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UNIVERSITY OF WISCONSIN-MADISON

Introducing Plasmo.jl (unofficial)

A Package for Graph-Based Modeling using JuMP

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JuMP Developers Meetup
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Plasmo.jl - What is it?

Platform for **S**calable **M**odeling and **O**ptimization
A Graph-based modeling and optimization framework

Key Features:

- Component models associated with nodes **and** edges
- Facilitates construction of hierarchical graphs (uses subgraphs)
- Modularization of component models
- Manipulate graph structure for solver interface
- Ease of modeling complex systems

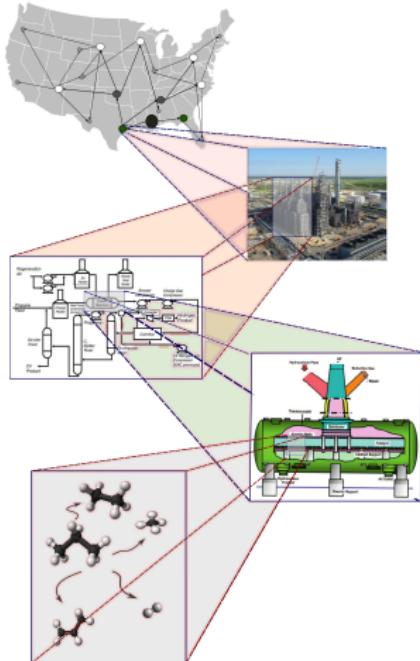


Overview

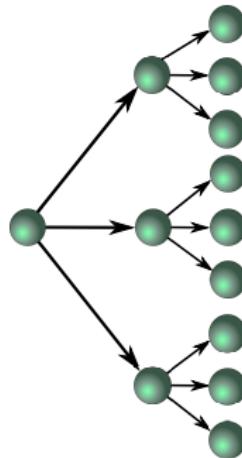
- Motivation - Complex systems
- Modeling Systems with Components (Graphs)
- Applications
- Design considerations
- Goals right now



