Module 5: Design of Sampled Data Control Systems

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GitHub: https://github.com/JuliaOli/Controle-II.git

Embasamento teórico

The main difference is that the **lag compensator** adds negative phase to the system over the specified frequency range, while a **lead compensator** adds positive phase over the specified frequency. A **Bode plot** of a phase-**lag compensator** has the following form

A principal diferença é que o **lag compensator** adiciona fase negativa ao sistema na faixa de frequência especificada, enquanto um **lead compensator** adiciona fase positiva à frequência especificada.

Um lead compensator típico possui a seguinte função de transferência:

$$C(s) = K\frac{\tau s + 1}{\alpha \tau s + 1}, \text{onde}, \alpha \pm < 1$$

Onde $\frac{1}{\alpha}$ é a razão entre as frequências do break point do zero do pólo (limite). A magnitude do lead compensator é:

$$K = \frac{\sqrt{1 + \omega^2 r^2}}{\sqrt{1 + \alpha^2 \omega^2 r^2}}$$

E a fase contribuída pelo lead compensator é dada por:

$$\phi = \tan^{-1}\omega\tau - \tan^{-1}\alpha\omega\tau$$

Pode ser demonstrado que a frequência em que a fase é máxima é dada por:

$$\omega_{\max} = \frac{1}{\tau\sqrt{\alpha}}$$

The maximum phase corresponds to:

$$\alpha = \left(\frac{1 - \sin(\phi_{\max})}{1 + \sin(\phi_{\max})}\right)$$

A magnitude de C (s) em $\omega_{\max}$:

$$\frac{K}{\sqrt{\alpha}}$$

Lecture Note 6: Compensator Design Using Bode Plot

Exemplo 1

```
num = 1;
den = [1 1 0];
```

```
G = tf(num,den);
H = 1;
% phase margin (PM) is at least 45 degrees
% error for a unit ramp input is â‰¤ 0.1.
% s â†’ 0, C(s) â†’ K
% Steady state error for unit ramp input is 1/K
% 1/K = 0.1
K = 10
```

K = 10

```
syms w_g
equ = 1 -100/(w_g^2 *(1+w_g^2)) == 0
```

equ =

$$1 - \frac{100}{w_g{}^2\,\left(w_g{}^2 + 1\right)} = 0$$

```
equ_sol = solve(equ, w_g);
W_G = equ_sol(2) %deu certo aqui
```

W_G =

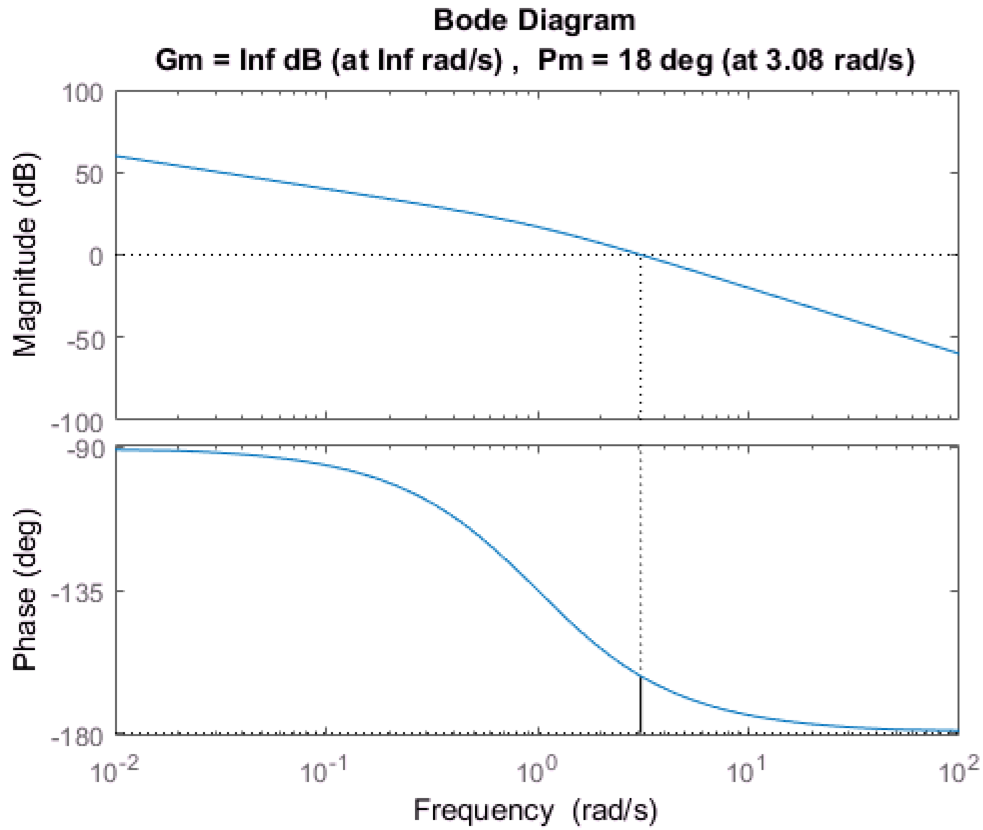
$$\frac{10\,\sqrt{2}}{\sqrt{\sqrt{401} + 1}}$$

```
phase = -90 - atan(W_G)*180/pi
```

phase =

$$-\frac{180\,\mathrm{atan}\left(\frac{10\,\sqrt{2}}{\sqrt{\sqrt{401} + 1}}\right)}{\pi} - 90$$

```
%margem de fase do sistema sem compensao para o valor de K dado. PM = 18
margin(G*K)
```




```
% Defining The maximum phase corresponds to sin(Ï†max)

PM = 18;

%the additional phase lead required to maintain PM=45 at  Ï‰ g = 3.1 rad/sec is
phi_max = 45 - PM;
phi_max = phi_max + 10
```

```
phi_max = 37
```

```
alpha = (1-sin(phi_max*180/pi))/(1+sin(phi_max*180/pi))
```

```
alpha = 0.2578
```

```
% finding Ï„

w_max = 4.41;

tetazinho = 1/(w_max*(alpha)^(1/2))
```

```
tetazinho = 0.4466
```

