B. Monte Carlo Basin Bifurcation Analysis

The additional results and the sensitivity study on initial conditions that is presented here, follows the appendix of the publication [MG5].

B.1. Dependence on initial conditions

During the sampling step of MCBB there is often a natural choice, given the parametrization and coordinates used, for the distribution of the initial conditions, and it is typical in basin studies to use a uniform distribution in a box. However, there is no a priori reason to expect that the limit of infinite box size converges, but experience shows that often plausible ranges for the box are naturally given by the system and the results don't depend heavily on box size (or even on substituting a normal distribution for the box). To further investigate this, the distribution of the initial conditions of the frequencies of the second order Kuramoto model is changed which is presented in Sec. III.B. Fig. B.1 shows results for uniform distributions with different bounds ($[-\pi, \pi]$, as shown in the main text, $[-5\pi, \pi]$ and $[-10\pi, 10\pi]$) and a normal distribution with mean 0 and standard deviation π. All other parameters (e.g. the clustering parameter ϵ_{DB}) are kept constant. Qualitatively the results are very similar: they show an unsynchronized regime (violet), a fully synchronized regime (blue) and several partially synchronized states. Quantitatively they differ. The broader the distributions of the frequencies gets, the later the fully synchronized states becomes the only existing asymptotic states. This behaviour can be expected from a second order Kuramoto system: with initial frequencies very far apart from each other, it will synchronize less well.

B.2. BBClustering

For the MCBB results presented in this thesis DBSCAN (see Sec. 2.3.5) is used to find the classes of asymptotic states. In the following, a specialized density-based clustering algorithm, Basin Bifurcation Clustering (BBClustering), is outlined that could be used in future research to replace DBSCAN. The basis of BBClustering is an additional continuation of the integration. As described in Sec. 4.1.4, $\delta_i^{\pm} = D(\rho^i, \rho^{i\pm})$ is the distance of the i-th trial to a continuation with the control parameter shifted by $\pm\delta p \approx \pm < \min_j(||p^{(i)} - p^{(j)}||) >_i$. BBClustering is based on DBSCAN but computes

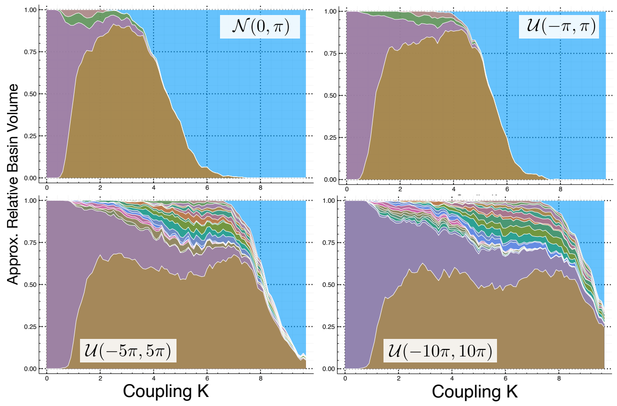

FIGURE B.1.: Results for the second order Kuramoto with different distributions of initial conditions.

the neighbourhood of each sample through the results of the continuation. A sample j is considered to be a neighbour of i if

$$D_{ij} \cdot \left(\frac{\delta p}{||p_i - p_j||} \right) < k \cdot \frac{1}{2} (\min_{\pm}(\delta_i^{\pm}) + \min_{\pm}(\delta_j^{\pm})). \tag{B.1}$$

As sketched in Fig. B.2, this criterion is based on the rationale that the continuation response $\delta^{\pm}$ will be strongly asymmetric when the state becomes unstable in one direction. Two samples belong to the same asymptotic state when the distance D_{ij} is about the same as the mean of the smaller continuation response. k is a parameter remaining in this clustering method, governing how fine or coarse the clusters should be resolved. In contrast to the standard DBSCAN algorithm, BBClustering features an adaptive neighbourhood criterion based on the continuation response. The remaining part of the algorithm is identical to DBSCAN and as outlined in Sec. 2.3.5. MCBB.jl (see Sec. B.4) features an implementation of BBClustering.

B.3. Additional Results and Resources

B.3.1. Logistic Map

While MCBB is designed with high-dimensional systems in mind, it also works in the fringe case of a one dimensional system such as the logistic map $x_{n+1} = rx_n(1 - x_n)$. Fig. B.3 shows the approximate relative basin volume computed with MCBB compared to the bifurcation diagram. It was computed using the mean, standard deviation and Kullbach-Leibler divergence as statistics with the weights 1, 0.5 and 0.5. The major bifurcation points are reproduced. The stable regions inside the chaotic regime form separate clusters, while most of the chaotic regime is grouped into to distinct clusters, one before and one after the larger stable region around $r \approx 3.8$.

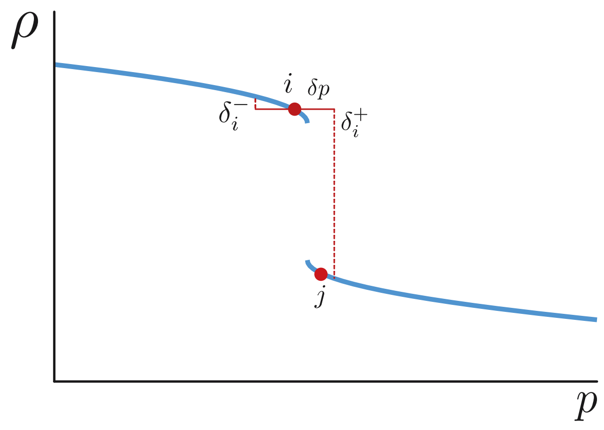

FIGURE B.2.: Sketch of the rational behind the neighourhood criterion of BBClustering.

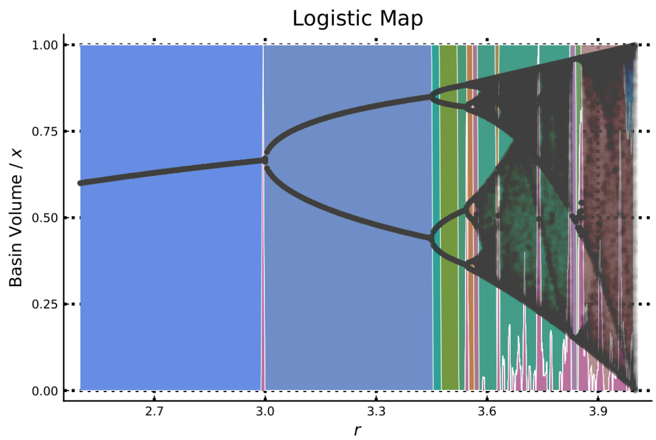

FIGURE B.3.: Basin Volume computed with MCBB and Bifurcation diagram of a logistic map.

B.3.2. Stuart-Landau Oscillator Network

Additionally to the results presented in Sec. 4.1.9 for the Stuart-Landau oscillator network, one can also further inspect the other clusters. In Fig. B.4 and B.5 show results for the classes of asymptotic states not shown in the main text. These are the outlier cluster, the travelling wave (TW) and oscillation death (OD) states. The Julia package (see Sec. B.4) also allows for further other visualizations and inspections of the measures and the clusters. The documentation of the package explains these in more detail.