Exercise 1: Planetary Energy Balance

1 Calculation of global mean temperature

Calculate first the incoming solar radiation ($E_{absorbed}$) and the re-radiated energy from the Earth ($E_{emitted}$), given a mean temperature of 288 K. Based on the Stefan-Boltzmann law for black body radiation, calculate now the **global mean surface temperature** for fixed incoming solar radiation and albedo.

$$E_{emitted} = \sigma T^4$$
$$E_{absorbed} = K \cdot (1 - \alpha)$$

$\sigma = 5.670373 \cdot 10^{-8}$ W m^{-2} K^{-4}
$\alpha = 0.31$
$K = 1361$ W m^{-2}
$R_e = 6.371 \cdot 10^3$m

where σ is the Stefan-Boltzmann constant, α the planetary albedo, K the solar constant and R_E the radius of the Earth.

Hint: Because we know from the conservation of energy that emitted radiation must be equal to the absorbed radiation, we can equal the incoming sunlight (absorbed energy) with the emitted energy radiated back from the Earth surface (assuming that the Earth is a black body). Note that we can approximate the area of incoming sunlight radiation with the area of a circle with the Earth radius (see Figure below). Think about that the emitted radiation comes from the entire surface of the Earth.

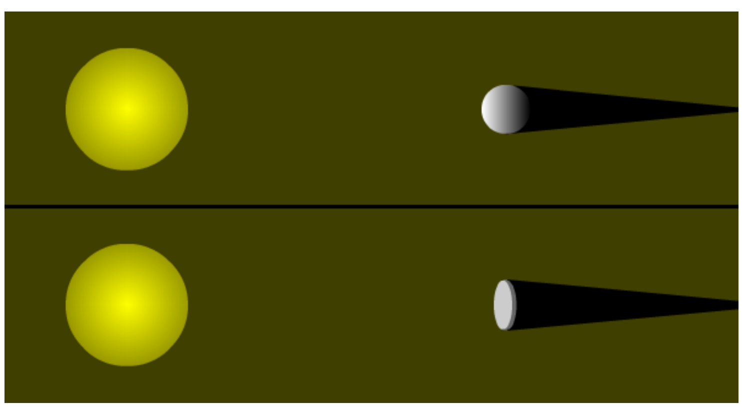


Figure 1: Sunlight intercepted by our spherical Earth (Credits: NCAR)

Questions

What is the global mean temperature?
Why is it so much colder than the observed global mean temperature?

2 Global mean temperature as a function of albedo

As you can see in your radiative balance equation, the global mean temperature is a function of the planetary albedo. Use an excel sheet to create a simple model based on your equation and calculate the global mean surface temperature T for different albedo values. Make a graph of T as a function of α.

3 Solar irradiance and greenhouse gas effect

A way to account for the greenhouse gas effect is to add a parameter to your radiative balance equation, which describes the amount of energy which is kept in the atmosphere due to greenhouse gases rather than radiated back to space:

$$4\sigma T(1-x)$$

Question

Which value x gives you the observed global mean surface temperature?

Recap from your lecture that solar irradiance varies over longer time period (Milankovitch cycles). Test in your spreadsheet model, what happens to the global mean surface temperature as you change the solar constant (e.g. to min/max values of the figure below or other values you can find)!

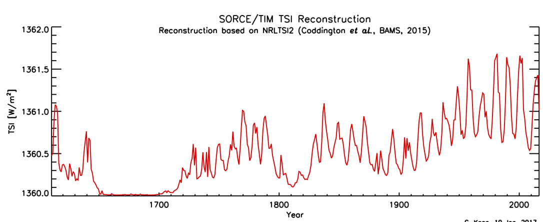


Figure 2: Reconstructed solar irradiance.