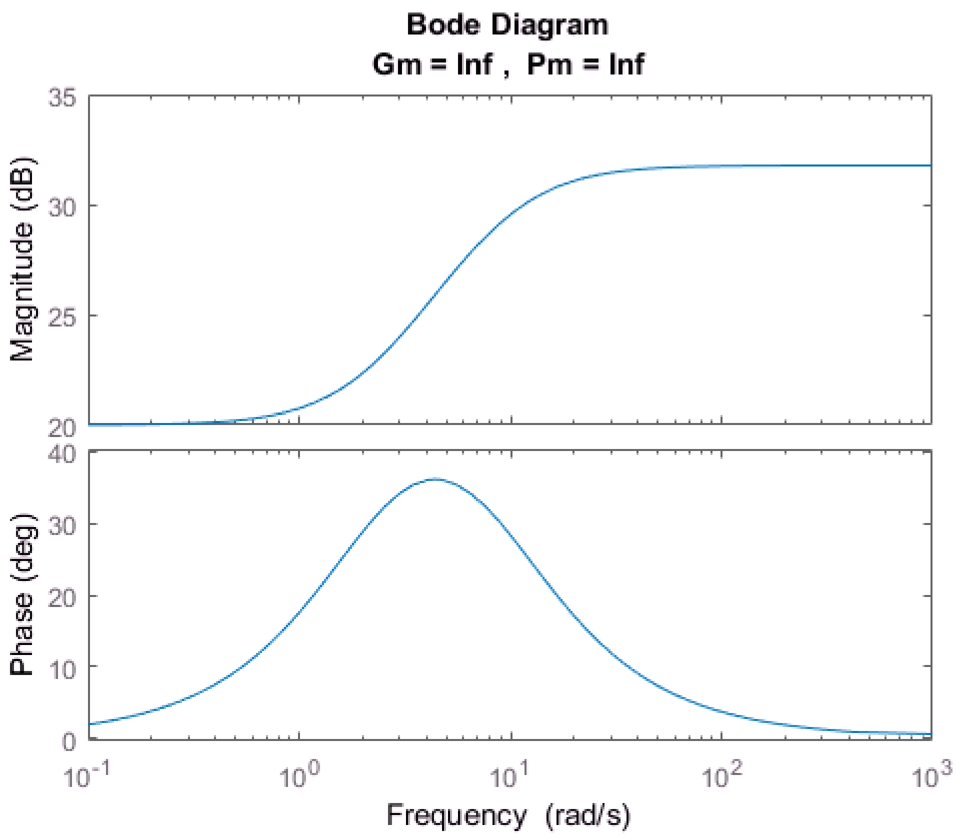
```
% Lead Compensator

num = [tetazinho 1];
den = [tetazinho*alpha 1];

C = tf(num, den)
```

```
C =

  0.4466 s + 1
  ------------
  0.1151 s + 1

Continuous-time transfer function.
```

```
C = K*C
```

```
C =

  4.466 s + 10
  ------------
  0.1151 s + 1

Continuous-time transfer function.
```

```
margin(C)
```



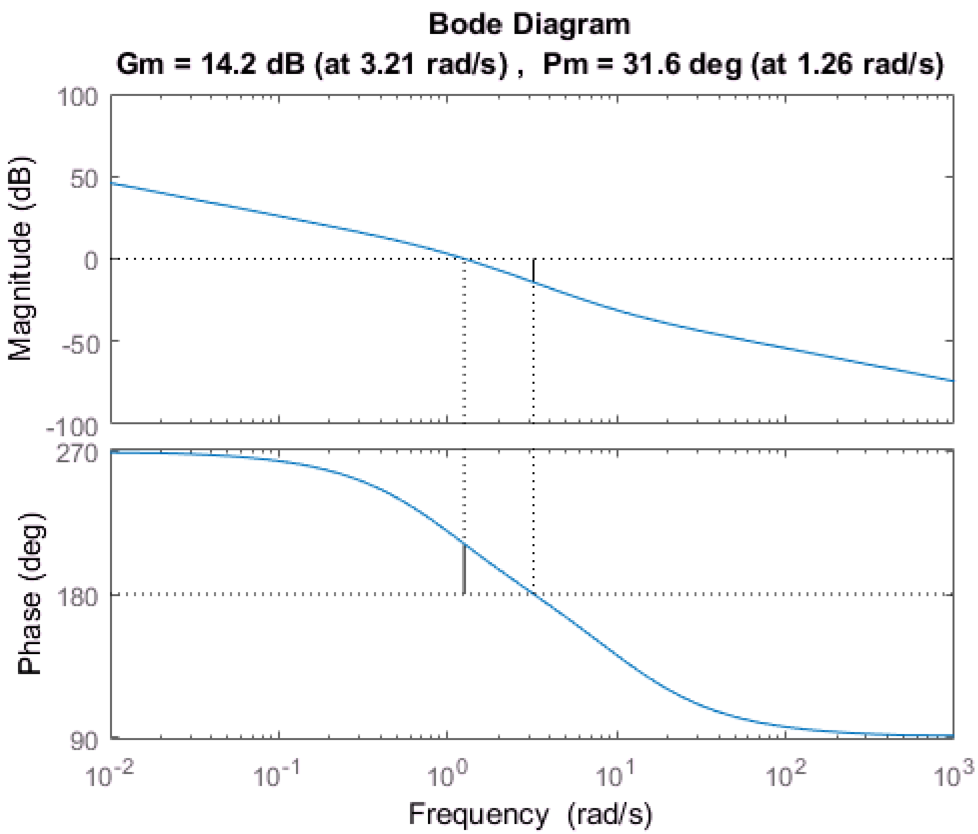
Exemplo 2

```
%%
Ts = 0.2;
num = [0.0187 0.0175];
den = [1 -1.8187 0.8187];
G_z = tf(num, den, Ts)
```

```
G_z =

    0.0187 z + 0.0175
```

```
    ----------------------
    z^2 - 1.819 z + 0.8187

  Sample time: 0.2 seconds
  Discrete-time transfer function.
```

```
%The bi-linear transformation will transfer G z (z) into w-plane, as
%We need first design a phase lead compensator so that PM of the compensated system is at
%least 50 0 with K v = 2 . The compensator in w-plane is

num = [-1/3000 - 29/300 1];
den = [1 1 0];
G_w = tf(num, den);
K = 2;
margin(G_w*K)
```




```
%%
PM = 31.6;

%the additional phase lead required to maintain PM=45 at  Ï‰ g = 3.1 rad/sec is
phi_max = 50 - PM
```

```
phi_max = 18.4000
```

```
phi_max = phi_max + 11.6
```

```
phi_max = 30
```

```
alpha = (1-sin(phi_max*pi/180))/(1+sin(phi_max*pi/180))
```

```
alpha = 0.3333
```

```
%%
w_max = 1.75 %nem sei de onde veio
```

```
w_max = 1.7500
```

```
tetazinho = 1/(w_max*(alpha)^(1/2))
```

```
tetazinho = 0.9897
```