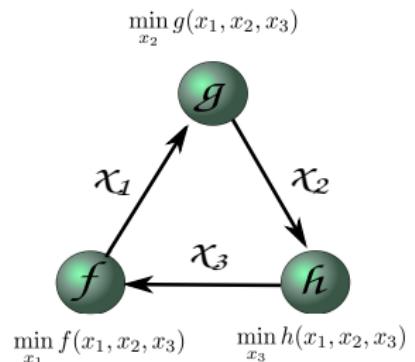
Group Research Theme:Complex Systems



Multi-scale systems



Multi-stage stochastic programs

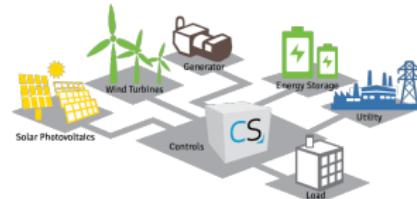


Asynchronous systems



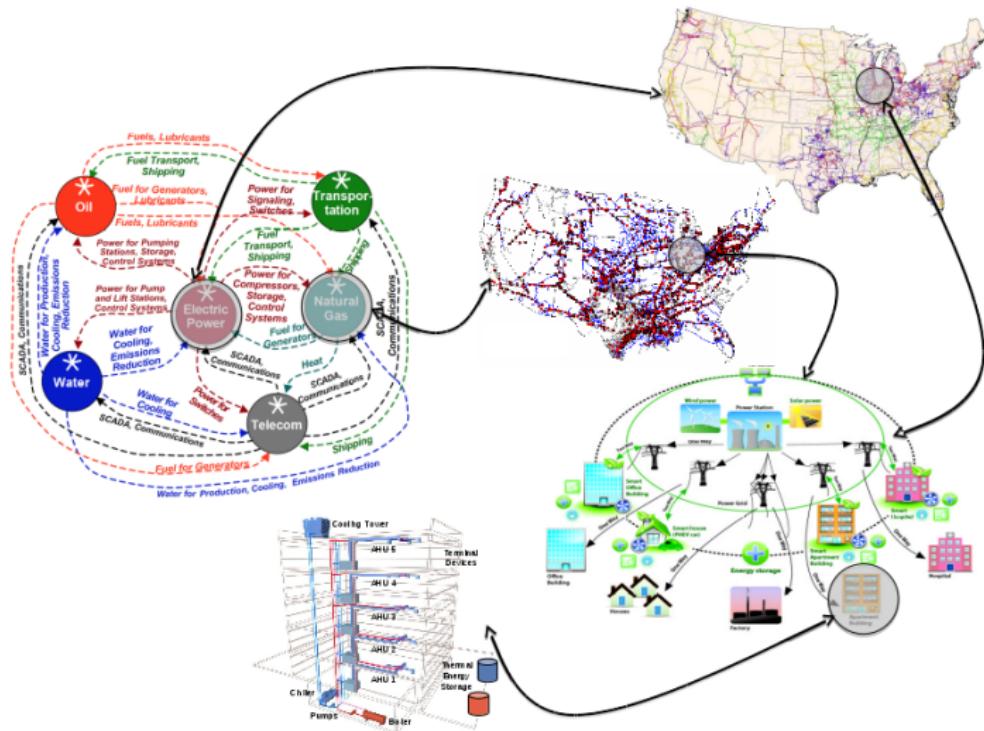
Types of Problems

- Nonlinear (nonconvex) optimization
- Stochastic programming
- Model predictive control
- Some Applications
 - ▶ Energy storage systems
 - ▶ Connected infrastructure
 - ▶ Microgrids



