

Part I

Q1: In Figure 10.6, the purpose of the 10-dB coupler is to split the signal from the sweeper into a larger portion, that will be bounced off the target and eventually returned (greatly attenuated), and a smaller portion that will later be compared with this returned signal for the radar calculation. The two horn antennas are for sending the signal in the direction of the target and capturing the reflected signal from the target. The isolator ensures that the received signal does not re-enter the 10-dB coupler from the coupled port, which would send signal into the isolated and input ports. The three-port tee combines the transmitted and received signals, and the diode detector mixes them and outputs at a frequency that is the difference of the two input waves. Finally, the functionality of the sweeper is to generate the transmitted signal, and the functionality of the oscilloscope is to view the intermediate frequency signal.

The cross-sectional length of the WR-90 waveguide is 22.86 mm, so the cutoff frequency is

$$f_c = \frac{c_0}{2a} = \frac{3 \cdot 10^8}{2 \cdot 0.02286} = 6.56 \text{ GHz}.$$

At 10 GHz, the wavelength in free space is 3 cm. The horn's aperture has dimensions 7.4 cm x 5.7 cm, so its area is 0.004218 m². We assume that its effective area is the same as its geometric, and that its efficiency is 1. We can then calculate the gain as $G = \frac{4\pi A}{\lambda^2} = \frac{4\pi \cdot 0.004218}{0.03^2} = 58.9 = 17.7 \text{ dB}$. We

calculate the far field distance as $r = \frac{2d^2}{\lambda} = \frac{2(0.057^2 + 0.074^2)}{0.03} = 58.2 \text{ cm}$.

Q2: The frequency range and period of the VCO sweep will affect the output waveform for a given target distance. This is because for a given distance, the signal will take the same amount of time to return from the target. However, if the rate of frequency change (the slope k) is larger, then the observed frequency difference will be greater. Setting a larger frequency range or shorter time period would make k greater.

Q3: The time rate of change is $k = \frac{1 \text{ GHz}}{0.02 \text{ s}} = 50 \frac{\text{GHz}}{\text{s}}$. For an object 50 cm away, the total trip is 1 m and it takes the signal $t = \frac{d}{c_0} = \frac{1 \text{ m}}{3 \cdot 10^8 \text{ m/s}} = 3.33 \text{ ns}$ to travel this distance and enter the receiver horn. This corresponds to a frequency difference of $\Delta f = kt = 50 \frac{\text{GHz}}{\text{s}} * 3.33 \text{ ns} = 167 \text{ Hz}$.

Q4: Because of the square-law behavior of the diode, the output contains a term which is the product of cosines of the two input frequencies ($A \cos(\omega_{Tx} t) \cos(\omega_{Rx} t)$ where A is a constant). This can be rewritten as the sum of two cosines, where one has the frequency $\omega_{Tx} - \omega_{Rx}$ and the other $\omega_{Tx} + \omega_{Rx}$. The difference frequency is what we look at on the oscilloscope and use to calculate the distance. Over the period of the diode (20 ms) we see a high frequency section and a low frequency section. The low frequency represents the distance to the object, because it is where the Tx and Rx signals being compared are from the same "chirp", so we can calculate the delay. The high frequency component appears when the transmitted chirp starts over at 8 GHz. The signal being received is still from the last chirp (near 9 GHz), so the frequency difference is very large and does not represent the distance.



Figure 1: One period of the diode output signal with the horns 65 cm apart.

Q5: If the two horns are touching, we expect the difference frequency to be 0 Hz since the distance is 0 m so there should be no delay between the transmitted and received signals.

Q6: We measured the calibration constant to be 472 Hz. This was the frequency difference displayed when the horns were touching. The received signal travels further, because it still needs to go through both antennas and their cables, and it travels further by a distance of $d = \frac{\Delta f c_0}{k} = \frac{472 * 3 * 10^8}{50 * 10^9} = 2.8 \text{ m}$.

Q7: With the copper plate 45 cm from the antenna, we observed a frequency difference of 943 Hz, which corresponds to a calculated radar target distance of $d = \frac{(\Delta f - 472)c_0}{2k} = \frac{(943 - 472) * 3 * 10^8}{2 * 50 * 10^9} = 1.4 \text{ m}$ with the calibration constant taken into account. This is much higher than the result we should have obtained which may be due to error in measuring the calibration constant or interference between the two signals that we did not consider.

Q8: We then took measurements up to 1 m and plotted the frequency observed on the oscilloscope versus the target distance. Figure 2 shows that our results are much different from what we expected. Above 65 cm, we did not observe any change in the frequency difference. One possible cause of error is the orientation of our setup (if we had the signal reflecting in such a way that it was not received well by the other horn) and another is interfering signals from elsewhere in the lab.

