

Part I

When we turn TRANSFORM OFF, we see that the start and stop frequencies are 5.5 MHz and 1.1 GHz respectively, and that changing the start and stop times does not affect this.

Q1: Some VNAs may automatically adjust the frequency range to ensure that the sampling rate is fast enough for the time range that you wish to observe. This does not appear to be the case for the VNA we are using.

Q2: We first checked the settings of the VNA to find that it assumes the velocity to be the speed of light in vacuum. We then measured the round-trip time delay for both the short and open load at the end of the GR cable, which was 9.405 ns and 9.593 ns respectively. We then calculated the length of the cable to be $L = \frac{c_0 t}{2\sqrt{2.26}} = \frac{3*10^8*9.5*10^{-9}}{2\sqrt{2.26}} = 94.8 \text{ cm}$. We also measured the cable with a meter stick and got $L = 94.5 \text{ cm}$, so both methods could have worked, but TDR was probably more accurate.

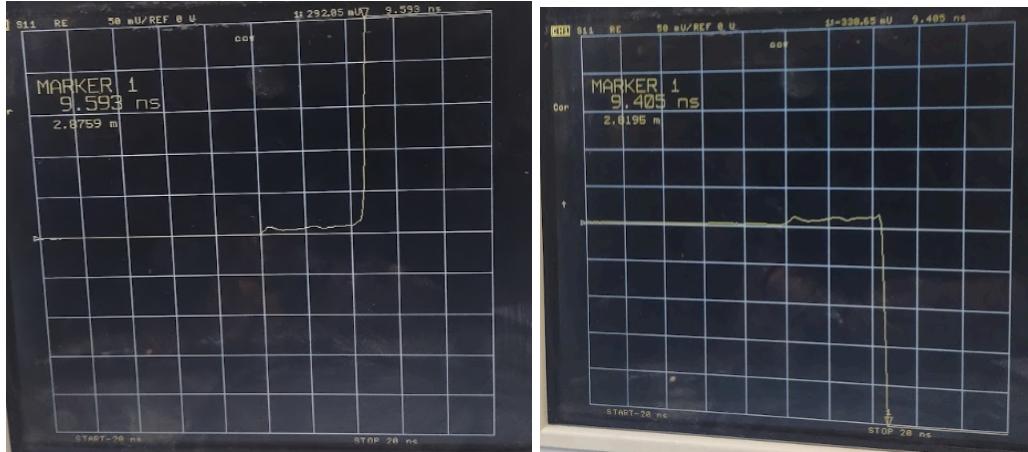


Figure 1: VNA display for time-domain reflectometry of open load (left) and short load (right).

Q3: The shortest and longest lengths of the stub are 6 cm and 26 cm according to the markings on the stub. We calculated them to be $L_1 = \frac{c_0 t}{2\sqrt{2.26}} = \frac{3*10^8*413*10^{-12}}{2\sqrt{2.26}} = 4.1 \text{ cm}$ and $L_2 = \frac{c_0 t}{2\sqrt{2.26}} = \frac{3*10^8*1.605*10^{-9}}{2\sqrt{2.26}} = 16.0 \text{ cm}$. The stub termination is not perfect; if it was (and the pulse was a perfect step) then we would see the voltage change instantaneously from the incident voltage to 0 without hitting any other values.

