*10^8}{10^4} = 31.6\ km$. The transmission line still would have length of 1.6 meters, which would be $\frac{1.6}{31600} = 5.06 * 10^{-5}\lambda$. Finally, in this scenario, we find $\frac{\Delta z}{\lambda} = \frac{0.05}{31600} = 1.6 * 10^{-6}$.

Q2: The wavelength would be $\lambda = \frac{c}{f} = \frac{3*10^8}{10^4} = 30\ km$. To have the same length in wavelengths as the artificial line, the actual transmission line would need to be 30 kilometers long.

Q3: The characteristic impedance of the artificial transmission line is $Z_0 = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{20*10^{-3}}{200*10^{-9}}} = 316\ \Omega$.

It does not depend on the value of Δz selected because $\sqrt{\frac{L'}{C'}} = \sqrt{\frac{L_\Delta}{\Delta z} * \frac{\Delta z}{C_\Delta}} = \sqrt{\frac{L_\Delta}{C_\Delta}}$. That is, the ratio of the lumped inductance to capacitance equals the ratio of the per-unit inductance to capacitance.

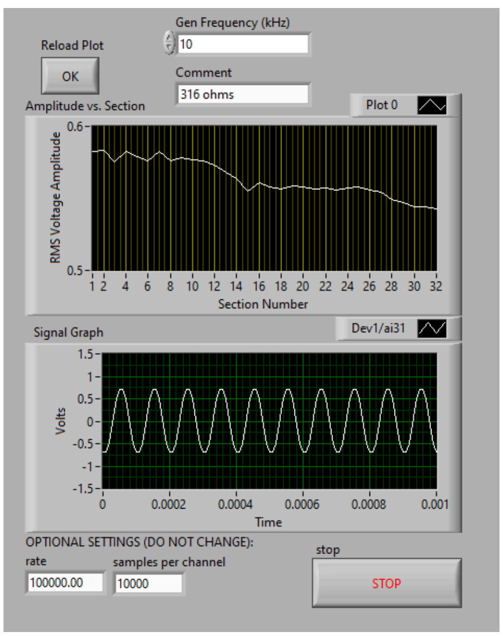

Figure 1: Voltage along line with load of 316 Ω and frequency of 10 kHz.

Q4: Although the line is theoretically matched, the voltage at the load is smaller than the input voltage due to the losses in the line. For our calculations, we have assumed that the line is lossless (made up of only inductors and capacitors) but there is significant resistance in the wires, especially at high frequencies. Much of this resistance is in the windings of the inductors (each coil contains a lot of length of wire).

Q5: Our line was fairly well matched at 316 Ω, though not perfect. A lossless matched line would have VSWR = 1, which would mean that the minimum and maximum voltages across the line are equal, which would mean that the voltage plotted over the length of the line makes a flat line. In addition to the losses, we can see that there is some reflection at 316 Ω since there are two slight, though distinct, standing waveforms. We experimented with adjusting the load resistance, and found that our best match was achieved with a load resistance of 322 Ω.

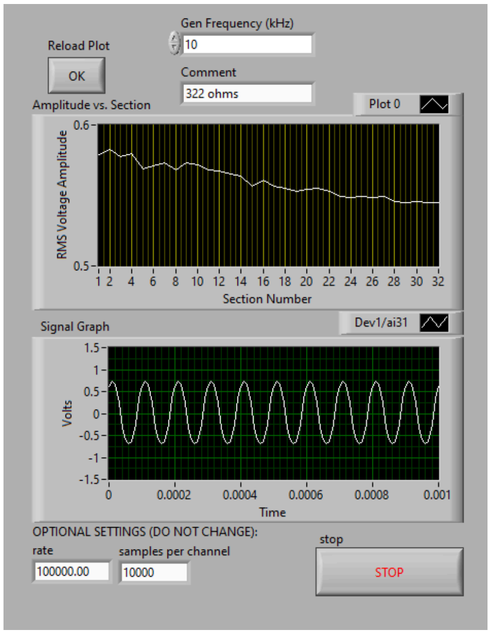

Figure 2: Voltage along line with load of 322 Ω and frequency of 10 kHz.