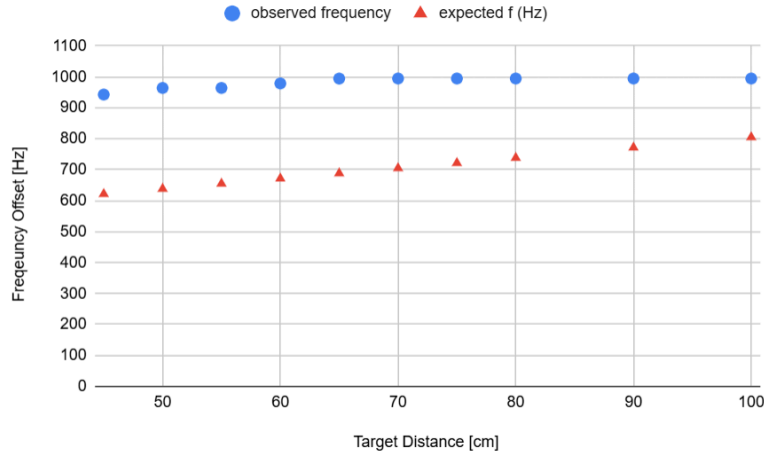


Figure 2: Frequency differences observed on the oscilloscope (the calibration constant is included) over target distance, and expected frequency differences.

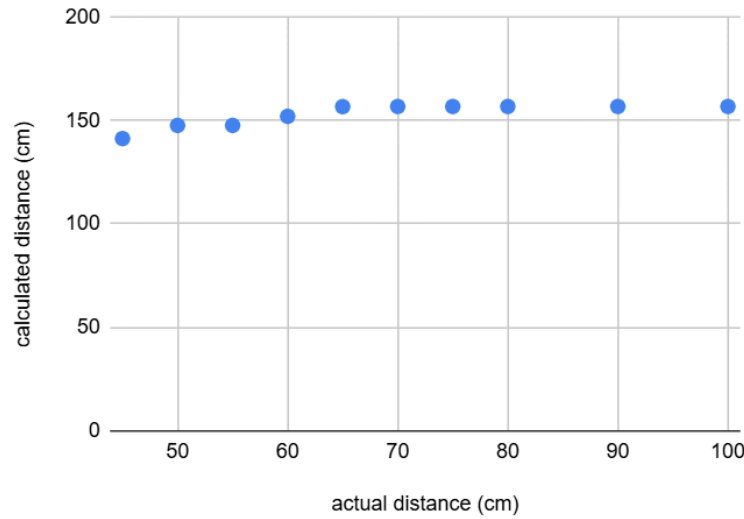


Figure 3: Distance calculated by frequency distances from radar setup versus distance measured by ruler.

Q9: The Doppler shift that we measured was 43.44 Hz. The calibration constant does not apply here because the transmitted frequency is constant, so the distance to the target does not impact the frequency difference at all and we do not need to include the waveguide lengths. Instead, the difference between the transmitted and received frequencies is due solely to the Doppler effect and the relative velocity between the target and antennas. We calculate the speed of the plane to be $v = \frac{f_d c}{2f} = \frac{43.44 * 3 * 10^8}{2 * 8.5 * 10^9} = 0.77 \text{ m/s}$.

Part II

Q10: With different tuner positions, we are able to change the signal amplitude by over a factor of 2, which is enough that the scope automatically resizes the vertical axis. The tuner is an adjustable short stub, which allows us to match impedances over a range of frequencies. Making the best match at the frequency we are observing means that the level of the received signal will be the highest.

Q11: We then experimented with rolling a can down a ramp and measuring the Doppler shift. In each experiment, the incline was the same, but we changed the starting position of the can. We calculated the velocity of the can at the point directly in front of the horn using the Doppler frequency and using kinematics. The calculation using the Doppler shift was $v = \frac{c_0 f_d}{2f}$, where f is the transmitted frequency and f_d is the difference frequency on the oscilloscope where the amplitude is maximized (we assume this is when the can passes the mark directly in front of the antenna). The calculation derived from conservation of energy is $v = \sqrt{\frac{4}{3}gH}$, where g is the acceleration of gravity and H is the vertical height above the mark where the can begins. It is important to note that this equation was derived assuming that the can was a solid cylinder with consistent density. The actual can had a higher moment of inertia, meaning that more of its energy went into rotational motion than we anticipated. This means that our calculations are overestimates, which can be seen in Table 1 since $v_{\text{kinematics}}$ is higher than v_{doppler} in each case.