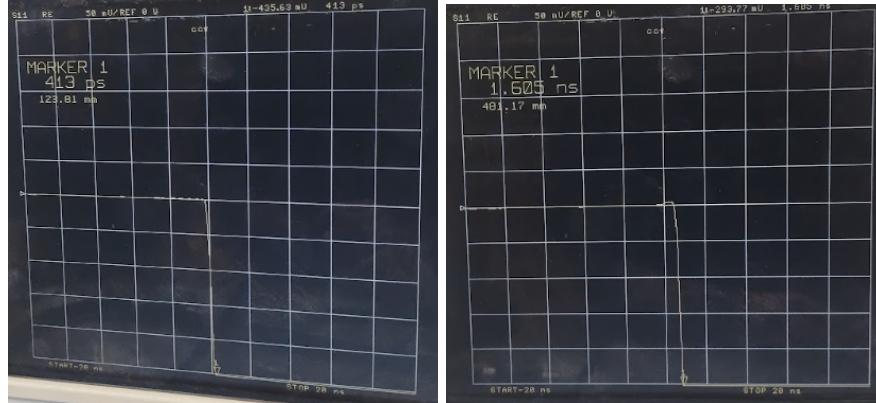


Figure 2: VNA display with short stub of length 6 cm (left) and 26 cm (right).

Q4: The “dirt” comes from parasitic inductance in the connectors. We can tell that it is inductive because it bumps the voltage up temporarily, which is indicative of higher impedance than we expect, such as an inductor initially acting as an open.

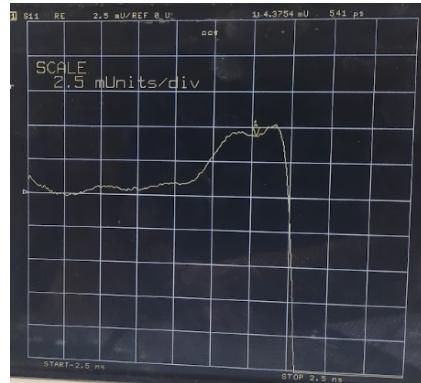


Figure 3: VNA display with short stub of length 26 cm zoomed in on edge.

Q5: The reflection coefficient should theoretically be $\Gamma = S_{11} = \frac{100-50}{100+50} = \frac{1}{3}$. We measured it to be 0.333 (Figure 5, left), which is in agreement with our calculation.

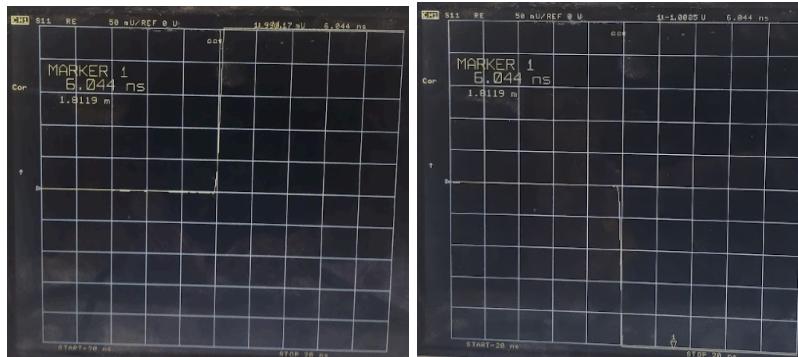


Figure 4: VNA display with open (left) and short (right) terminations. The observed reflection coefficients are $\Gamma_{\text{open}}=0.999$ and $\Gamma_{\text{short}}=-1.001$.