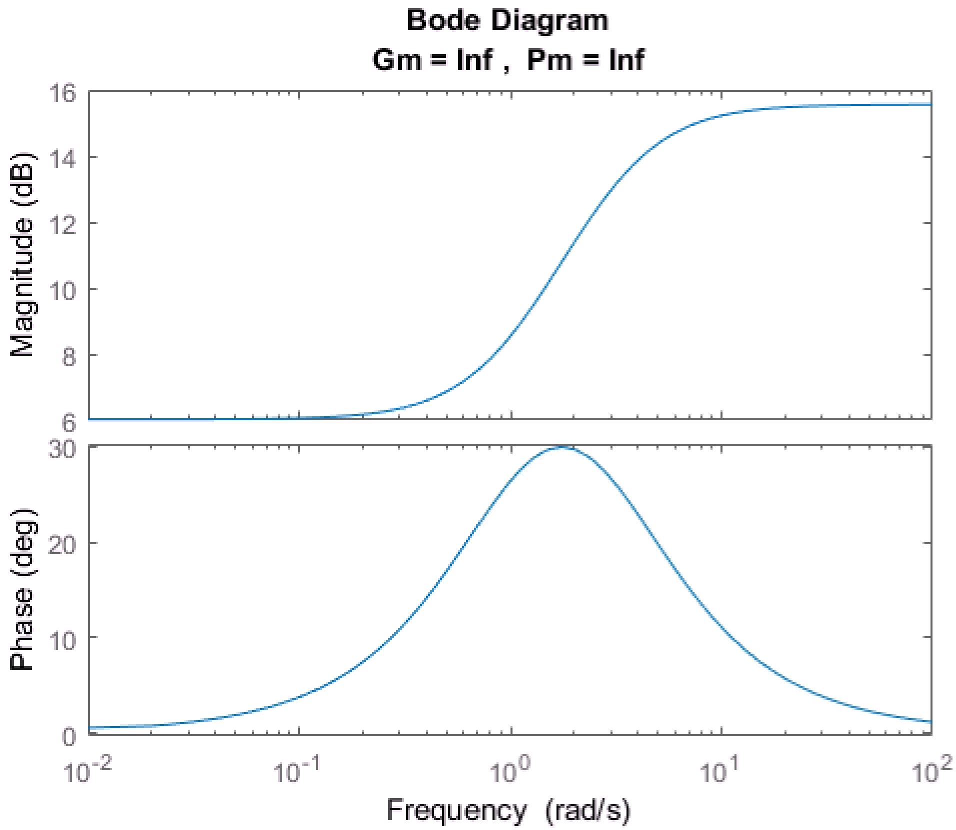
```
num = [tetazinho 1];
den = [tetazinho*alpha 1];

C = tf(num, den)
```

```
C =

  0.9897 s + 1
  ------------
  0.3299 s + 1

Continuous-time transfer function.
```

```
C = K*C
```

```
C =

  1.979 s + 2
  ------------
  0.3299 s + 1

Continuous-time transfer function.
```

```
margin(C)
```



Lecture Note 7: Lag Compensator Design

Exemplo 1

```
%%
num = 1;
den = [0.5 1.5 1];
G = tf(num,den)
```

```
G =

           1
  -------------------
  0.5 s^2 + 1.5 s + 1

Continuous-time transfer function.
```

```
H = 1;

%Steady state error for unit step input is 1/(1 + lims?0 C(s)G(s)) =
%1/( 1+ C(0)) = 1/( 1 + K?)
%Thus, 1/( 1 + K?) = 0.1, or, K = 9.

K_alpha = 9;
syms w_g

Mag = 0.715*w_g^2 - 1.5*w_g - 1.43 == 0;
Mag_sol = solve(Mag, w_g);
W_G = Mag_sol(2)
```

```
W_G =
```

