Classical Physics Models

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If you're getting some cold feet to jump in to DiffEq land, here are some handcrafted differential equations mini problems to hold your hand along the beginning of your journey.

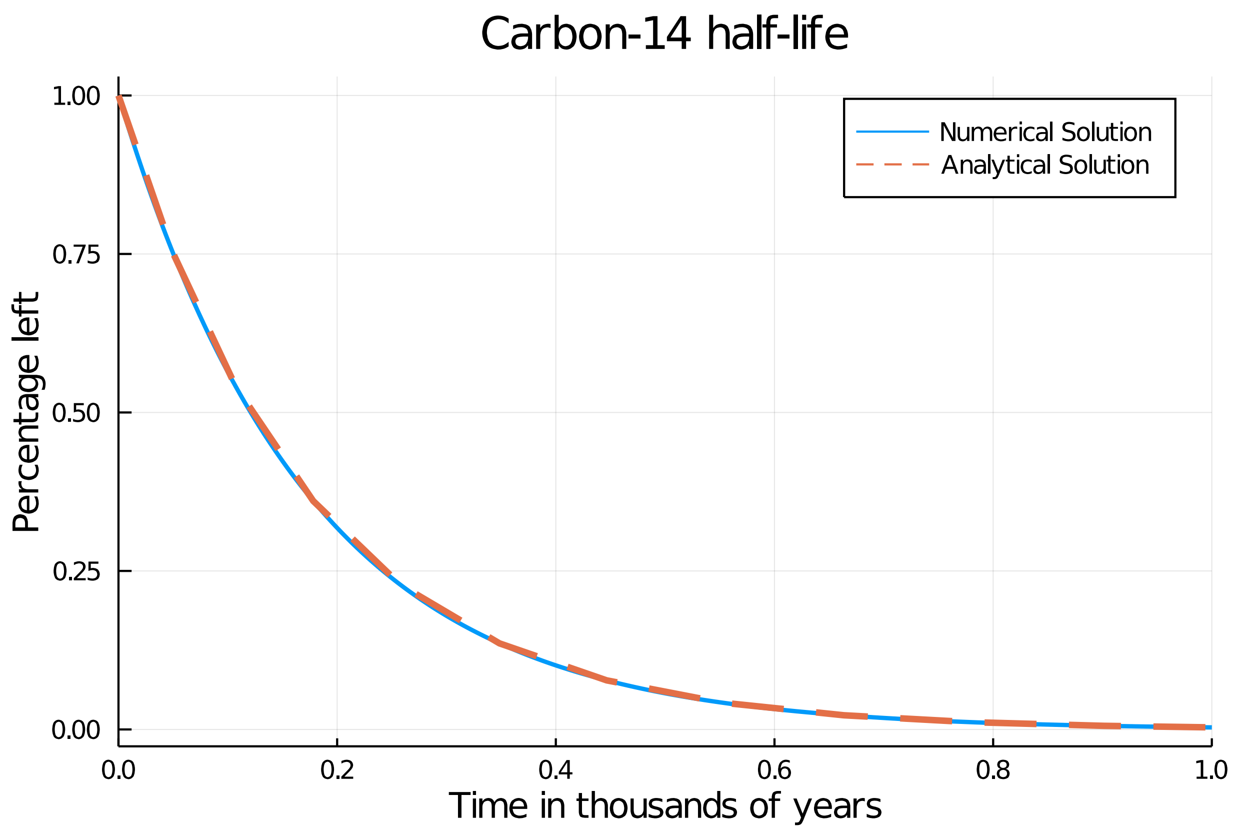
0.1 First order linear ODE

Radioactive Decay of Carbon-14

$$f(t,u) = \frac{du}{dt}$$

The Radioactive decay problem is the first order linear ODE problem of an exponential with a negative coefficient, which represents the half-life of the process in question. Should the coefficient be positive, this would represent a population growth equation.

```
using OrdinaryDiffEq, Plots
gr()

#Half-life of Carbon-14 is 5,730 years.
C_1 = 5.730

#Setup
u_0 = 1.0
tspan = (0.0, 1.0)

#Define the problem
radioactivedecay(u,p,t) = -C_1*u

#Pass to solver
prob = ODEProblem(radioactivedecay,u_0,tspan)
sol = solve(prob,Tsit5())

#Plot
plot(sol,linewidth=2,title ="Carbon-14 half-life", xaxis = "Time in thousands of years",
yaxis = "Percentage left", label = "Numerical Solution")
plot!(sol.t, t->exp(-C_1*t),lw=3,ls=:dash,label="Analytical Solution")
```



0.2 Second Order Linear ODE

Simple Harmonic Oscillator Another classical example is the harmonic oscillator, given by

$$\ddot{x} + \omega^2 x = 0$$

with the known analytical solution

$$\begin{aligned} x(t) &= A\cos(\omega t - \phi) \\ v(t) &= -A\omega \sin(\omega t - \phi), \end{aligned}$$

where

$$A = \sqrt{c_1 + c_2} \qquad \text{and} \qquad \tan\phi = \frac{c_2}{c_1}$$

with c_1, c_2 constants determined by the initial conditions such that c_1 is the initial position and ωc_2 is the initial velocity.

Instead of transforming this to a system of ODEs to solve with `ODEProblem`, we can use `SecondOrderODEProblem` as follows.

```
# Simple Harmonic Oscillator Problem
using OrdinaryDiffEq, Plots

#Parameters
ω = 1
```