

# Classical Physics Models

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If you're getting some cold feet to jump in to DiffEq land, here are some handcrafted differential equations mini problems to hold your hand along the beginning of your journey.

## 0.1 First order linear ODE

### Radioactive Decay of Carbon-14

$$f(t, u) = \frac{du}{dt}$$

The Radioactive decay problem is the first order linear ODE problem of an exponential with a negative coefficient, which represents the half-life of the process in question. Should the coefficient be positive, this would represent a population growth equation.

```
using OrdinaryDiffEq, Plots
gr()
```

```
#Half-life of Carbon-14 is 5,730 years.
C_1 = 5.730
```

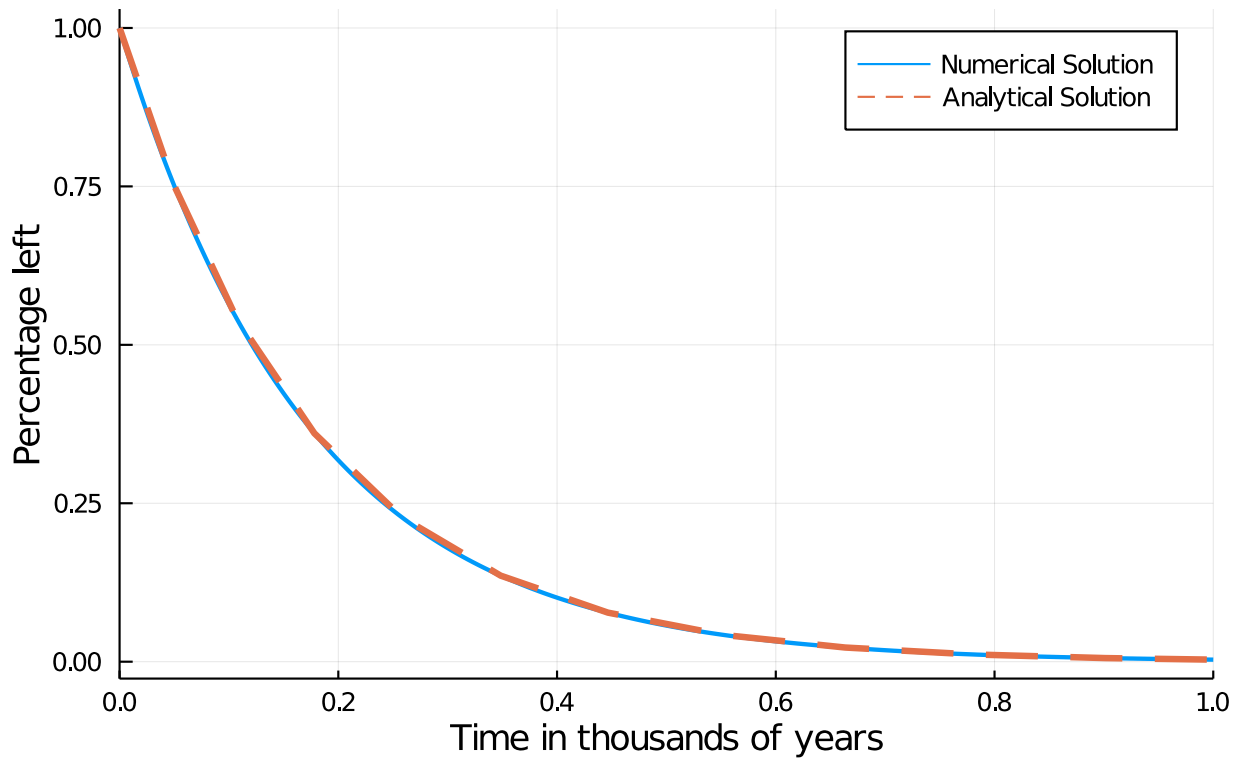
```
#Setup
u_0 = 1.0
tspan = (0.0, 1.0)
```

```
#Define the problem
radioactivedecay(u,p,t) = -C_1*u
```

```
#Pass to solver
prob = ODEProblem(radioactivedecay,u_0,tspan)
sol = solve(prob,Tsit5())
```

```
#Plot
plot(sol,linewidth=2,title="Carbon-14 half-life", xaxis="Time in thousands of years",
yaxis="Percentage left", label="Numerical Solution")
plot!(sol.t, t->exp(-C_1*t),lw=3,ls=:dash,label="Analytical Solution")
```

## Carbon-14 half-life



## 0.2 Second Order Linear ODE

**Simple Harmonic Oscillator** Another classical example is the harmonic oscillator, given by

$$\ddot{x} + \omega^2 x = 0$$

with the known analytical solution

$$\begin{aligned} x(t) &= A \cos(\omega t - \phi) \\ v(t) &= -A\omega \sin(\omega t - \phi), \end{aligned}$$

where

$$A = \sqrt{c_1^2 + c_2^2} \quad \text{and} \quad \tan \phi = \frac{c_2}{c_1}$$

with  $c_1, c_2$  constants determined by the initial conditions such that  $c_1$  is the initial position and  $\omega c_2$  is the initial velocity.

Instead of transforming this to a system of ODEs to solve with `ODEProblem`, we can use `SecondOrderODEProblem` as follows.

```
# Simple Harmonic Oscillator Problem
using OrdinaryDiffEq, Plots

#Parameters
ω = 1
```