We then increased the frequency gradually to experimentally determine the cutoff frequency. We noticed that when we set the frequency to 100 kHz, the graph became very choppy as the voltage jumped up and down over short lengths of line. This was because the sampling frequency was the same as the wave frequency. Rather than measure the RMS voltage, it measured the same phase of the wave for each sample, so the measured voltage at each point on the line is just some value between the maximum and minimum. By increasing the sampling rate to 110000 samples per second we were able to get actual RMS values and smooth out our graph.

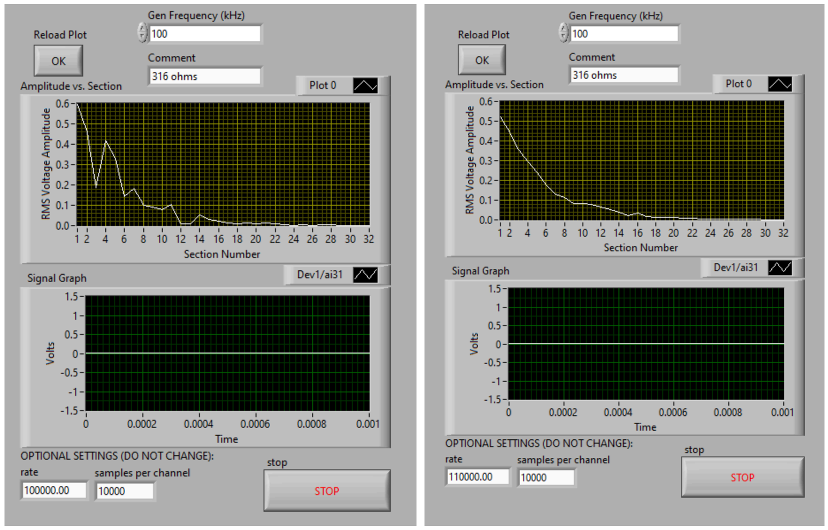

Figure 3: Voltage along line with load of 316 Ω and frequency of 100 kHz with sampling rate of 100000 samples per second (left) and 110000 samples per second (right).

Q6: Based on our observations, we estimated the cutoff frequency to be approximately 100 kHz, as that is where we saw the voltage drop sharply to practically zero early in the line, instead of a gradual decrease. This is about equal to our theoretically calculated value $f_c = \frac{1}{\pi\sqrt{L_\Delta C_\Delta}} = \frac{1}{\pi\sqrt{10^{-3}10^{-8}}} = 100.7\ kHz$.

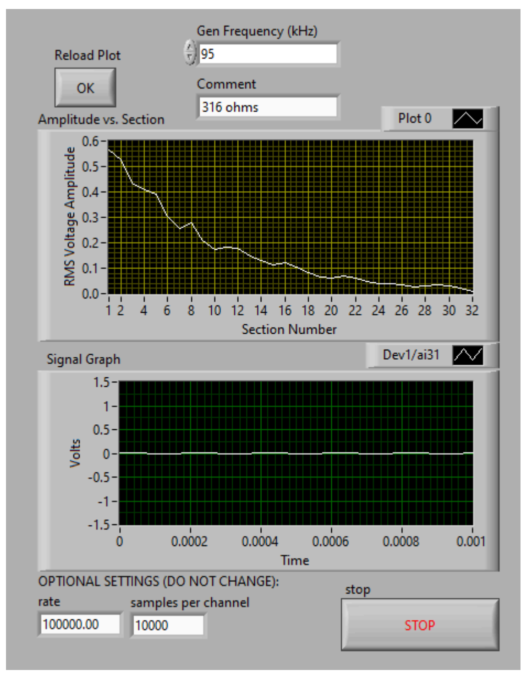

Figure 4: Voltage along line with load of 316 Ω and frequency of 95 kHz. Although we are nearing the cutoff frequency we still see transmission across the line, which is significantly different from when we increased the frequency above 100 kHz.