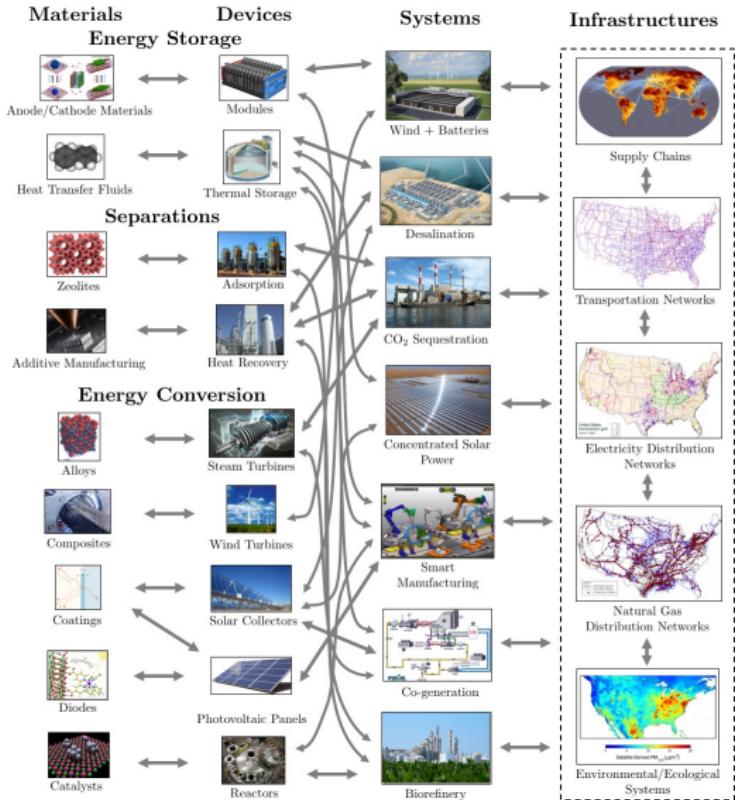


Hierarchical Networks



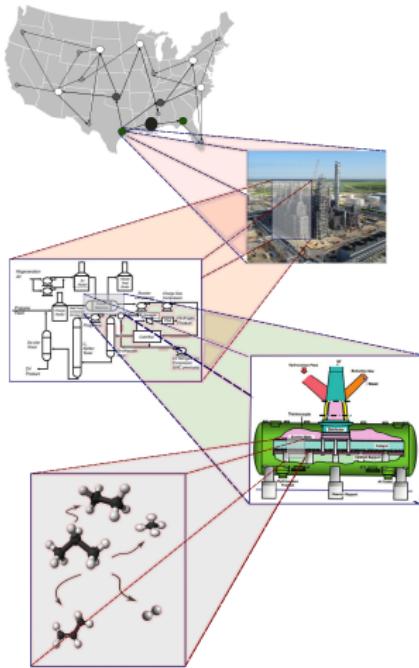


Technology Landscapes





Challenges with complex physical systems



- Millions of constraints and variables can make the computation intractable
 - ▶ Generally apply ad-hoc methods to perform some model reduction
- Millions of system connections makes model instantiation non-trivial
 - ▶ Multiple scenarios
 - ▶ Solution inspection
- Modeling asynchronicity in large communicating systems is non trivial
 - ▶ Decentralized control



Some existing modeling frameworks

- Modelica
 - ▶ Components, hierarchies, architectures (highly abstracted)
 - ▶ Designed for simulation (Optimica extension does some optimization)
 - ▶ Write connectors for coupling (I always found this difficult)



- gProms

- ▶ Equation oriented chemical flowsheeting software
- ▶ Custom modeling language
- ▶ Commercial



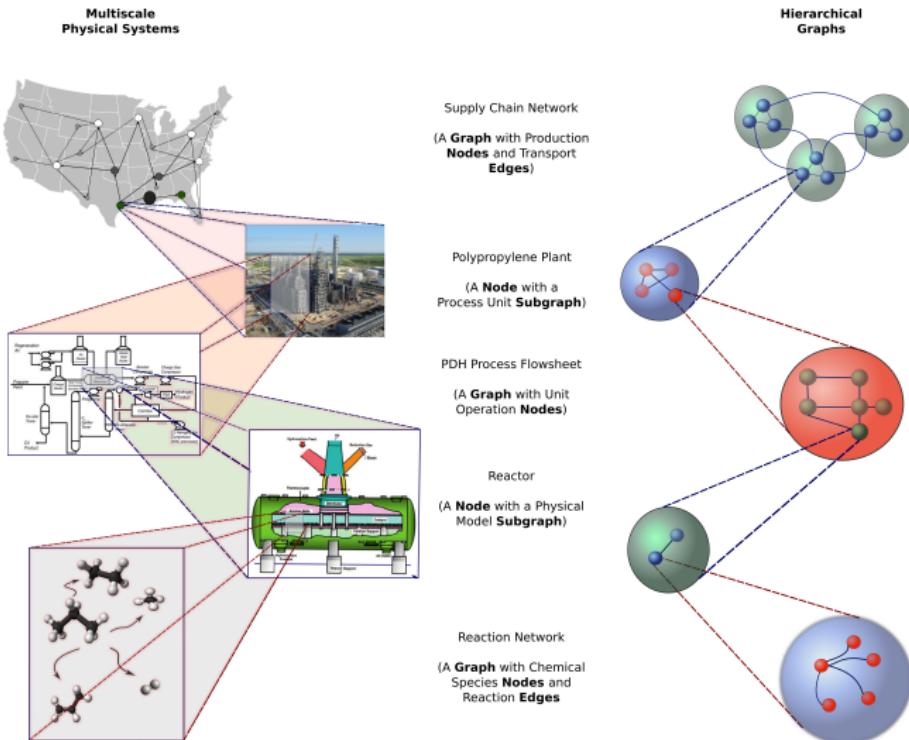


Revisiting our Goals

- Model encapsulation
- Modularity and reuse
- Navigate solutions to complex optimization problems
- Facilitate modeling of communicating systems