$$\frac{\sqrt{2}\ \sqrt{31699}}{143}+\frac{150}{143}$$

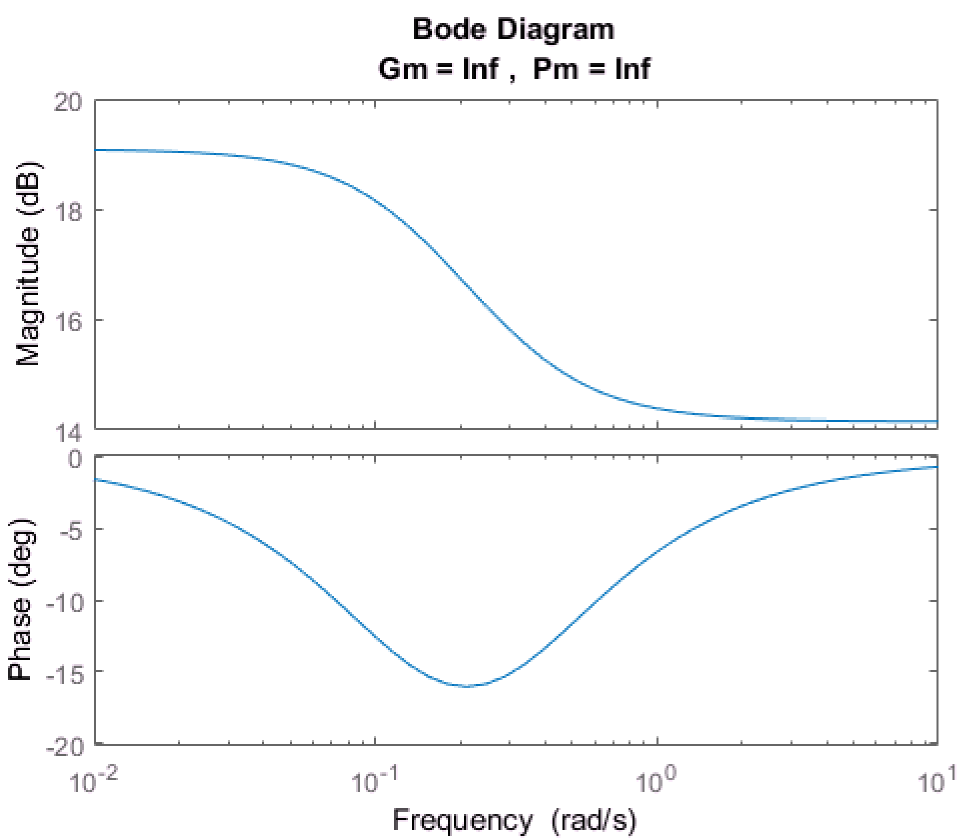
```
W_G = 2.8;

%Novo K
K = 5.1;
alpha = 9/K;
tau = 10/W_G
```

```
tau = 3.5714
```

```

% Lead Compensator

num = [tau 1];
den = [tau*alpha 1];

C = tf(num, den)
```

```
C =

  3.571 s + 1
  -----------
  6.303 s + 1

Continuous-time transfer function.
```

```

margin(C*9)
```



Exemplo 2

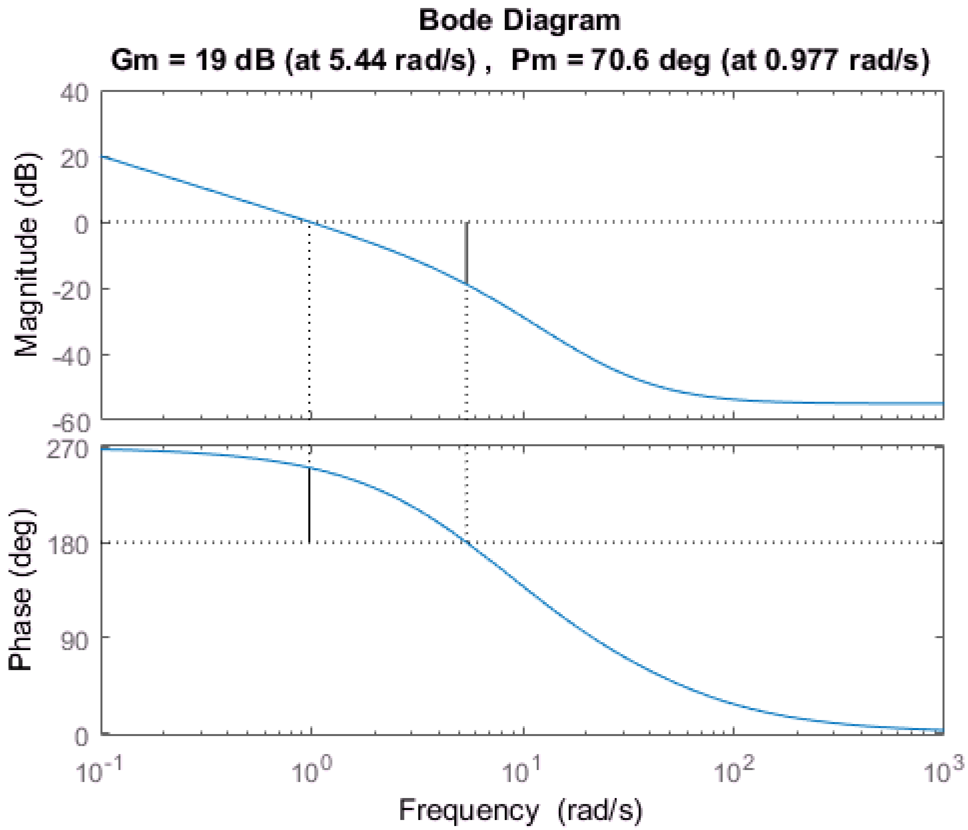
```
%%
s=tf('s');
gc=1/(s*(1+0.1*s)*(1+0.2*s));
gz=c2d(gc,0.1,'z2h')
```

```
gz =

  0.005824 z^2 + 0.01629 z + 0.002753
  -----------------------------------
  z^3 - 1.974 z^2 + 1.198 z - 0.2231

Sample time: 0.1 seconds
Discrete-time transfer function.
```

```
aug=[0.1 1];

gwss = bilin(ss(gz),-1,'S_Tust',aug);
gw=tf(gwss)
```

```
gw =

  0.001756 s^3 - 0.06306 s^2 - 1.705 s + 45.27
  --------------------------------------------
     s^3 + 14.14 s^2 + 45.27 s - 6.032e-13

Continuous-time transfer function.
```

```


%Since Gw(0) = 1, Kα = 9 for 0.1 steady state error.
margin(gw)
```



Lecture Note 8: Lag-lead Compensator

Exemplo 1

Phase margin (PM) is at least 45 degrees, crossover frequency around 10 rad/sec and the velocity error constant Kv is 30.