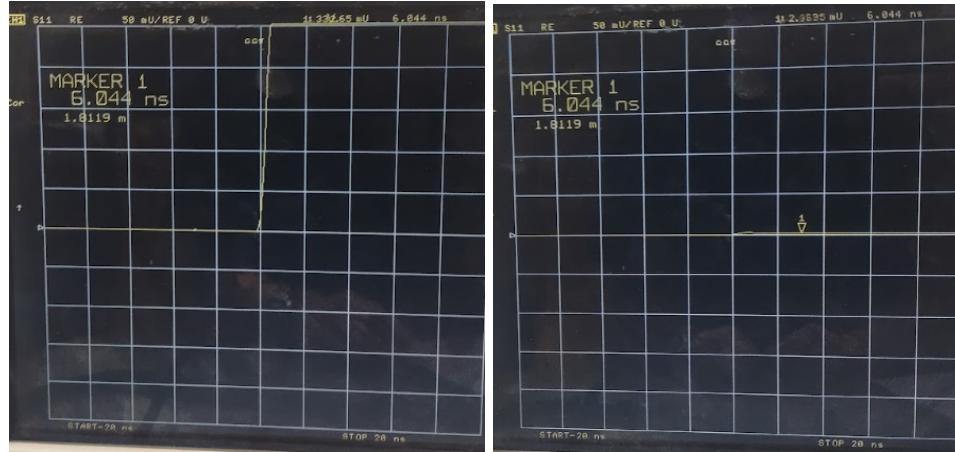


Figure 5: VNA display with $100\ \Omega$ (left) and $50\ \Omega$ (right) terminations. The observed reflection coefficients are $\Gamma_{100}=0.333$ and $\Gamma_{50}=0.002$.

Q6: We see that the connector being loose increases the parasitic effects at the connector. There is especially a greater reflection due to the equivalent series inductance at the connector

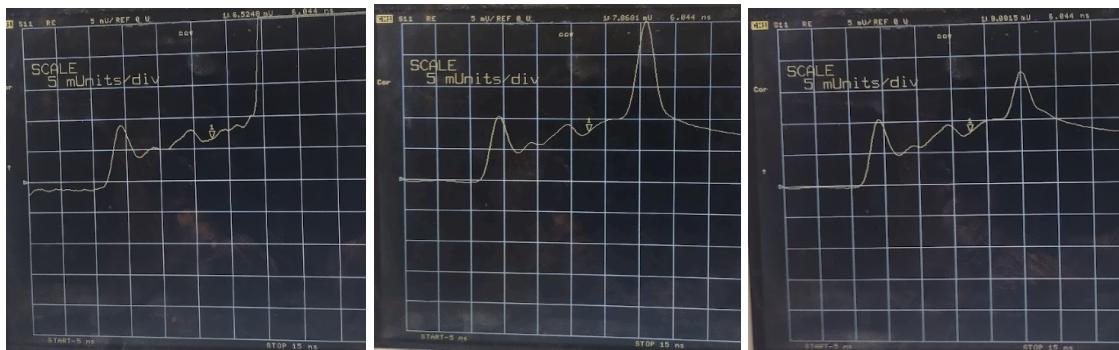


Figure 5: VNA display zoomed in on reflection with matched load unconnected (left), loosely connected (center), and tightly connected (right).

Q7: The value of the load is theoretically $Z = 50||100 = \frac{5000}{150} = 33.3\ \Omega$. We computed it to be $Z = 50 \frac{1+\Gamma}{1-\Gamma} = 50 \frac{1-0.138}{1+0.138} = 37.9\ \Omega$. The plot of S_{21} should theoretically rise to the voltage amplitude of the incident wave after S_{11} drops down when it hits the parallel load. This is what we see happen in our plot, but we also noticed that it rises gradually over the course of many nanoseconds, which might be due to the repeated reflections at the $50\ \Omega / 100\ \Omega$ boundary.

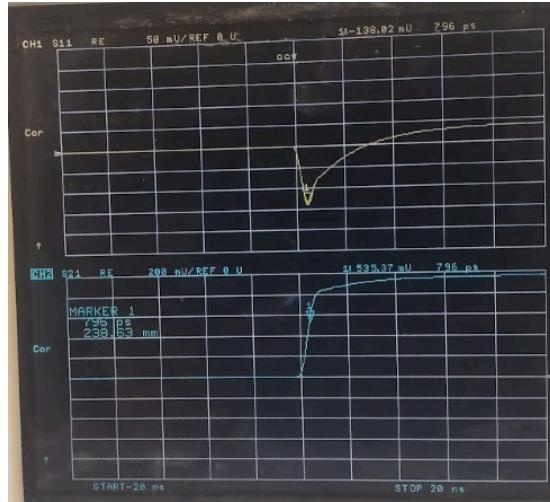


Figure 6: S_{11} and S_{21} with tee between ports 1 and 2, and 100Ω load connected to side arm of tee.