Part II

Q7: We first find the impedance of the parallel resistor and inductor, and then set the series resistance to the real part and the series inductor impedance to the imaginary part. This allows us to find the series resistance and inductance that our parallel line is equivalent to.

$$Z = \left(\frac{1}{R_1} + \frac{1}{j\omega L_1}\right)^{-1}$$

$$Z = \left(\frac{R_1 + j\omega L_1}{j\omega R_1 L_1}\right)^{-1}$$

$$Z = \left(\frac{j\omega R_1 L_1}{R_1 + j\omega L_1}\right)\left(\frac{R_1 - j\omega L_1}{R_1 - j\omega L_1}\right)$$

$$Z = \left(\frac{j\omega R_1^{\ 2} L_1 + R_1(\omega L_1)^2}{R_1^{\ 2} + (\omega L_1)^2}\right)$$

$$R_2 = \frac{R_1(\omega L_1)^2}{R_1^{\ 2} + (\omega L_1)^2} \qquad j\omega L_2 = \frac{j\omega R_1^{\ 2} L_1}{R_1^{\ 2} + (\omega L_1)^2}$$

$$R_2 = \frac{150(2\pi * 10^4 * 10^{-3})^2}{150^2 + (2\pi * 10^4 * 10^{-3})^2} \qquad L_2 = \frac{150^2 * 10^{-3}}{150^2 + (2\pi * 10^4 * 10^{-3})^2}$$

$$R_2 = 22.4\,\Omega \qquad L_2 = 851\ \mu H$$

Q8: Now that we have the equivalent series lumped elements, we find the per-unit resistance and inductance as $R' = \frac{22.4\,\Omega}{0.05\,m} = 448\ \frac{\Omega}{m}$ and $L' = \frac{0.851\,mH}{0.05\,m} = 17.0\ \frac{mH}{m}$. We then calculate the characteristic impedance to be $Z_0 = \sqrt{\frac{j\omega L' + R'}{j\omega C'}} = \sqrt{\frac{j2\pi * 10^4 * 17 * 10^{-3} + 448}{j2\pi * 10^4 * 200 * 10^{-9}}} = (297.7 - j59.9)\ \Omega$. To achieve a matched load, we the decade resistor to 298 Ω and the decade capacitor to $C = \frac{1}{\omega * 59.9} = \frac{1}{2\pi * 10^4 * 59.9} = 266\ nF \approx 0.27\ \mu F$, and we connect them in series.

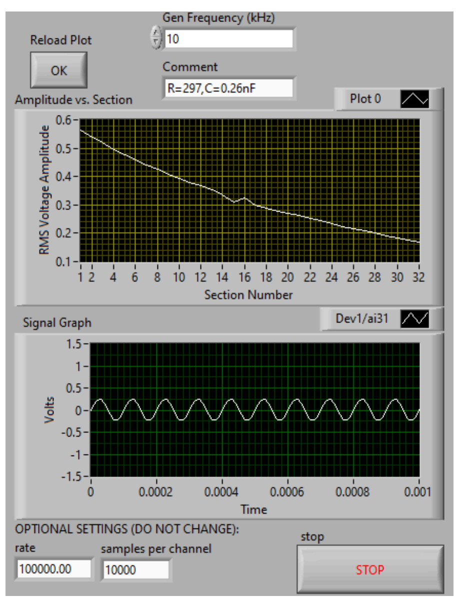

Figure 5: Voltage across line with added losses and matched load.