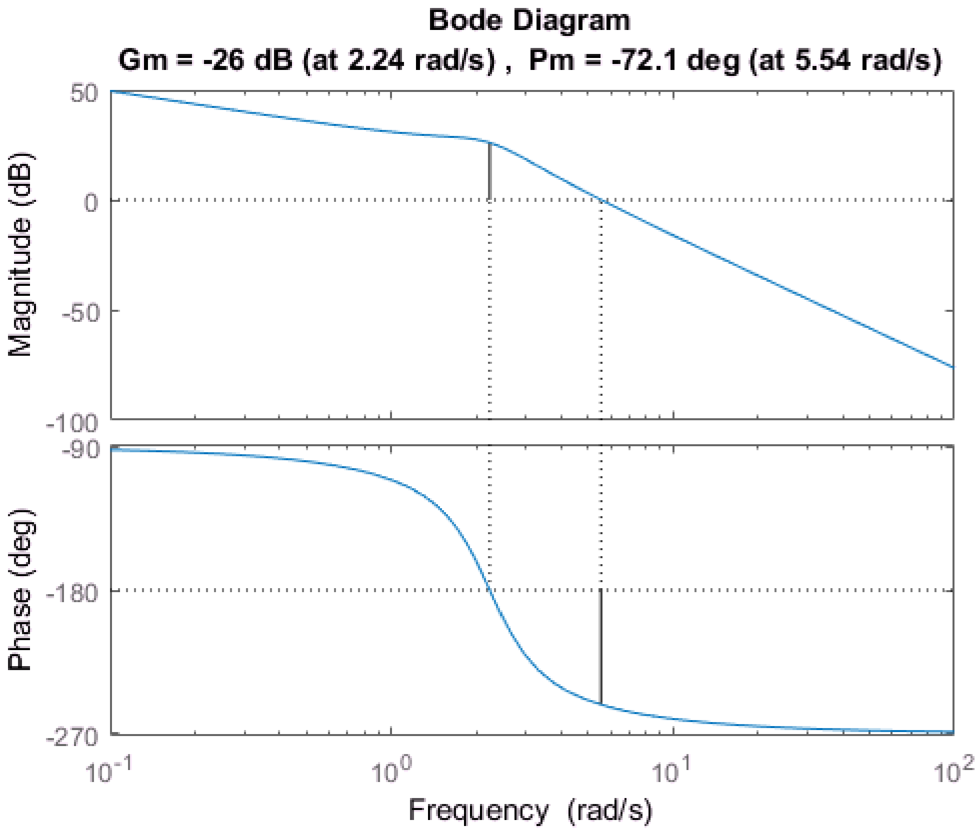
```
%% Exemplo 1
num = 1;
den = [0.2 0.3 1 0];
G = tf(num,den)
```

```
G =
 
            1
  ---------------------
  0.2 s^3 + 0.3 s^2 + s
 
Continuous-time transfer function.
```

```
K = 30
```

```
K = 30
```

```
%G = G*K
margin(G*K)
```



Dados apresentados na questão não batem com os produzidos

**Since the PM of the uncompensated system with K is negative. We need a lead compensator to compensate for the negative PM and achieve the desired phase margin.

We design the lead part first. From Figure 2, it is seen that at 10 rad/sec the phase angle of the system is −198o. Since the new ωg should be 10 rad/sec, the required additional phase at ωg, to maintain the specified PM, is 45−(180−198) = 63º. With safety margin 2º.***

```
phi_max = 63;
alpha = (1-sin(phi_max*pi/180))/(1+sin(phi_max*pi/180))
```

```
alpha = 0.0576
```

```
w_max = 10;
tau = 1/(w_max*(alpha)^(1/2))
```

```
tau = 0.4165
```

```

num = [tau 1];
den = [tau*alpha 1];
C_lead = tf(num, den)
```

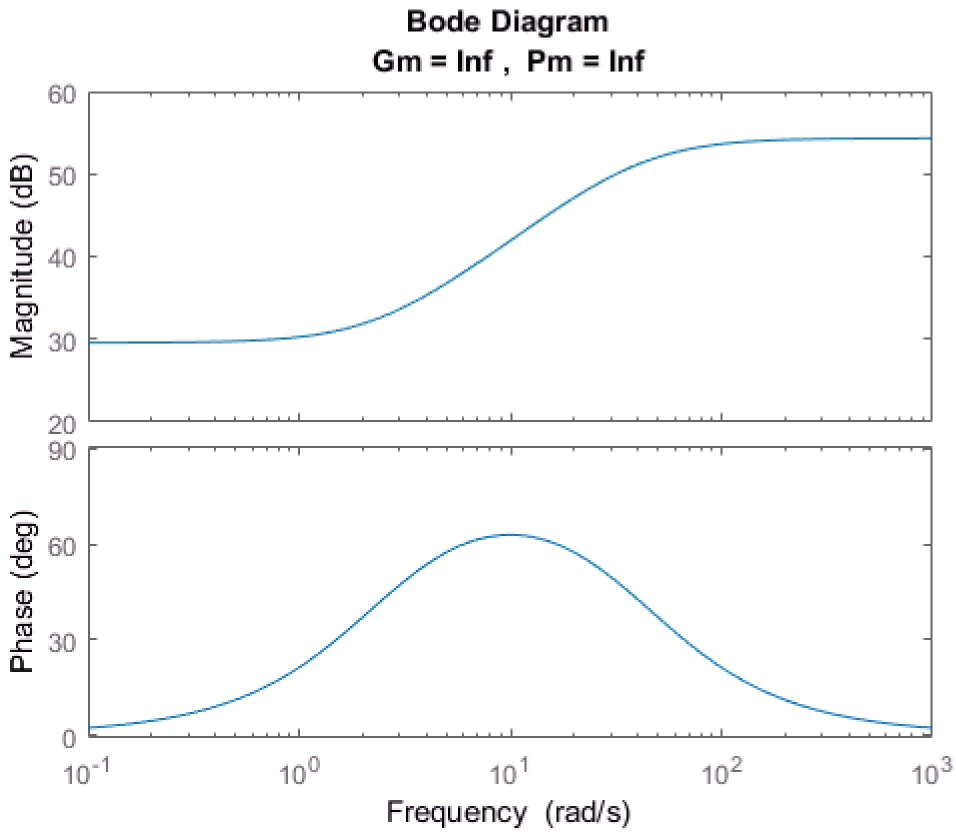
```
C_lead =

  0.4165 s + 1
  -------------
  0.02401 s + 1

Continuous-time transfer function.
```