Q8: This line does not have a uniform characteristic impedance along its length. Since $Z_L = Z_0 \frac{1+\Gamma}{1-\Gamma}$, the impedance of the line is greater when the reflection is greater. It may not be as extreme of a change as the plot of S_{11} shows in the reflection coefficient, however, because there are also parasitic effects at the connectors and repeated reflections wherever the impedance changes.

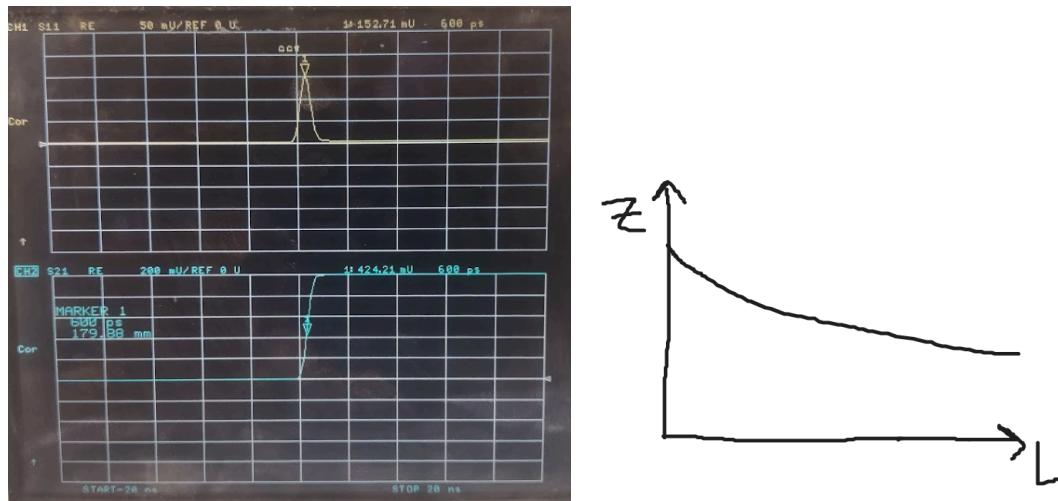


Figure 7: Non-constant Z_0 load connected to ports 1 and 2 (left) and rough sketch of Z_0 over the length of line (right).

Part II

1. A. The measured time delay is 46 ns. The phase velocity is

$$v_p = \frac{3 \cdot 10^8}{\sqrt{\epsilon_r}} = \frac{3 \cdot 10^8}{\sqrt{2.5}} = 1.9 \cdot 10^8 \text{ m/s}, \text{ and the length of the cable based on the time delay is}$$

$L = v_p t = 8.73 \text{ m}$. We also estimated the length of the cable by measuring the diameter of one of its windings with a yardstick and multiplying the circumference by the number of windings. In that case, we got $L = 0.28\pi * 10 = 8.80 \text{ m}$.

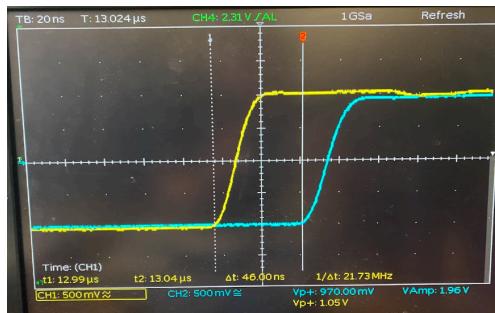


Figure 1: Scope display with 1 MHz square wave source and roughly 10 m piece of coaxial cable between channels 1 and 2, 50Ω load.