The Power of Abstraction - Graphs

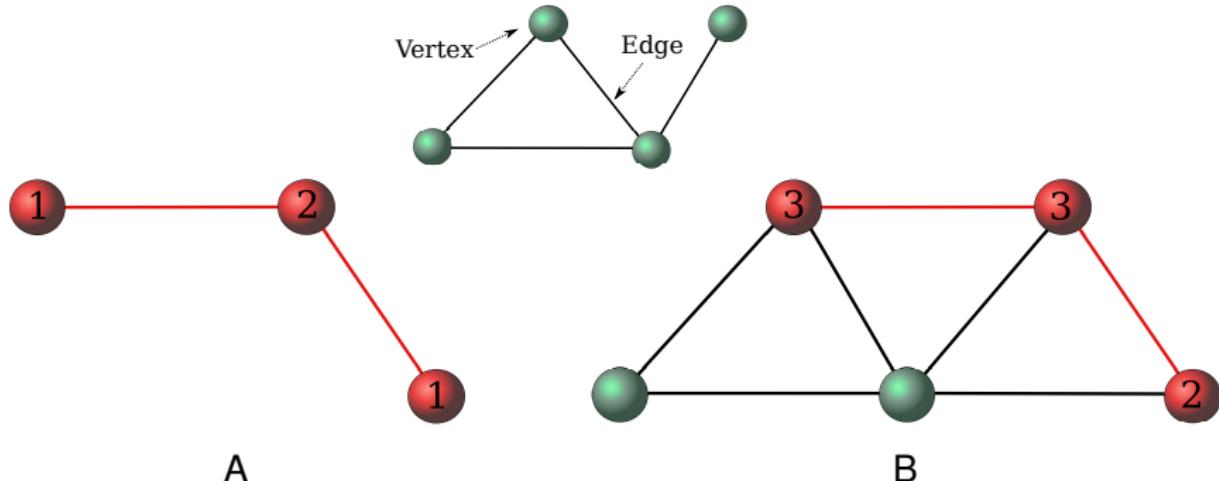




Relevant Graph Concepts

Graph Definition

A graph (G) is a finite set $V(G)$ of vertices (nodes) and a finite family $E(G)$ of pairs of elements of $V(G)$ called edges



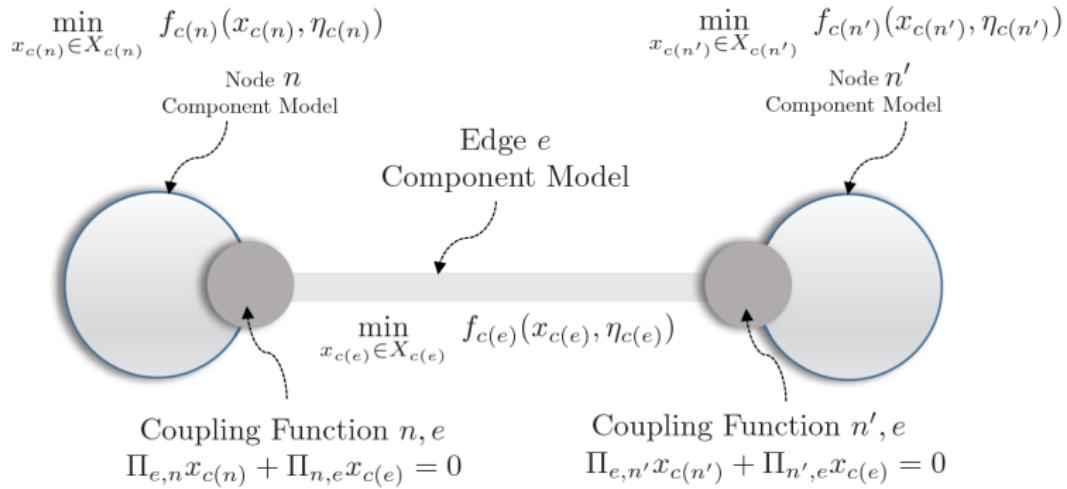
- A is a subgraph of B

- The degree of a node is specific to its graph



Graph Based Modeling

Plasmo associates model components with nodes and edges





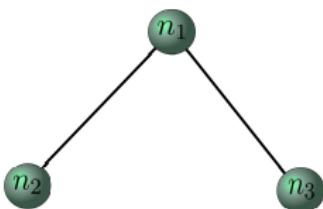
Plasmo.jl - Key Features

Key Features (some in progress):

- Associates models (JuMP Models) and linking information (constraints) with nodes and edges within a graph
- Exploits a subgraph abstraction to enable hierarchies of models (multiple graphs defined on a set of nodes)
- Uses [LightGraphs.jl](#) as the graph backend
- Accesses model information on nodes and edges
- Provides interfaces with structured solvers (PIPS, etc...)