A introdução desse compensador aumentará a frequência de crossover de ganho, onde a característica da fase será diferente da designada.

Resposta de frequência do sistema no Exemplo 1 com apenas um compensador de avanço.

**Novamente é apresentado resultados diferentes

```
margin(K*C_lead)
```



Em altas frequências, a magnitude da parte do compensador de atraso é 1 / α.

$$20\log_{10}\alpha_1 = 12.6 \Longrightarrow \alpha_1 = 4.27$$

```
%%
syms alpha_1
alpha_high = 20*log10(alpha_1) == 12.6;
alpha_solve = solve(alpha_high)
```

alpha_solve = $10^{63/100}$

```
alpha_solve = 4.2
```

alpha_solve = 4.2000

```
tau = 1/0.25
```

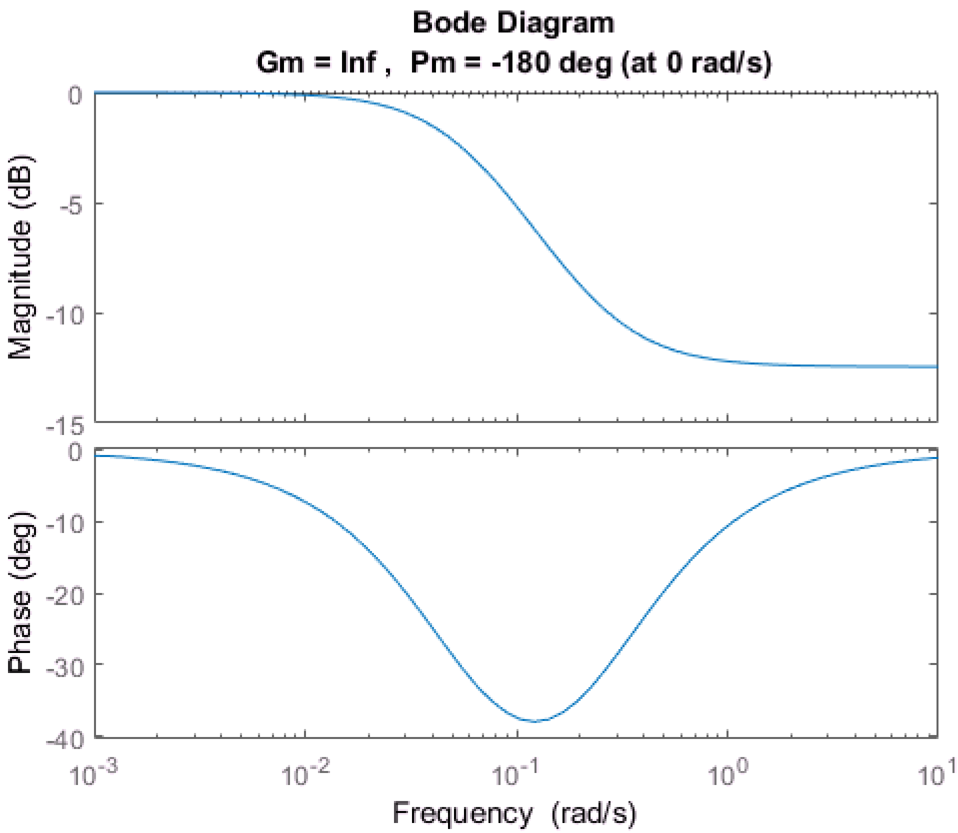
tau = 4

```
num = [tau 1];
```

```
den = [tau*alpha_solve 1];
C_comp = tf(num, den)
```

```
C_comp =

   4 s + 1
  ----------
  16.8 s + 1

Continuous-time transfer function.
```