- B. After removing the 50Ω load, we captured the display in Figure 2. We see that there is still a delay between the pulse at the source and the pulse at the load, but that the amplitude of each wave is twice what it was in the previous problem, and that channel 1 “pauses” on both the rising and falling edge; it changes to its halfway point, flattens out, and then continues moving. This is because the open load has a reflection coefficient of 1, so the total voltage, after the reflected wave returns, is twice that of the incident wave. The flat section of the rising edge of channel 1 is after the incident wave has been sent, but before the reflected wave has returned. Channel 2’s rising edge is in the middle of this because it is when the incident wave reaches the load and gets reflected.

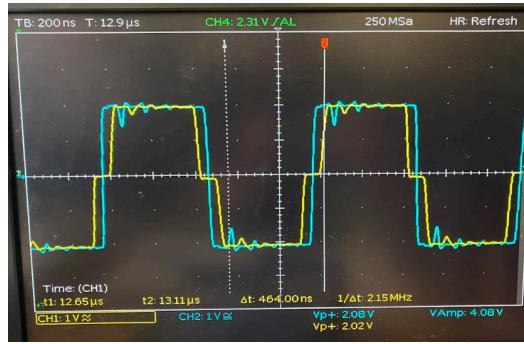


Figure 2: Scope display with 1 MHz square wave source and roughly 10 m piece of coaxial cable between channels 1 and 2, 100Ω load.

- C. The effect of the parallel capacitance is to make the cable appear longer. This is because the capacitance is initially a short circuit, so when the incident wave hits, the impedance drops, and

it takes slightly longer to reach its final value (when the capacitor is charged). This makes us measure a longer travel time through the cable.

- 2. A.** Since we still have the same load and the same length of cable as in the previous question, we see that the amplitudes and the delays on both edges are the same. The only difference now is the period of the pulse, so there is more time between the rising and falling edges. In hindsight, we placed the markers in Figure 3 in the wrong spots; the second marker should have been where channel 2 begins rising to measure the one-way trip time, or where channel 1 begins rising the second time to measure the round-trip time. Using the ticks on the horizontal axis, we can estimate the round trip time as $(1 - \frac{1}{3.5}) * 136 \text{ ns} = 97 \text{ ns}$, which is about twice the one-way time we measured in the previous question.

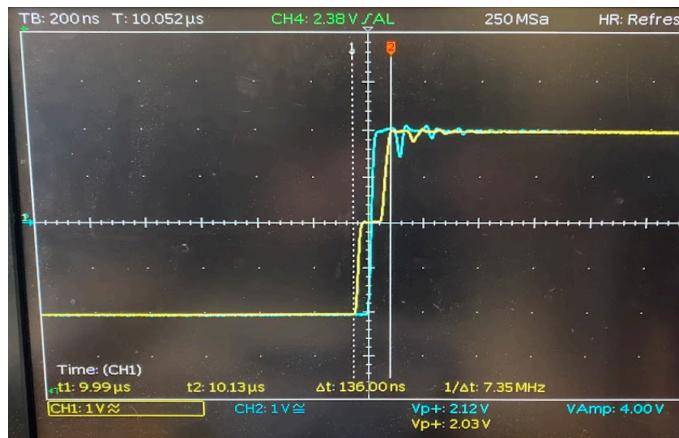


Figure 3: Scope display with 10 kHz 50% duty cycle pulse source and roughly 10 m piece of coaxial cable between channels 1 and 2, 100 Ω load.

- B.** We got no reflection when the variable resistor was set to 50 Ω . This tells us that the characteristic impedance of the cable is 50 Ω , because it was matched to the 50 Ω load.

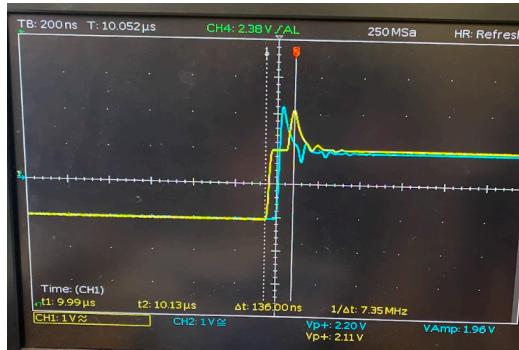


Figure 4: Scope display with 10 kHz 50% duty cycle pulse source, variable resistor set to 50 Ω .