Temperature change over time is given by:

$\triangle$T = ($E_{absorbed}$ - $E_{emitted}$) $\cdot$ time / heat capacity

Surface heat capacity: $4.0 \cdot 10^8$ Jm^{-2}K^{-1}

year	incoming radiation (W/m^2)	Re-radiated energy (W/m^2)	ΔT	T
0	390.1	390.1	0	288
1	397.8			
2	397.8			
3	397.8			
4	397.8			
5	397.8			
6	397.8			
7	397.8			
8	397.8			
9	397.8			
10	397.8			
11	397.8			
12	397.8			
13	397.8			
14	397.8			
15	397.8			

You can create a very simple climate model and look at the response in temperature if you change the forcing (e.g. different incoming solar radiation). In your spreadsheet, implement the following table and calculate the re-radiated energy, temperature change and new surface temperature for each year. You know from the calculations before that ...

- re-radiated temperature depends on temperature
- temperature change depends as response to different incoming solar radiation is given by the equation above
- the new temperature T is calculated by simply adding $\triangle$ T to the old T

Question

If you increase the incoming solar radiation by 2% (which corresponds approximately to the same forcing as double CO_2 in the atmosphere), what is the change in temperature during the first 15 years? Do you reach an equilibrium state?

4 1D-Energy Balance Model with diffusion

In our simple energy balance model based on the Stefan-Boltzmann law for black body radiation, we have not accounted for varying incoming solar radiation at different latitudes yet. We will know have a look at a more advanced energy balance model, which takes this difference into account and includes a diffusion parameterization, which means that heat transport from lower to higher latitudes can happen. Follow the steps below.

Questions

Because this energy balance model takes into account heat diffusion, the latitudinal temperature gradient is reduced. This means that at each time step, the warmer regions around the equator get colder and the colder polar regions get warmer. What should happen if we run this model for a longer time?

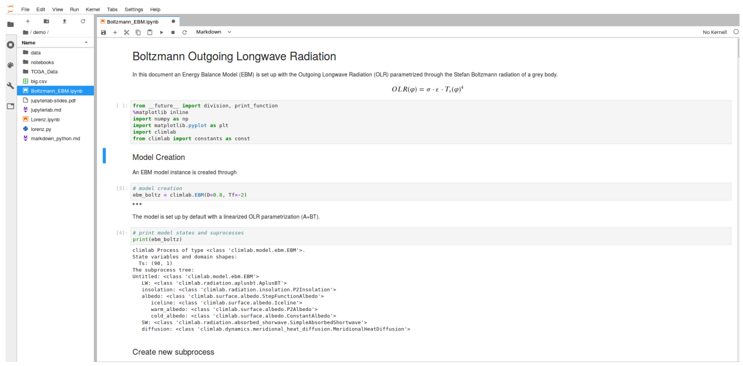
Note that this simplified model uses a so-called parameterization to describe the heat transport between latitudes. According to what you have learned in your lectures, what are the most important processes for heat transport to balance the equator-pole difference in received energy?

Run notebook online:

- Go to `https://jupyter.org/try` and click on *Try JupyterLab*
- Upload the file *Boltzmann_EBM.ipynb* and click on it in the Name list
- You should now be able to modify the cells and run the code
- Follow further instructions in the notebook.

Run notebook on lab computer:

- Open OSGeo4W Shell from Start>OSGeo4W
- Type **py3_env** (sets the environment to python3)
- Type **pip install jupyter** (will take some time)
- Type **pip install –upgrade jupyter_client**
- For this exercise: Type **pip install climlab**
- For this exercise: Type **pip install attrdict**
- Type **jupyter notebook**



Jupyter notebook Basics:

"Jupyter-notebook is an open-source web application that allows you to create and share documents that contain live code, equations, visualizations and narrative text." (https://jupyter.org/)

- There two general modes when you work with a jupyter-notebook. Type *esc* come to the "out-of-cells" mode where you can insert and delete new cells. You are in the "editing" mode when you click on one cell. Here you can modify the text/code or run the cells (=execute the code). There are **text cells** which contain text in markdown language and **code cells** which contain Python code. When you run a *textit* code cell, the output of the code appears below it.
- To run a cell: type *Ctrl + c*
- To insert new cells above/below a selected cell: type *a/ b*
- To delete cells: type *dd*