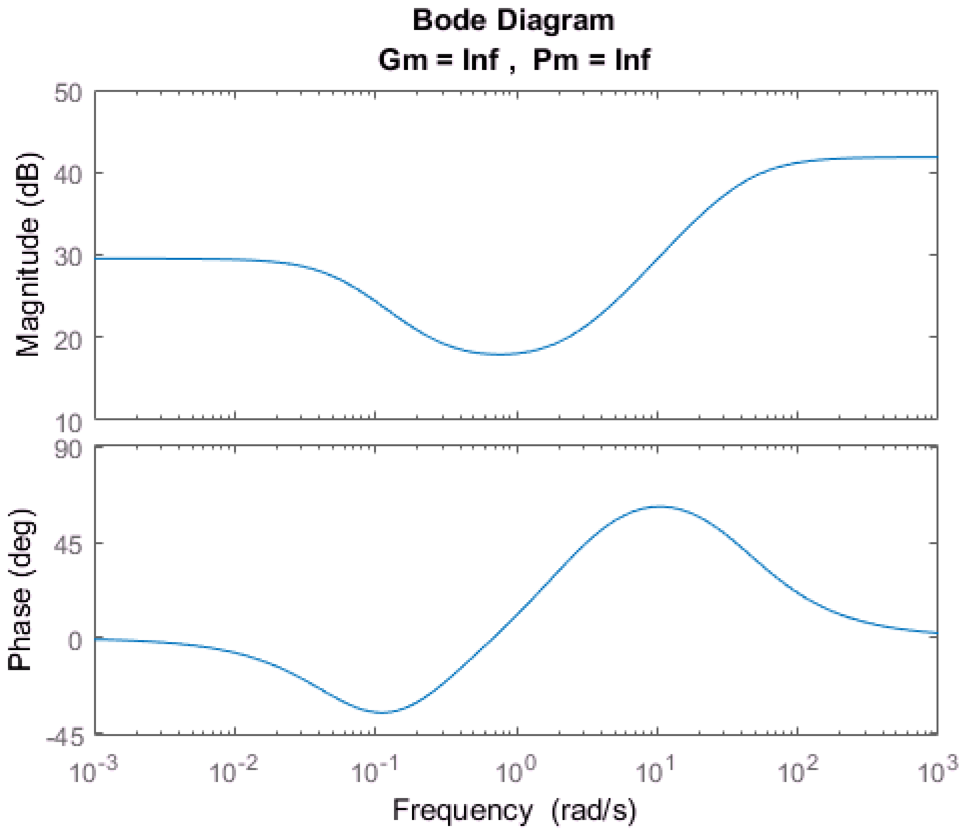
```
margin(C_comp)
```




```
C_final = K*C_comp*C_lead
```

```
C_final =

  49.98 s^2 + 132.5 s + 30
  ------------------------
  0.4033 s^2 + 16.82 s + 1

Continuous-time transfer function.
```

```
margin(C_final)
```



Exemplo 2

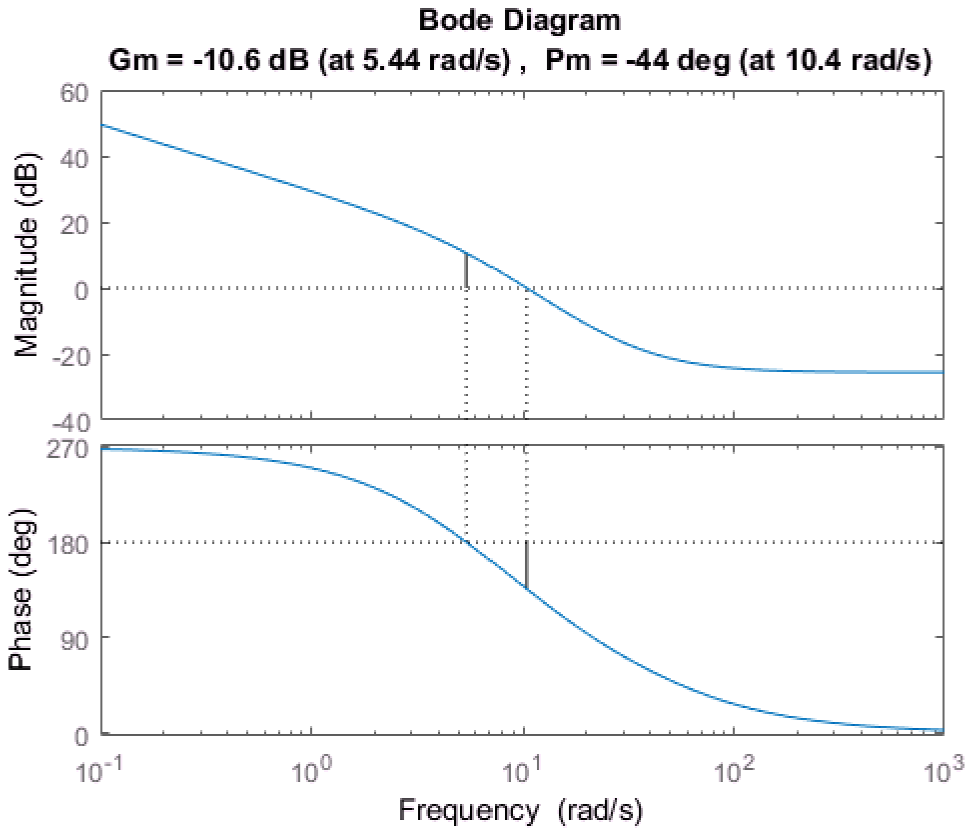
```
%%
s= tf('s');
gc=1/(s*(1+0.1*s)*(1+0.2*s));
gz=c2d(gc,0.1,'zoh')
```

```
gz =

  0.005824 z^2 + 0.01629 z + 0.002753
  -----------------------------------
  z^3 - 1.974 z^2 + 1.198 z - 0.2231

Sample time: 0.1 seconds
Discrete-time transfer function.
```

```
aug=[0.1,1];
gwss = bilin(ss(gz),-1,'S_Tust',aug);
gw=tf(gwss)
```

```
gw =

  0.001756 s^3 - 0.06306 s^2 - 1.705 s + 45.27
  --------------------------------------------
     s^3 + 14.14 s^2 + 45.27 s - 6.032e-13

Continuous-time transfer function.
```

```
margin(30*gw)
```




```
phi_max = 66;
alpha_2 = (1-sin(phi_max*pi/180))/(1+sin(phi_max*pi/180))
```

```
alpha_2 = 0.0452
```