Plasmo - Old syntax (still supported)



```
#Create a graph
graph = PlasmoGraph()
n1 = add_node!(graph)
n2 = add_node!(graph)
n3 = add_node!(graph)
edge1 = add_edge!(graph, n1, n2)
edge2 = add_edge!(graph, n1, n3)
#Set component models
setmodel!(n1, simple_model())
setmodel!(n2, simple_model())
setmodel!(n3, simple_model())
#provide linking information
setcouplingfunction!(graph, edge1, couplenodes)
setcouplingfunction!(graph, edge2, couplenodes)
model = generate_model(graph)

function couplenodes(m::Model, graph, edge)
    @constraint(m, getconnectedfrom(graph, edge)[:, x]
    == getconnectedto(graph, edge[:, x]))
end
```

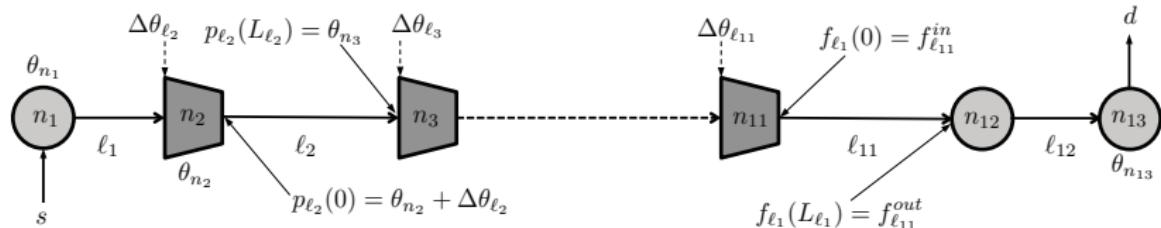


New Syntax - In Progress

```
#Create a graph
m = GraphModel()
n1 = add_node!(m)
n2 = add_node!(m)
n3 = add_node!(m)
edge1 = add_edge!(m, n1, n2)
edge2 = add_edge!(m, n1, n2)
#Set component models
setmodel!(n1, simple_model())
setmodel!(n2, simple_model())
setmodel!(n3, simple_model())
#link the two models
@linkconstraint(edge1, getconnectedfrom(m, edge1)[:, x]
== getconnectedto(edge1)[:, x])
@linkconstraint(edge2, getconnectedfrom(m, edge2)[:, x]
== getconnectedto(edge2)[:, x])
solve(m)
#solve_pips(m) #solve with PIPS NLP if the structure is correct
```



Gas Networks



- \mathcal{N} : Set of nodes in the gas network (junctions)
- \mathcal{L} : Set of links (pipelines)
- $\mathcal{S} \subseteq \mathcal{N}$: Set of gas supplies
- $\mathcal{D} \subseteq \mathcal{N}$: Set of gas demands
- $\mathcal{L}_a \subseteq \mathcal{L}$: Set of active links (pipelines with compressors)
- $\mathcal{L}_p \subseteq \mathcal{L}$: Set of passive links (pipelines without compressors)



Gas Networks

The equations for networks are analogous to the single pipeline from before, but we include new indices for the sets.

