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Groups and smooth geometry using LieGroups.jl

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ABSTRACT

`LieGroups.jl` is a Julia package that provides both an interface to define as well as work with Lie groups, the corresponding Lie algebra as well as group actions.
This paper presents the main features of the interfaces and how that is integrated within the `JuliaManifolds` ecosystem. We further present an overview on existing Lie groups implemented in `LieGroups.jl` as well as how to get started to use the package in practice.

Keywords

Julia, Riemannian manifolds, Lie groups, Differential geometry

1. Introduction

[3], [2].

2. Notation

3. The interface

3.1 Lie groups

[4]

3.2 Lie algebras

3.3 Group actions

4. Implemented Lie groups

Table 1. : Implemented Lie groups in `LieGroups.jl` as of version 0.1.6

Group	Symbol	comment/code
`CircleGroup()`	$\mathbb{S}^1$	3 representations
`GeneralLinearGroup(n, F)`	$\mathrm{GL}(n,\mathbb{F})$	$\mathbb{F} \in \{\mathbb{R},\mathbb{C}\}$
`HeisenbergGroup(n)`	`H(n)`	
`OrthogonalGroup(n)`	$\mathrm{O}(n)$	
`PowerLieGroup(G, n)`	$\mathcal{G}^n$	`G^n`
`ProductLieGroup(G1, G2,...)`	$\mathcal{G}_1 \times \mathcal{G}_2 \times \ldots$	`G1` $\times$ `G2` $\times \ldots$
Semidirect product group	$\mathcal{G}_1 \ltimes \mathcal{G}_2$	`G1` $\ltimes$ `G2`
	$\mathcal{G}_1 \rtimes \mathcal{G}_2$	`G1` $\rtimes$ `G2`
`SpecialEuclideanGroup(n)`	$\mathrm{SE}(n)$	
`SpecialGalileanGroup(n)`	$\mathrm{SGal}(n)$	
`SpecialLinearGroup(n, F)`	$\mathrm{SL}(n,\mathbb{F})$	$\mathbb{F} \in \{\mathbb{R},\mathbb{C}\}$
`SpecialOrthogonalGroup(n)`	$\mathrm{SO}(n)$	
`SpecialUnitaryGroup(n)`	$\mathrm{SU}(n)$	
`SymplecticGroup(n)`	$\mathrm{Sp}(2n)$	
`TranslationGroup(n; field=𝔽)`	$\mathbb{F}^n$	$\mathbb{F} \in \{\mathbb{R},\mathbb{C},\mathbb{H}\}$
`UnitaryGroup(n)`	$\mathrm{U}(n)$	

5. An example how to use `LieGroups.jl`

6. References

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