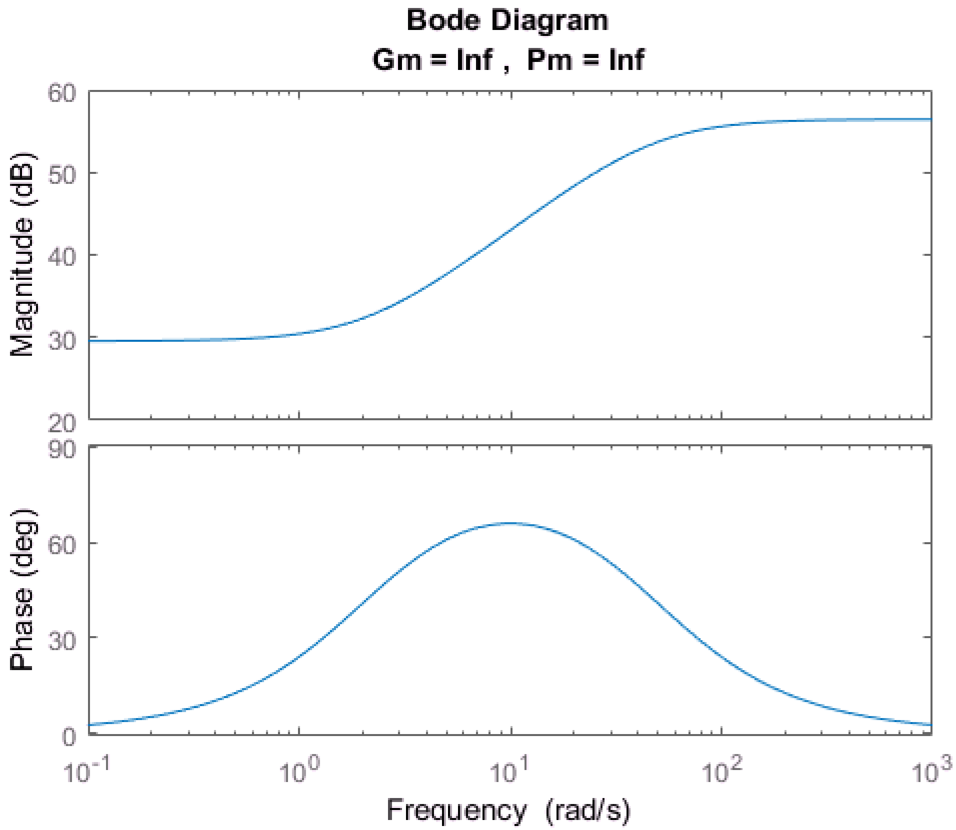
```
w_max = 10;
tau = 1/(w_max*(alpha_2)^(1/2))
```

```
tau = 0.4705
```

```
num = [tau 1];
den = [tau*alpha_2 1];
C_lead = tf(num, den)
```

```
C_lead =

  0.4705 s + 1
  -------------
  0.02126 s + 1

Continuous-time transfer function.
```

```
margin(30*C_lead)
```




```
syms alpha_1
alpha_high = 20*log10(alpha_1) == 14.2;
alpha_solve = solve(alpha_high)
```

alpha_solve = $10^{71/100}$

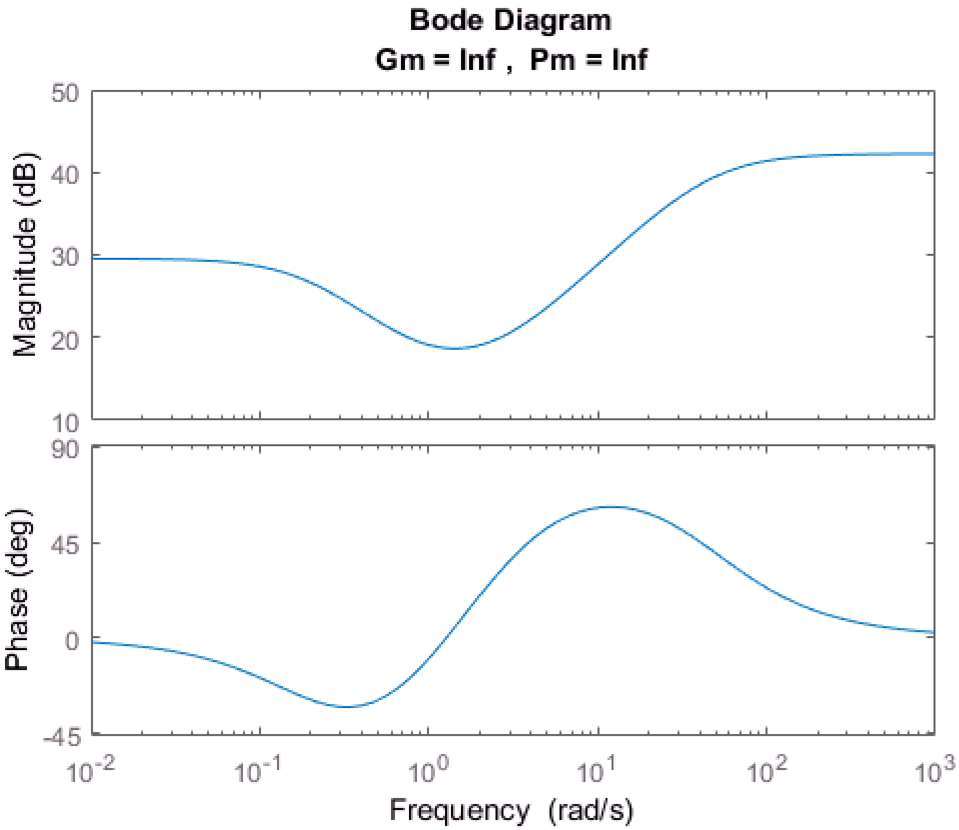
```
alpha_solve = 5.12;
tau = 1
```

```
tau = 1
```

```
num = [tau 1];
den = [tau*alpha_solve 1];
C_comp = tf(num, den)
```

```
C_comp =

    s + 1
  ----------
  5.12 s + 1

Continuous-time transfer function.
```

```
margin(30*C_lead*C_comp)
```


Bode Diagram
Gm = Inf , Pm = Inf
Magnitude (dB)
50
40
30
20
10
Phase (deg)
90
45
0
-45
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/s)