Q9: We first calculate the attenuation coefficient from the measurements in Figure 5. We use the points (4, 0.5) and (28, 0.2) as they are easiest to read from the graph. The distance between the two points is made up of 24 sections of 5 cm each, so it is 1.2 m.

$$0.2 = 0.5e^{-1.2\alpha}$$
$$0.4 = e^{-1.2\alpha}$$
$$-1.2\alpha = ln\, 0.4$$
$$\alpha = \frac{ln\, 0.4}{-1.2} = 0.76$$

We then calculate the theoretical expected value of the attenuation coefficient.

$$\alpha = Re[\sqrt{(j\omega C')(j\omega L' + R')}]$$
$$\alpha = Re[\sqrt{(j2\pi * 10^4 * 200 * 10^{-9})(j2\pi * 10^4 * 17 * 10^{-3} + 448)}]$$
$$\alpha = 0.75$$

We find that the expected and measured values are almost identical. The difference is likely due to the losses in the artificial transmission line that we are neglecting.

Part III

Q11: To make the artificial transmission line half of a wavelength long, we need to double the wavelength. We can do this by halving the frequency since the phase velocity remains the same and $v_{ph} = \lambda f$. The frequency we need is thus $\frac{10\, kHz}{2} = 5\, kHz$. When we examine a quarter wavelength line, we will need to again halve the frequency.

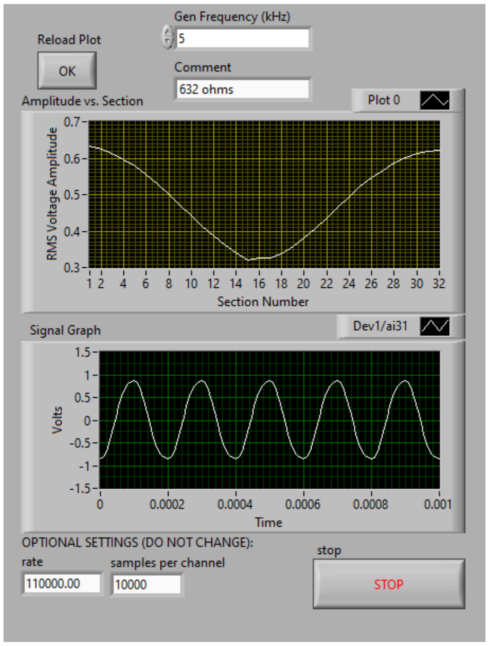

Figure 6: Voltage along line for load impedance of 632 Ω and frequency of 5 kHz. As we expect, we see half of a wavelength.

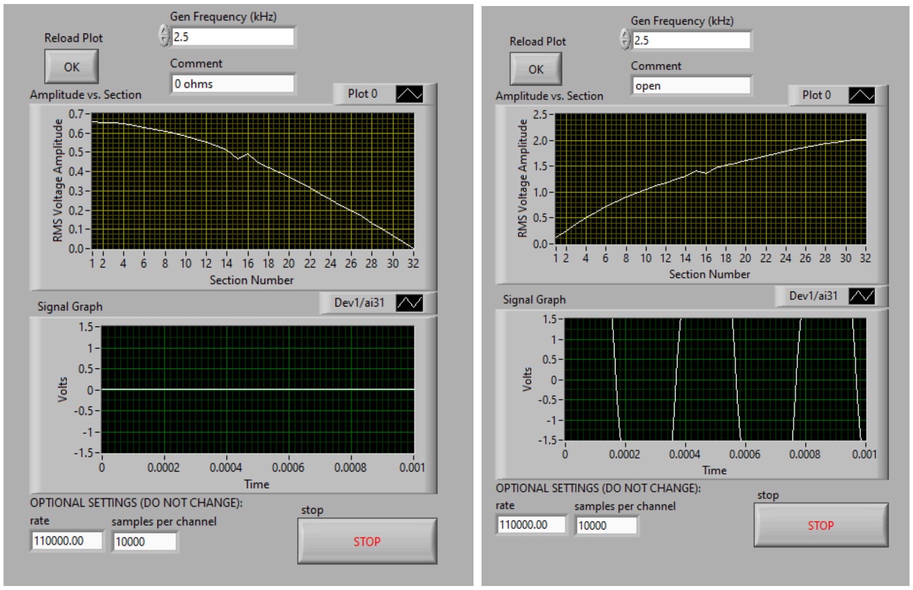

Figure 7: Voltage along line for frequency of 2.5 kHz with short load (left) and open load (right).