- C.** For the 75 Ω load, we found that the reflection coefficient was $\Gamma_{75} = \frac{0.38}{1.8} = 0.211$, which allows us to calculate the resistance as $Z_L = Z_0 \frac{1+\Gamma_{75}}{1-\Gamma_{75}} = 50 \frac{1.211}{0.789} = 76.7 \Omega$. For the 25 Ω

load, we found that the reflection coefficient was $\Gamma_{25} = \frac{-0.620}{2.015} = 0.307$, which gives us the resistance $Z_L = Z_0 \frac{1+\Gamma_{25}}{1-\Gamma_{25}} = 50 \frac{0.692}{1.307} = 26.5 \Omega$ for this scenario. We found that load values measured with TDR were both only slightly above the settings of the variable resistor. We also tried a short load and found that as expected, the total voltage became 0 after the reflected wave returned.

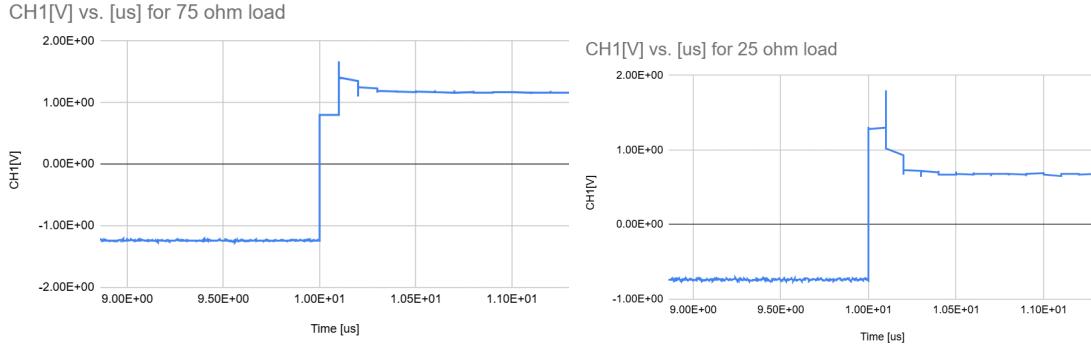


Figure 5: Scope output (exported and plotted externally) for variable resistor set to 75Ω (left) and 25Ω (right).

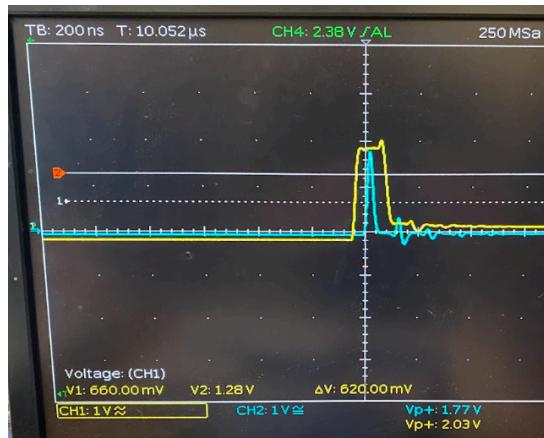


Figure 6: Scope display with 10 kHz 50% duty cycle pulse source, short load.

D. We first set the decade capacitor to 50 nF, and using the points 0 V at $10.8 \mu\text{s}$ and 0.6 V at $12.25 \mu\text{s}$, calculated the time constant to be $\tau = \frac{-1.45*10^{-6}}{\ln \frac{0.12}{0.72}} = 8.09 * 10^{-7} \text{ s}$, so the measured capacitance is $C = \frac{\tau}{(25||50)} = \frac{8.09*10^{-7}}{16.67} = 48.5 \text{ nF}$. We then set the decade capacitor to 500 nF, and using the points 0.1 V at $11.9 \mu\text{s}$ and 0.248 V at $14.7 \mu\text{s}$, calculated the time constant to be $\tau = \frac{-2.8*10^{-6}}{\ln \frac{0.478}{0.62}} = 1.02 * 10^{-5} \text{ s}$, so the measured capacitance is $C = \frac{\tau}{(25||50)} = \frac{1.02*10^{-5}}{16.67} = 612 \text{ nF}$.