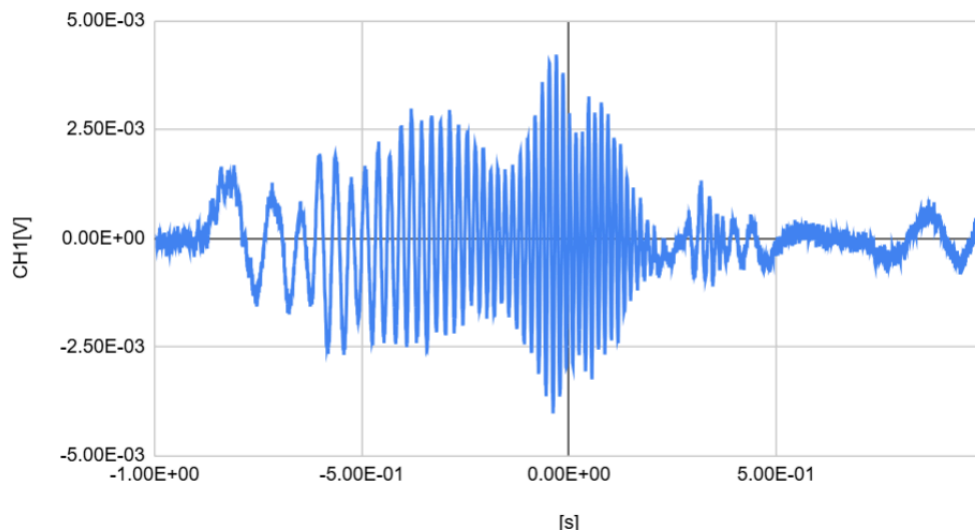


Figure 4: Oscilloscope output for can beginning 12 cm above the mark. We measured the Doppler shift at the point with maximum amplitude to calculate the velocity directly in front of the horn.

Initial height (m)	$v_{\text{kinematics}}$ (m/s)	frequency_doppler (Hz)	v_{doppler} (m/s)
0.06	0.8858893836	39.0625	0.732421875
0.12	1.252836781	64.1025641	1.201923077
0.17	1.491174034	71.42857143	1.339285714

Table 1: Results from the can experiment. The transmitted frequency was 8 GHz in each case.

Q12: We then experimented with swinging a basketball on a string attached to the ceiling. In each trial, we released the basketball at the same point, where the string was taut but the basketball was 8 cm above where it would be at its lowest point. Since there was no rotational kinetic energy (and since we neglect air resistance), all of the gravitational potential energy goes into its translational velocity. Therefore, in this experiment we calculated the maximum velocity of the basketball (velocity at the lowest point in the

trajectory) as $v = \sqrt{2gH}$. The velocity was calculated from the Doppler shift in the same manner as the previous problem. The only difference was that we measured the Doppler shift at the point of maximum frequency (rather than maximum amplitude) because this is the point at which the ball is at the lowest point and has the maximum velocity with which we will compare the expected velocity.

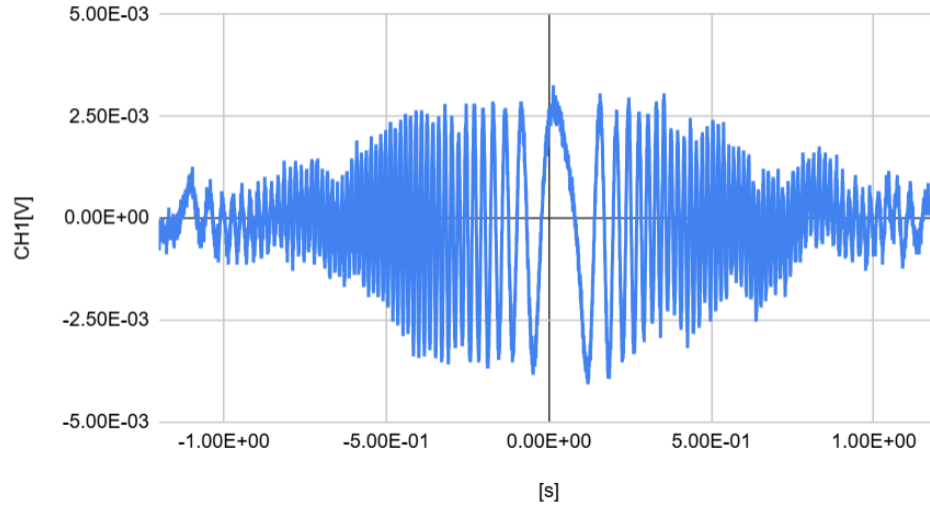


Figure 4: Oscilloscope output for basketball pendulum with transmitted frequency of 8 GHz.

Our two trials in this experiment both began with the ball at the same initial height, so we expect both to have the same velocity. The difference between the two trials was the frequency transmitted by the radar system; we experimented with both 8 GHz and 12 GHz. As we expected, the Doppler shift frequency $f_d = \frac{2vf}{c_0}$ was greater for the 12 GHz transmitted frequency, but when we use the Doppler effect to calculate the velocity, we obtained about the same answer.

frequency_transmitted (GHz)	Initial height (m)	v_kinematics (m/s)	frequency_doppler (Hz)	v_doppler (m/s)
8	0.08	1.252836781	71.42857143	1.339285714
12	0.08	1.252836781	100	1.25

Table 2: Results from the basketball experiment.