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Groups and smooth geometry using LieGroups.jl

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ABSTRACT

`LieGroups.jl` is a Julia package that provides both an interface to define as well as work with Lie groups, the corresponding Lie algebra as well as group actions.
This paper presents the main features of the interfaces and how that is integrated within the `JuliaManifolds` ecosystem. We further present an overview on existing Lie groups implemented in `LieGroups.jl` as well as how to get started to use the package in practice.

Keywords

Julia, Riemannian manifolds, Lie groups, Differential geometry

1. Introduction

In many situations, one encounters data that does not reside in a vector space. For example in robotics, the configuration space of a rigid body in three-dimensional space is given by the special Euclidean group $\mathrm{SE}(3)$, consisiting of all translations and rotations. A subset of these is the space of rotations, given by the special orthogonal group $\mathrm{SO}(3)$, or more generally $\mathrm{SO}(n)$ in n-dimensional space.
These spaces are examples of Lie groups, formally defined as a smooth manifold equipped with a group structure.
The package `LieGroups.jl` aims to provide an easy access to using Lie groups within the Julia programming language [4] by providing an interface to define and work with Lie groups, as well as providing a library of Lie groups, that can directly be used.
This paper aims to provide an overview of the main features of `LieGroups.jl`. After introducing the necessary notation in Section 2, we present the interface in Section 3, split into the Lie group, the Lie algebra, and group actions. In Section 4, we present an overview of currently implemented Lie groups. Finally, in Section 5, we demonstrate how to get started and use `LieGroups`.

2. Notation

3. The interface

[3], [2].

3.1 Lie groups

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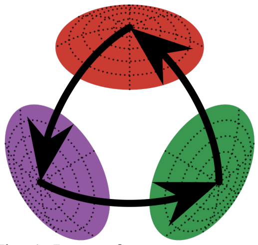

Fig. 1: Logo of `LieGroups.jl`.

Table 1. : Implemented Lie groups in `LieGroups.jl` as of version 0.1.6

Group	Symbol	comment/code
`CircleGroup()`	$\mathbb{S}^1$	3 representations
`GeneralLinearGroup(n, F)`	$\mathrm{GL}(n,\mathbb{F})$	$\mathbb{F}\in\{\mathbb{R},\mathbb{C}\}$
`HeisenbergGroup(n)`	`H(n)`	
`OrthogonalGroup(n)`	$\mathrm{O}(n)$	
`PowerLieGroup(G, n)`	$\mathcal{G}^n$	`G^n`
`ProductLieGroup(G1, G2,...)`	$\mathcal{G}_1\times\mathcal{G}_2\times\ldots$	`G1` $\times$ `G2` $\times\ldots$
Semidirect product group	$\mathcal{G}_1\ltimes\mathcal{G}_2$	`G1` $\ltimes$ `G2`
	$\mathcal{G}_1\rtimes\mathcal{G}_2$	`G1` $\rtimes$ `G2`
`SpecialEuclideanGroup(n)`	$\mathrm{SE}(n)$	
`SpecialGalileanGroup(n)`	$\mathrm{SGal}(n)$	
`SpecialLinearGroup(n, F)`	$\mathrm{SL}(n,\mathbb{F})$	$\mathbb{F}\in\{\mathbb{R},\mathbb{C}\}$
`SpecialOrthogonalGroup(n)`	$\mathrm{SO}(n)$	
`SpecialUnitaryGroup(n)`	$\mathrm{SU}(n)$	
`SymplecticGroup(n)`	$\mathrm{Sp}(2n)$	
`TranslationGroup(n; field=F)`	$\mathbb{F}^n$	$\mathbb{F}\in\{\mathbb{R},\mathbb{C},\mathbb{H}\}$
`UnitaryGroup(n)`	$\mathrm{U}(n)$	

3.2 Lie algebras

3.3 Group actions

4. Implemented Lie groups

5. An example how to use `LieGroups.jl`

[1]

6. References

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