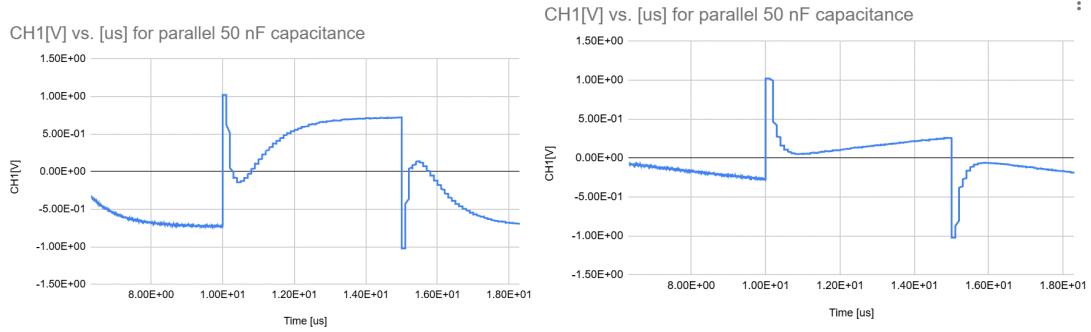


Figure 7: Scope output (exported and plotted externally) for variable capacitor set to 50 nF (left) and 500 nF (right).

3. We found that when we reattached the long cable, the amplitude of the sine wave on channel 1 increased slightly, due to the constructive interference of the incident and reflected waves. Although it was theoretically matched, it was not perfect due to the parasitics of the cable and connectors.

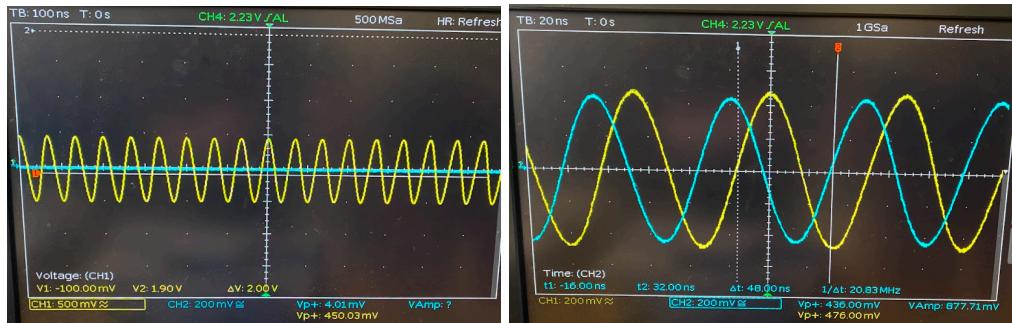


Figure 8: Scope display of 15 MHz sine wave without long cable (left) and with long cable (right).

We measured the change in phase to be $360\left(\frac{t_{delay}}{t_{period}}\right) = 360\left(\frac{48.0}{66.8}\right) = 259^\circ$. The wavelength at this frequency is $\lambda = \frac{v_p}{f} = \frac{1.9*10^8}{15*10^6} = 12.6 \text{ m}$, so the cable is 0.69 wavelengths long. Therefore, we should expect a phase change of $0.69*360^\circ=248^\circ$.

Finally, we calculated the attenuation constant, after observing the amplitudes of the sine waves to be 476 mV on channel 1 and 436 mV on channel 2. We got $\alpha = \frac{\ln \frac{436}{476}}{-10} = 0.009 \text{ Np/m}$.