Mass and Momentum Balances on a Network

$$\frac{\partial p_\ell(t, x)}{\partial t} + \frac{c^2}{A_\ell} \frac{\partial f_\ell(t, x)}{\partial x} = 0, \quad \ell \in \mathcal{L}$$

$$\frac{\partial f_\ell(t, x)}{\partial t} + \frac{2c^2 f_\ell(t, x)}{A_\ell p_\ell(t, x)} \frac{\partial f_\ell(t, x)}{\partial x} - \frac{c^2 f_\ell(t, x)^2}{A_\ell p_\ell(t, x)^2} \frac{\partial p_\ell(t, x)}{\partial x} + A_\ell \frac{\partial p_\ell(t, x)}{\partial x} = -\frac{8c^2 \lambda A_\ell}{\pi^2 D_\ell^5} \frac{f_\ell(t, x)}{p_\ell(t, x)} |f_\ell(t, x)|, \quad \ell \in \mathcal{L}$$

Compressor Power

$$P_\ell(t) = c_p \cdot T \cdot f_{in,\ell} \left(\left(\frac{p_{in,\ell}(t) + \Delta p_\ell(t)}{p_{in,\ell}(t)} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right), \quad \ell \in \mathcal{L}_a$$

Boundary Conditions

$$p_\ell(0, t) = p_{in,\ell}(t) + \Delta p_\ell(t), \quad \ell \in \mathcal{L}_a$$

$$p_\ell(0, t) = p_{in,\ell}(t), \quad \ell \in \mathcal{L}_p$$

$$p_\ell(L_\ell, t) = p_{out,\ell}(t), \quad \ell \in \mathcal{L}$$

Node Conservation

$$\sum_{\ell \in \mathcal{L}_n^{rec}} f_{out,\ell}(t) - \sum_{\ell \in \mathcal{L}_n^{snd}} f_{in,\ell}(t) + \sum_{i \in \mathcal{S}_n} g_i(t) - \sum_{j \in \mathcal{D}_n} d_j^{gas}(t) = 0, \quad n \in \mathcal{N}$$

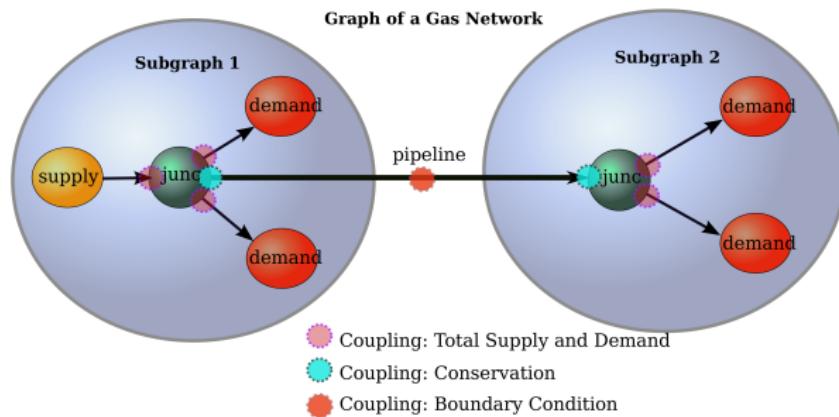
Supply and Demand

$$f_{deliver,n}(t) \leq f_{demand,n}(t), \quad n \in \mathcal{D}$$



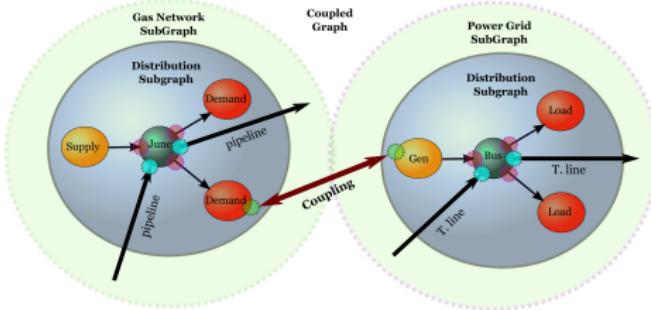
Graph Based Modeling - Gas Networks

- The subgraph abstraction allows multiple couplings on the same node
- This can be used to build modular systems and couple them at higher levels





Graph Based Modeling - Coupled Networks



```
m = GraphModel()
graph = getgraph(m)
add_subgraph!(graph, power_network)
add_subgraph!(graph, gas_network)
generator = getnode(power_network, :gen)
demand = getnode(gas_network, :demand)
link = add_edge!(graph, generator, demand)
@linkconstraint(link, getconnectedfrom(graph, link)[:Pgnd] <=
getconnectedto(graph, link)[:fdeliver])
```