Q12: If the line was perfect, we would see 0 voltage at the load for the short, and twice the amplitude of the input voltage at the source (where it sees an open circuit). We would see the opposite for the open load: twice the input voltage amplitude at the load, and 0 voltage at the source. The shape of the curve in either case would be the portion of a squared sine wave from a maximum to adjacent minimum.

Q13: In the case of a half-wavelength line, the voltage is maximized at both the load and the source, and minimized in the middle. In the case of a quarter-wavelength line, the voltage is also maximized at the load, but the curve is stretched by a factor of 2 along the horizontal axis so that the minimum is at the source. The generator "sees" an open circuit in the former case, and a short in the latter.

The currents are opposite because they are maximized at a short and minimized at an open, so when voltage is high, currents are low. (If plotted over the line, current magnitude would also make a sinusoid, but with a quarter-wavelength shift on the horizontal axis.) In either case current would be zero at the load. For a half-wavelength line it would also be zero at the source, and maximized in the middle. For a quarter-wavelength line it would be maximized at the source. The maximum amplitudes of current and voltage are the same for either line.

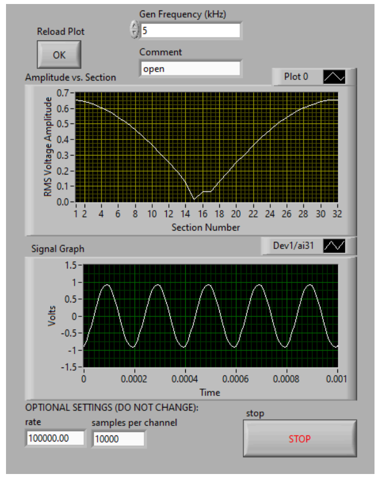

Figure 8: Voltage along line for frequency of 5 kHz with open load.

Q14: We calculate the capacitance $C = \frac{1}{\omega Z_0} = \frac{1}{2\pi * 10^4 * 316} = 50\ nF$ and use this value on the decade capacitor as the impedance.

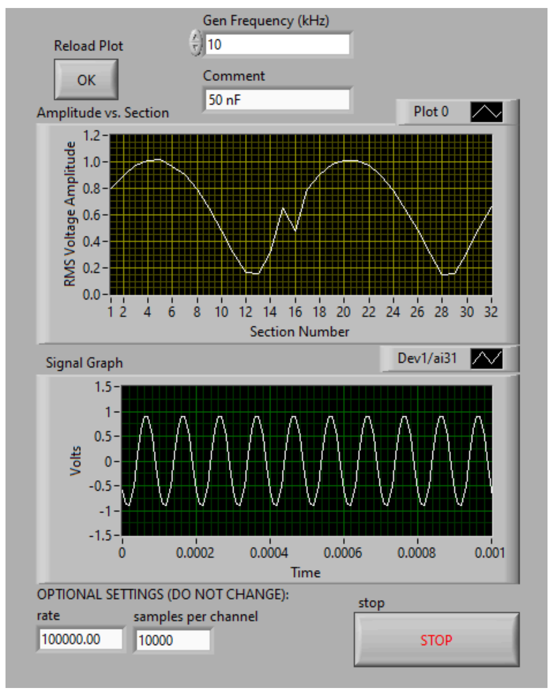

Figure 9: Voltage along line with frequency of 10 kHz and the load being a capacitor of 50 nF.