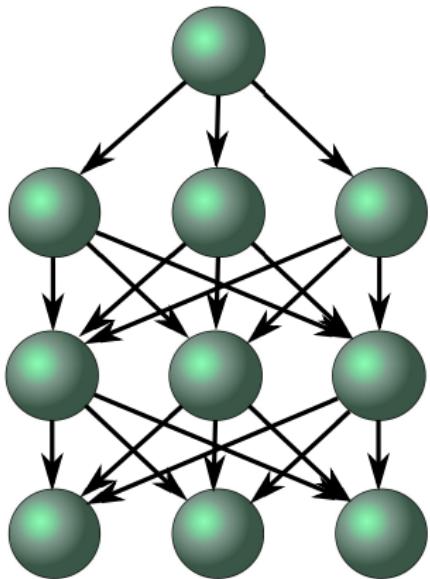
Graph Based Modeling - Coupled Networks

Key Findings:

- Infrastructure models (graphs) can be developed independently and coupled within larger systems (graphs)
- Illinois Case Study: 7% more gas delivered to generators; 27% revenue increase versus uncoordinated case
- Uncoordinated case simulated by solving successive optimization problems



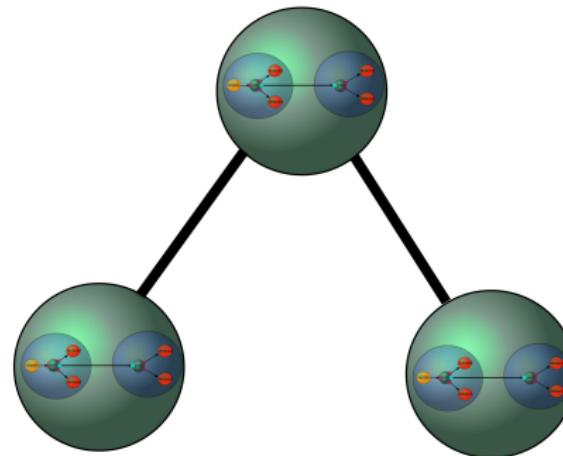
Multistage Stochastic Programming



- Our graph abstraction corresponds to the node-based abstraction in multistage stochastic programming
- Component models associated within nodes (scenarios)
- Link constraints propagate transition from stage to stage



Embedding Graph Models

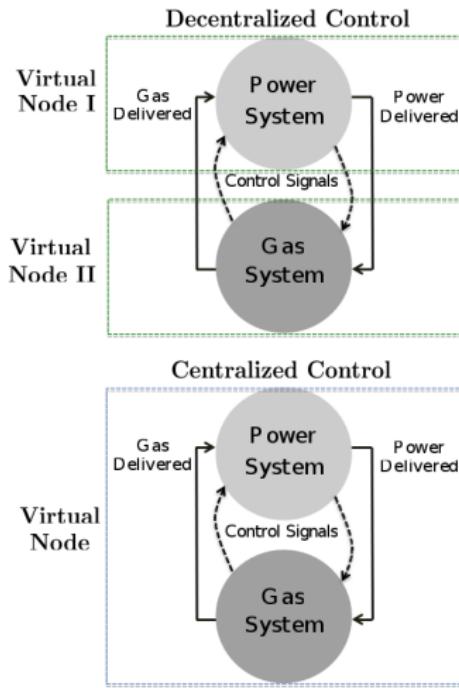


- Graph models can themselves be embedded as models in nodes or edges
- Simplifies construction of multiple layers in systems



Future Direction

- Generalize the model interface if possible (strictly uses JuMP)
- Find suitable abstraction for computational workflows
 - ▶ Decentralized control
 - ▶ Algorithmic strategies (e.g. scheduling and operations)
 - ▶ Graph partitioning and model reduction
 - ▶ Initialization strategies
- Simulation interfaces





Goals right now

- Finalize physical model abstraction
- Push first version to github
- Figure out a suitable graph communication abstraction